1. Introduction

Every physicist knows that the electron, proton, and neutron have spin magnetic dipole moments, as do many other particles and nuclei with non-zero spin. Can an elementary particle or nucleus also have a spin electric dipole moment (EDM)? In this review we try to explain why other particles and nuclei with non-zero spin. Can an EDM cannot exist unless both parity (P) and time reversal (T) invariance are violated. This can be seen from the non-relativistic Hamiltonian for the interaction of an EDM with an electric field \( \mathbf{E} \), which is \( H_{\text{EDM}}^{\text{NR}} = -d \cdot \mathbf{E} \). For an elementary particle or nucleus in a non-degenerate state, the spin angular momentum \( \mathbf{J} \) is the only vector available to define a direction. Thus \( \mathbf{d} \) must be collinear with \( \mathbf{J} \), and:

\[
H_{\text{EDM}}^{\text{NR}} = -d \cdot \mathbf{E} = -d \frac{\mathbf{J}}{J} \cdot \mathbf{E}.
\] (1)

However \( \mathbf{E} \) is a T-even polar vector while \( \mathbf{J} \) is a T-odd axial vector. Thus \( H_{\text{EDM}}^{\text{NR}} \) is odd under \( T \) and \( P \) transformations. The same conclusion is of course true for the relativistic generalization of \( H_{\text{EDM}}^{\text{NR}} \).

Until now no EDM has been observed, and it is obvious from the present experimental upper limits, summarized in Table I, that EDMs must be extremely small. For example, the upper limit on the electron EDM \( d_e \) is \( 8.3 \times 10^{-17} \mu_B \), where \( \mu_B \) is the Bohr magneton. Nevertheless, EDMs may be non-zero, because \( P \) and \( T \) are in fact violated in nature. Parity nonconservation [as well as the violation of charge conjugation (C) invariance] occurs in the weak interaction. Furthermore, combined charge-parity (CP) violation is observed in neutral \( K \) meson and \( B \) meson decays. 5 It is assumed CP invariance, for which we have very strong confidence, then this CP violation is equivalent to T violation. Thus the weak interaction and the mechanism or mechanisms causing CP violation could act jointly to generate EDMs by \( P,T \)-odd radiative corrections to the \( P, C, T \) conserving electromagnetic interaction.

Unfortunately, given the present state of our knowledge, such radiative corrections cannot be calculated with confidence. Instead, they depend on uncertain theoretical models of CP violation. According to the standard model, while CP violation in quantum chromodynamics (QCD) can in principle be large, CP violation in the electroweak sector is described phenomenologically by a single phase that appears in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. 24 This gives a satisfactory account of \( K \) and \( B \)-meson CP violation data. 25 It can be shown that in this description the neutron EDM \( d_n \) appears only at the three-loop level of perturbation theory 28 while the electron EDM \( d_e \) appears only at the four-loop level 25 (and there are additional suppressions). Thus the standard model predict EDM predictions: \( d_n \approx 10^{-32} \text{ e cm}, d_e \leq 10^{-38} \text{ e cm} \) (where \( e = 4.8 \times 10^{-10} \text{ esu} \) is the unit of electronic charge) are many orders of magnitude smaller than the present experimental limits. Indeed, if the standard model mechanism of CP violation is the only one, then given present or foreseeable experimental capabilities, future observation of any EDM is very unlikely.

However, there are good reasons to think that additional mechanisms exist for CP violation. It is generally accepted that if the universe initially was symmetric in baryon–antibaryon number, the presently observed baryon–antibaryon asymmetry could not have developed without a much larger CP violation than is predicted by the standard model. 65 Furthermore, in many theories that attempt to go beyond the standard model, predicted EDMs are relatively large. For example, in various supersymmetric theories, many new hypothetical particles and couplings appear, and along with them exist new CP violating phases. Thus in many such models the electron and neutron EDMs already appear at the

