Parton distributions and their implication to LHC physics

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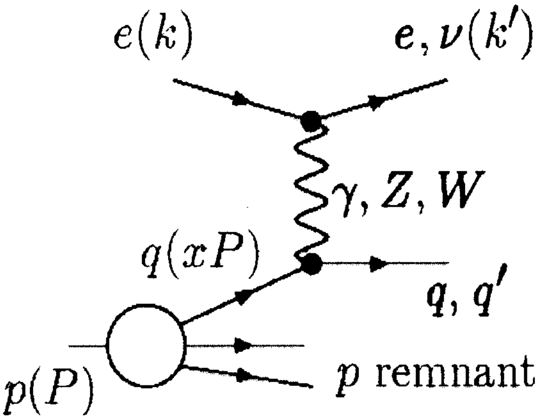
Contents

Deep Inelastic Scattering (DIS)

DIS is a straightforward way to probe p structure



- Virtuality: $Q^2 \equiv -(k-k')^2$
  $\Rightarrow$ spatial resolution of probe
- Bjorken variable: $x \equiv Q^2/2p \cdot q$
  $\Rightarrow$ mom. fraction of struck parton

Inclusive DIS cross-sections $\Leftrightarrow$ Structure functions (SFs)

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \times \{Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2)\}$$

($Y_\pm = 1 \pm (1-y)^2$, $y \equiv p\cdot k/p\cdot q$; inelasticity parameter)

⋆ SFs parameterise the structure of the target as 'seen' by the virtual photon

⋆ two SFs are needed $\Leftarrow$ two status of γ^* (L and T):

$$F_2 = \frac{Q^2}{4\pi^2\alpha}(\sigma_L + \sigma_T),\ F_L = \frac{Q^2}{4\pi^2\alpha}(\sigma_L)$$

SFs $\Leftrightarrow$ Parton Distribution Functions (PDFs)

Naive Quark-Parton-Model:

$$F_2(x) = x\sum_q e_q^2(q(x) + \bar{q}(x))$$
$$F_L(x) = 0$$

⋆ $q(x)dx$ represents the probability that a quark q carries momentum fraction between x and $x+dx$.

⋆ γ^* scatters incoherently off the quark constitutes.

⋆ F_2 depends only on x (not Q^2: "scaling")

Parton distributions in QCD

"collinear divergence" arises from soft gluon(k_T=0) emission:

$$F_2|_{\text{div}}(x,Q^2) = x\Sigma_{q,\bar{q}} e_q^2 \frac{\alpha_s}{2\pi} P(x) \int_{\kappa}^{k^2_{\text{max}}=Q^2/x} \frac{d|k^2|}{|k^2|}$$

- ⋆ *collinear divergence indicates sensitivity to long-range part of the strong interaction, therefore incalculable in perturbative theory.*
  ...but can factorise the divergence into a "renormalised" quark distribution: $q(x) \rightarrow q(x,\mu^2)$, *(J.Stirling)*