---

**Table I. Experimental limits on EDMs and the \( \tau \) weak dipole moment (WDM).**

| Particle | EDM \( |d| \) (e cm) | WDM \( |d| \) (e cm) | Ref. |
|-----------------|-----------------|-----------------|-----|
| \( \bar{n}_e \) | \( 2 \times 10^{-29} \) | 35 |
| \( e^- \) | \( 1.6 \times 10^{-27} \) | 23 |
| \( \mu^\pm \) | \( 7 \times 10^{-19} \) | 29 |
| \( \tau^\pm \) | \( 1.1 \times 10^{-17} \) | \( 5.8 \times 10^{-18} \) | 34 |
| \( p \) | \( 5.4 \times 10^{-24} \) | 19 |
| \( n \) | \( 3 \times 10^{-26} \) | 13 |
| \( \Lambda^0 \) | \( 1.5 \times 10^{-16} \) | 20 |
| \( ^{199} \text{Hg atom} \) | \( 2 \times 10^{-28} \) | 17 |

\( e = 4.8 \times 10^{-10} \text{ esu} \); \( F = \text{form factor} \)

---

*E-mail: commins@physics.berkeley.edu*
one-loop level, and as a result predictions of $d_a$ and $d_n$ are close to present experimental limits. Thus discovery of an EDM by practical experimental methods is a real possibility within the foreseeable future, and such a discovery would provide definite evidence for physics beyond the standard model.

The search for EDMs of the neutron and of nuclei is important for a related issue of fundamental significance in QCD: the “strong CP problem”. A CP-odd term exists in the effective Lagrangian density for QCD, characterized by the “QCD CP-violating parameter” $\Theta$. It is given by the well-known expression: $L_{\text{CP}} = -\kappa \frac{\mu_B}{2} \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \gamma^\nu \Psi F_{\mu\nu}$. Here $\Psi$ is the Dirac field for the fermion, $\bar{\Psi} = \Psi^\dagger \gamma^0$ is the Dirac conjugate field, and $\kappa$ is a suitable constant. Rewriting eq. (2) in terms of $\mathbf{E}$ and $\mathbf{B}$ fields, we obtain:

$$ L_{\text{Pauli}} = -\kappa \frac{\mu_B}{2} \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \gamma^\nu \Psi F_{\mu\nu}. $$

(2)

where $\mathbf{F}_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} = \begin{pmatrix} 0 & \mathbf{E} & \mathbf{E} \\ -\mathbf{E} & 0 & -\mathbf{B} \\ \mathbf{E} & \mathbf{B} & 0 \end{pmatrix}$ is the electromagnetic field tensor, and $\kappa$ is a suitable constant. This Lagrangian density results in the Hamiltonian density

$$ H_{\text{Pauli}} = -\kappa \mu_B \bar{\Psi} \left( \mathbf{E} \cdot \mathbf{B} - i \mathbf{E} \cdot \mathbf{B} \right) \Psi $$

(4)

and

$$ \mathcal{H}_{\text{Pauli}} = -\kappa \mu_B \bar{\Psi} \left( \mathbf{E} \cdot \mathbf{B} - i \mathbf{E} \cdot \mathbf{B} \right) \Psi $$

(5)

and $\mathcal{H}_{\text{Pauli}}$ of eq. (6) are each $P$- and $T$-invariant. We can render them $P,T$-odd by replacing $\mathbf{E}$ by $-\mathbf{B}$ and $\mathbf{B}$ by $-\mathbf{E}$, which is equivalent to the replacement of $\mathbf{F}_{\mu\nu}$ by the tensor $-\mathbf{F}_{\mu\nu} \ast$, where:

$$ \mathbf{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} \mathbf{F}^{\alpha\beta} = \begin{pmatrix} 0 & B_x & B_y \\ -B_y & 0 & -E_z \\ B_z & E_y & 0 \end{pmatrix} $$

(7)

and $\mathbf{F}_{\mu\nu} \ast$ is the completely antisymmetric unit 4-tensor. Alternatively we obtain the same Lagrangian density by replacing $\sigma^{\mu\nu}$ in eq. (2) with $i\sigma^{\mu\nu}\gamma^5$ (where, as usual, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$), with no change in $F_{\mu\nu}$. Making this latter transformation and replacing $\kappa \mu_B$ by $d$ we obtain the EDM Lagrangian density:

$$ \mathcal{L}_{\text{EDM}} = -\frac{d}{2} \bar{\Psi} \varepsilon_{\mu\nu\gamma} \gamma^\gamma \gamma^\nu \Psi F_{\mu\nu} = \bar{\Psi} \left[ \mathbf{E} \cdot \mathbf{E} + i \mathbf{E} \cdot \mathbf{B} \right] \Psi $$

(8)

which was first described by Salpeter. In the non-relativistic limit the first term on the right-hand side (r.h.s.) of eq. (9) reduces to the r.h.s. of eq. (1). However when the particle of interest is relativistic, the full expression on the r.h.s. of eq. (9) must be used, and this has significant consequences, as we shall see.

3. The Neutron EDM

In all neutron EDM experiments use is made of the fact that non-relativistic polarized neutrons in collinear $\mathbf{E}$ and $\mathbf{B}$ fields undergo Larmor precession with frequency $\nu = [2\mu_B B_{\text{ref}} \pm 2d_{n,\text{ref}}]/h$, where the sign corresponds to parallel (antiparallel) $\mathbf{E}$ and $\mathbf{B}$ fields. Thus the presence of an EDM is revealed by an electric field-dependent shift in $\nu$ proportional to the $T$-odd pseudoscalar $\mathbf{E} \cdot \mathbf{B}$. The earliest experiments employed neutron beams and the Ramsey method of magnetic resonance were performed at the Institut Laue–Langevin (ILL) in Grenoble, where the following result was obtained:

$$ d_n = [\pm 0.2 \pm 1.5(\text{stat}) \pm 0.7(\text{syst})] \times 10^{-26} \text{ cm.} $$

(10)

The statistical uncertainty on the r.h.s. of eq. (10) is mainly due to the limited number of ultra-cold neutrons that could be generated and stored, while the systematic uncertainty arises for the most part from a geometric phase effect caused by unintended gradients in $\mathbf{B}$. New methods are needed if $d_n$ is to be determined to much better precision. Several novel projects are under development, including one at ILL and another at the Los Alamos National Laboratory (LANL). In the latter experiment, ultra-cold polarized neutrons are produced and stored in a bath of superfluid $^4$He containing a dilute solution of nuclear–spin-polarized $^3$He. A neutron can only exchange momentum with $^3$He when the free neutron dispersion curve and the phonon–roton dispersion curve of superfluid $^4$He intersect (see Fig. 1), since energy and momentum must both be conserved in the collision. Intersections occur at $E = k = 0$ and at $k = k^*$, $E = E^*$ (corresponding to the temperature $T^* = E^*/k_B \approx 12$ K). Polarized neutrons enter the superfluid bath, and those with wavelength $\lambda = 2\pi/k^* \approx 11010-2$
0.89 nm are downscattered to form a polarized ultra-cold neutron sample. The probability of subsequent up-scattering by absorption of a \(^4\text{He}\) excitation is very small at the bath operating temperature \(T \leq 500 \text{ mK}\), since the Boltzmann factor for these excitations is \(\exp(-T^s/T) \ll 1\). Thus a relatively large density of ultra cold polarized neutrons can be accumulated in the bath. The polarized \(^3\text{He}\) acts simultaneously as a neutron spin analyzer and as a co-magnetometer. The cross section for the reaction \(\text{He}(n,p)\) \(+764\text{ keV}\) is very large, but only in the \(n^-\text{He}\) \(J=0\) state (where the spins of these two species are opposed). Thus observation of this spin-dependent reaction by means of the resulting scintillations in \(^3\text{He}\) provides a way to detect the precession of the neutron spins in the \(E\) and \(B\) fields. A number of subtle problems are associated with this ambitious experiment. If they can be overcome, an improvement in the present limit on \(\alpha_n\) of a factor of \(\approx 100\) seems possible.

The neutron and proton EDMs are expected to be roughly comparable in magnitude. However, as we discuss in the next section, the proton presents a completely different challenge to the experimenter because it is charged.

4. Atomic and Molecular EDMs

4.1 Schiff’s theorem

It has long been considered impractical to search for an EDM by placing a charged particle (e, p, bare nucleus, ...) in an electrostatic field, since the particle would quickly be accelerated out of the region of observation. (Recent proposals for storage ring searches for EDMs of charged particles are discussed in §5 and §6). What can we learn by applying an external electrostatic field to a neutral atom or molecule that contains a nucleus or unpaired electron with an EDM \(d\)? At first sight this approach appears useless, because in the limits where all atomic or molecular constituents are treated as point charges, and where non-relativistic quantum mechanics applies, the atom or molecule cannot possess an EDM \(d\) (cannot exhibit a linear Stark effect) to first order in \(d\). This is Schiff’s theorem\(^{15}\) which can be understood intuitively as follows: A neutral atom or molecule is not accelerated in a uniform external electric field. Thus the average force on each of the atomic or molecular constituents must be zero. In the non-relativistic, point charge limits, the only forces are electrostatic; hence the average electric field at each point charge must be zero. This happens because the external field is cancelled, on average, by the internal polarizing field.

We note in passing that Schiff’s theorem is not in conflict with existence of the so called “permanent” electric dipole moments of many polar molecules, familiar in chemistry and molecular spectroscopy. These moments do not violate \(P\) or \(T\), nor do they result in a linear Stark effect for sufficiently small applied electric fields, in the absence of degeneracy. They have entirely different observational signatures than exist for the EDMs of interest to us.

4.2 Nuclear EDMs

Schiff’s theorem is evaded for a nucleus if one takes into account magnetic hyperfine structure, and more importantly, for a nucleus of finite size if the nuclear EDM distribution is not the same as the charge distribution.\(^{16}\) In the latter case, a small residual EDM effect remains, which is expressed in terms of an additional \(P,T\)-odd electronic potential \(V_3 = -e\Sigma S \cdot \mathbf{S}^3(r_i)\) that must be included in the atomic or molecular Hamiltonian. Here \(r_i\) refers to the position of the \(i\)-th electron relative to the nuclear center-of-mass, and the “Schiff moment” \(S\) is a vector proportional to the nuclear EDM depending on the difference between the normalized charge and EDM distributions:

\[
S = \left( I_1, I_2 = I \right) \left[ \frac{e}{10} \sum_{p=1}^7 \left( \frac{Z^2}{3} \, C_p \right) \right] \left( I_1, I_2 = I \right).
\]

where the sum is over all nuclear protons, and \(I\) is the nuclear spin. \(S\) can be generated by an intrinsic EDM of an unpaired nucleon, and/or by \(P,T\)-odd nucleon–nucleon (NN) interactions. Generally speaking, \(S\) is largest for the heaviest nuclei, and in particular it is enhanced in octupole-deformed nuclei such as \(^{228}\text{Ra}\) by roughly a factor of 10 to 100 compared to its value for a more symmetric heavy nucleus such as \(^{199}\text{Hg}\). In addition to the Schiff moment, a nucleus with nuclear spin \(I \geq 1\) can possess a magnetic quadrupole moment \(M\) originating from nucleonic EDMs and/or \(P,T\)-odd NN interactions. In a paramagnetic atom or molecule this would couple to the magnetic field resulting from the spin and spatial distribution of the unpaired electron. Because this interaction is magnetic, it would not be constrained by Schiff’s theorem.\(^{16}\)

The most sensitive nuclear EDM search to date\(^{17}\) was an optical pumping experiment carried out on the diamagnetic atom \(^{199}\text{Hg}\) (\(I=1/2\)). The result is:

\[
d(199\text{Hg}) \leq 2 \times 10^{-28} \text{ e cm}
\]

which gives the limit: \(d(199\text{Hg}) \leq 2 \times 10^{-28} \text{ e cm}\). Calculations relating \(d(199\text{Hg})\) to \(S\) yield the result \(d(199\text{Hg}) = -2.8 \times 10^{-12}[S/(e \text{ fm}^3)]\text{ e cm}^{18}\). From this and eq. (12) one obtains \(S(199\text{Hg}) = (3.8 \pm 1.8 \pm 1.4) \times 10^{-12} \text{ e fm}^3\). The largest contribution to \(S(199\text{Hg})\) is estimated to arise from a \(P,T\)-odd nucleon–nucleon interaction of the form \(\Xi G_F(\vec{p})(\vec{m}\vec{n}^2)/\sqrt{2}\), where \(G_F\) is Fermi’s coupling constant and \(\Xi\) is a dimensionless constant. However calculations
show that there is also a contribution from the intrinsic EDM of the proton; indeed the best current limit on the proton EDM: $|d_p| \leq 5.4 \times 10^{-24} \text{e cm}$, is inferred from the experimental result (12). Very significant improvements in our knowledge of $d_p$ may come from future storage-ring experiments; see §6. We note in passing that a relatively large upper limit also exists for the $\Lambda^0$ hyperon; see Table I.

4.3 The electron EDM

We next consider the unpaired atomic electron(s) in a paramagnetic atom or molecule. Sandars' analysis [which is based on the first term on the r.h.s. of eq. (9) including the factor $\gamma^0$] may be expressed in terms of the ratio $d_a/d_e$, or equivalently in terms of $E_{\text{eff}}$, the effective electric field experienced by $d_e$. It is convenient to write $E_{\text{eff}} = Q\Pi$ where $Q$ is a factor that includes relativistic effects as well as details of atomic (or molecular) structure, while $\Pi$ is the degree of polarization of the atom or molecule by the external electric field $E_{\text{ext}}$. For paramagnetic atoms with valence electrons in $s_{1/2}$ or $p_{1/2}$ orbitals, such as Cs and Tl in their ground states,

$$Q \approx 4 \times 10^{10} \text{V/cm} \times (Z/80)^3,$$

(13)

where $Z$ is the atomic number. Also, for such atoms $\Pi \approx 10^{-3}(E_{\text{ext}}/100 \text{V/cm})$, which is only $\approx 10^{-3}$ for the maximum attainable laboratory fields $E_{\text{ext}} \approx 100 \text{V/cm}$. Since for paramagnetic atoms in all practical situations, $\Pi$ is proportional to $E_{\text{ext}}$, the ratio $E_{\text{eff}}/E_{\text{ext}}$ is a constant, and is usually called the enhancement factor $R = d_a/d_e$. For the ground states of alkali atoms and for thallium, one finds $|R| \approx 10^2Z\alpha^2$ where $\alpha$ is the fine structure constant. Although in these cases $\Pi \ll 1$, $|R|$ can greatly exceed unity for sufficiently large $Z$. For example, for the thallium atom ($Z = 81$), one calculates $R = -585$.

Equation (13) also applies for a wide range of heavy polar diatomic paramagnetic molecules with valence electrons in $\sigma$ or $\pi$ orbitals, such as YbF in the ground $^2 \Sigma_{1/2}$ state, or PbO in the metastable $a(1) ^3 \Sigma_1$ state. (In these cases $Z$ is the atomic number of the heavy nucleus.) The main difference between atoms and molecules occurs in the factor $\Pi$. In a typical polar diatomic molecule, nearly complete polarization ($\Pi \approx 1$) can be achieved with relatively modest external fields: $(E_{\text{ext}} \approx 10^2-10^4 \text{V/cm})$ because of the very close spacing between adjacent spin-rotational levels of opposite parity. When $\Pi \approx 1$, $E_{\text{eff}}$ for a paramagnetic molecule such as YbF or PbO$^+$ is approximately 3 orders of magnitude larger than the maximum attainable with atoms.

$P.T$-odd electron–nucleon (eN) interactions can also contribute to $d_e$ in diamagnetic or paramagnetic atoms or molecules. These, as well as the $P.T$-odd NN interactions, can appear in one or several non-derivative coupling forms: “$\text{scalar}$”, “$\text{tensor}$”, and “$\text{pseudoscalar}$”. $P.T$-odd electron-electron interactions are also possible but these only yield an extremely small contribution. Finally, $C.T$-odd (P-even) eN and NN interactions, and possible $T$-odd beta decay couplings could cause a $P.T$-odd atomic or molecular EDM through radiative corrections involving the usual weak interactions of the standard model. For a paramagnetic atom or molecule the most important contribution to $d_e$ in addition to $d_e$ itself, is the scalar $P.T$-odd eN interaction.$^{16}$

The most sensitive search for $d_e$ to date employed $^{205}\text{Tl}$ in an atomic beam magnetic resonance experiment with separated oscillating fields. $^{23}$ The result is:

$$d_e = (6.9 \pm 7.4) \times 10^{-28} \text{e cm},$$

(14)

assuming $R(^{205}\text{Tl}) = -585$ and no contribution from the scalar $P.T$-odd eN interaction. Equation (14) yields the limit: $|d_e| \leq 1.6 \times 10^{-23} \text{e cm}$. At present, many new searches for $d_e$ are in progress, using cesium, francium, YbF, PbO$^+$, and the molecular ion $\text{HIF}^+$. $^{24}$ These experiments with free atoms or molecules employ various standard methods of atomic, molecular, and optical physics: laser and rf spectroscopy, optical pumping, atomic and molecular beams, ion trapping, atom trapping and cooling, etc. In another search for $d_e$, one applies a large electric field to the paramagnetic solid gadolinium gallium garnet (GGG). $^{25}$ In principle, the interaction of the EDMs of the unpaired electrons with the electric field at sufficiently low temperature can yield a net magnetization of GGG which can be detected by a superconducting quantum interference device (SQUID) magnetometer. (It has also been proposed separately that application of a sufficiently large electric field to a gaseous sample of diamagnetic diatomic molecules could generate an observable $P.T$-odd magnetization. $^{26}$) In a complementary experiment, application of an external magnetic field to the ferrimagnetic solid gadolinium iron garnet (GdIG) can yield an EDM-induced electric polarization of the sample, which is detectable in principle by ultra-sensitive charge measurement techniques. $^{27}$

The chances are good that at least one of the many new experimental searches for $d_e$ will improve the existing limit by at least a factor of 10 in the relatively near future. The experimental searches employing paramagnetic molecules (YbF, PbO$^+$, ...) are of particular interest because these molecules have very large $E_{\text{eff}}$ values.

5. The Muon EDM

In most theoretical models, including the standard model, the electron, muon and tau lepton EDMs are proportional or approximately proportional to their masses. Assuming this and given the present limit on $d_e$, one predicts $d_\mu < 3.3 \times 10^{-25} \text{e cm}$. However in some theoretical models $d_\mu$ could be larger than this by an order of magnitude or more. $^{28}$ This provides motivation for $d_\mu$ searches at the $10^{-24}$ e cm level. The best current limit on the muon EDM: $|d_\mu| \leq 7 \times 10^{-19} \text{e cm}$, was obtained in an experiment at the CERN muon storage ring in the 1970's, the primary purpose of which was a precise measurement of the muon $g$-factor anomaly $\alpha(\mu)/(g - 2)$. $^{29}$ Since then, storage ring technology has advanced considerably, resulting in a much more precise measurement of $\alpha(\mu)$ in recent years at Brookhaven. $^{30}$ This has led to new proposals, not only to improve the limit on $d_\mu$ but also for storage ring searches for the proton, deuteron, and $^3\text{He}$ EDMs. The deuteron EDM $d_D$ appears especially promising (see §6).

In order to understand the main features of these experiments, we consider a relativistic particle moving with velocity $\vec{v}$ in a horizontal plane, in electric and magnetic fields $\vec{E}$ and $\vec{B}$, where $\vec{B}$ is in the vertical direction (hence
\( \beta \cdot B = 0 \) and \( \beta \cdot \mathbf{E} = 0 \) also. It can be shown \(^{31}\) that the angular velocity \( \omega \) of spin precession with respect to the particle momentum is:

\[
\omega = -\frac{e}{m} \left[ a\mathbf{B} + \left( \frac{1}{\beta^2} \mathbf{y}^2 - a \right) \mathbf{B} \times \mathbf{E} + \frac{1}{2} \eta (\mathbf{E} + \mathbf{B} \times \mathbf{B}) \right]
\]

(15)

Here, \( \gamma = (1 - \beta^2)^{-1/2}, \ \eta = 2d(2mc/eh)^{-1} \) where \( d \) is the EDM, and in eq. (15) we employ units where \( h = c = 1 \). In the CERN and Brookhaven muon \( g-2 \) experiments the muon energy was chosen so that \( \beta \gamma^2 = a^{-1} \), and also \( \mathbf{E} \) was negligible. In this situation eq. (15) reduces to:

\[
\omega = \omega_a + \omega_e,
\]

(16)

where

\[
\omega_a = -\frac{e}{m} a\mathbf{B} \quad \text{and} \quad \omega_e = -\frac{e}{2m} \eta \mathbf{B} \times \mathbf{B}.
\]

In this case the spin precesses about \( \omega \) with frequency \( \omega = \sqrt{a^2 + \alpha^2} \approx \omega_a \) and has a small oscillatory vertical component (see Fig. 2). In the CERN experiment this was searched for by observing the angular distribution of electrons emitted above and below the horizontal orbit plane in polarized muon decay. However the precision was limited because the vertical spin component was so very small, as well as oscillatory, owing to the presence of \( \omega_e \). In a newly proposed muon EDM search \(^{32}\) one chooses \((\beta^2 \gamma^2)^{-1} \gg a \) and an electric field is applied of magnitude \( |\mathbf{E}| = B \gamma^2 a |\mathbf{B}| \) and in the direction \( \mathbf{B} \times \mathbf{B} \). In other words \( \mathbf{E} \) is radial and in the orbit plane; see Fig. 3. In this case \( \omega_a \) is eliminated and thus \( \omega \) is directed along \( \mathbf{E} \) with magnitude:

\[
\omega = \frac{e}{2m} \eta \beta (1 + \gamma^2 a) \mathbf{B}
\]

(17)

Consequently starting from a horizontal orientation the spin precesses very slowly in the vertical plane and the vertical spin component increases approximately linearly with time, becoming much larger than in earlier experiments. With this new scheme it may be possible to extend the limit on \( d_{\mu} \) to \( \approx 10^{-24} \text{e cm} \).

6. The Deuteron EDM

In a recently proposed storage ring deuteron EDM search, \(^{33}\) polarized deuterons with momentum \( p = 1.5 \text{ GeV/c} \) are to circulate in a specially designed ring with a magnetic field \( B = 2 \text{T} \) normal to the orbit plane, and with no applied electric field. In the instantaneous rest frame of the particle, the magnetic and electric fields are:

\[
\mathbf{B}' = \gamma \mathbf{B}
\]

(18)

and

\[
\mathbf{E}' = \gamma (\mathbf{B} \times \mathbf{B})
\]

(19)

For \( p = 1.5 \text{ GeV/c} \) and \( B = 2 \text{T} \), the rest frame electric field is \( \mathbf{E}' \sim 5 \times 10^9 \text{V/m} \). In contrast to nuclear EDM searches employing neutral atoms, this very large electric field is applied directly to the deuteron without any “Schiff” screening. As usual, the component of precession angular velocity due to the magnetic moment is directed along \( \mathbf{B} \), in the laboratory frame this is described by the formula \( \omega_a = -(e/m)a\mathbf{B} \). However, in this experiment a novel feature is introduced: the beam velocity is modulated at frequency \( \omega_a \), (with \( \Delta \beta/\beta \approx \pm 1\% \)). As is evident from eq. (19) this produces a component of \( \mathbf{E}' \) that oscillates in the plane perpendicular to \( \mathbf{B} \) at the same frequency as the magnetic spin precession. The interaction of \( \mathbf{d}_a \) with this oscillating component of \( \mathbf{E}' \) causes a spin reorientation analogous to that which occurs in conventional magnetic resonance. The net result in the laboratory frame is the development of a vertical component of spin polarization proportional to \( d_0 \) that is approximately linear in the time. Given the parameters we have mentioned, the rate of vertical polarization accumulation is \( \approx 10^{-9} \text{rad/s} \) for \( d_0 = 10^{-29} \text{e cm} \).

Detection of the deuteron spin polarization could be achieved as follows. A thin gas jet causes Coulomb scattering of a small fraction of beam deuterons on each turn around the ring. The scattered deuterons strike a thick carbon target in the shape of an annulus surrounding the beam. Elastic scattering of \( D \) on carbon is spin-dependent
due to the spin–orbit interaction, and at 1.5 GeV/c, the analyzing power is known to be better than 30%. Downstream from the carbon target is an array of scintillation detectors, also in the form of an annulus and segmented into four quadrants (left, right, up, down). The left–right asymmetry provides the EDM signal, while the up-down asymmetry gives information on the $g-2$ precession. Assuming an initial deuteron polarization of 95%, $10^{12}$ deuterons in the ring, and a polarization coherence time of 1000 s, as well as other parameters previously mentioned, it appears possible to achieve a statistical uncertainty in $d_\tau$ of 1000 s, as well as other parameters previously mentioned, as asymmetry gives information on the weak dipole moment (WDM) $d$. This result together with plausible theoretical assumptions leads to the following limit on the EDM $d_\tau$: 

$$|d_\tau| \leq 1.1 \times 10^{-17} \, \text{cm}. \hspace{1cm} (26)$$

8. Can a Neutrino Possess an EDM?

It is not yet known whether a neutrino and anantineutrino of the same mass eigenstate are distinct particles (“Dirac” neutrino and antineutrino) or whether a neutrino of given mass is self-conjugate (“Majorana” neutrino). Neutrino magnetic and electric dipole moments are described by 3 × 3 matrices $\mu_{ij}$, $d_{ij}$ respectively, where the diagonal elements $\mu_{ii}$, $d_{ii}$ refer to the static dipole moments of the $i$’th mass eigenstate. If neutrinos are of the Majorana type, the diagonal elements $\mu_{ii}$, $d_{ii}$ must be zero because under charge conjugation, the magnetic dipole and electric dipole operators change sign. Of course, no such restriction applies to Dirac neutrinos.

A neutrino EDM could cause anomalous ionization in a detector because of its interaction with atomic electrons. Making use of this, analysis of an experiment carried out by Cowan and Reines in 1957 to detect $\bar{\nu}_e$ radiated from a reactor yielded the result $|d_{\bar{\nu}}| \leq 2 \times 10^{-20} \, \text{cm}$, where $F$ is a form factor.

9. Conclusion

After a half century of search, there is still no experimental evidence for an EDM of an elementary particle, nucleus, atom, or molecule. However, widespread appreciation of the significance of EDM searches for the general problem of CP violation and the development of new and refined experimental techniques now generate more intense interest in EDM searches than ever before. The present experimental upper limits for $d_n$, $d_e$, and $d(199 \text{Hg})$ already provide serious constraints on various supersymmetric models of CP violation. Improvements of factors of 10–100 in the limits on $d_n$ and $d_e$, which may come in the relatively near future, should thus be very significant. Finally, success of the proposed deuteron storage ring experiment would bring the field of nuclear EDMs into an entirely new era.

24) See, for example, E. D. Commins and D. P. DeMille: to be published in Lepton Dipole Moments, ed. B. L. Roberts and W. Marciano (World Scientific, Singapore).

Eugene D. Commins was born in New York, N.Y., U.S.A. in 1932. He obtained his B.A. degree in 1953, and his Ph. D. from Columbia University in 1958. He was an Instructor in Physics at Columbia from 1958 to 1960, and since then has been a member of the Physics Faculty at the University of California, Berkeley. His experimental research has mainly focused on space inversion symmetry or parity (P) and time reversal invariance (T), investigated with the tools of low-energy atomic and nuclear physics.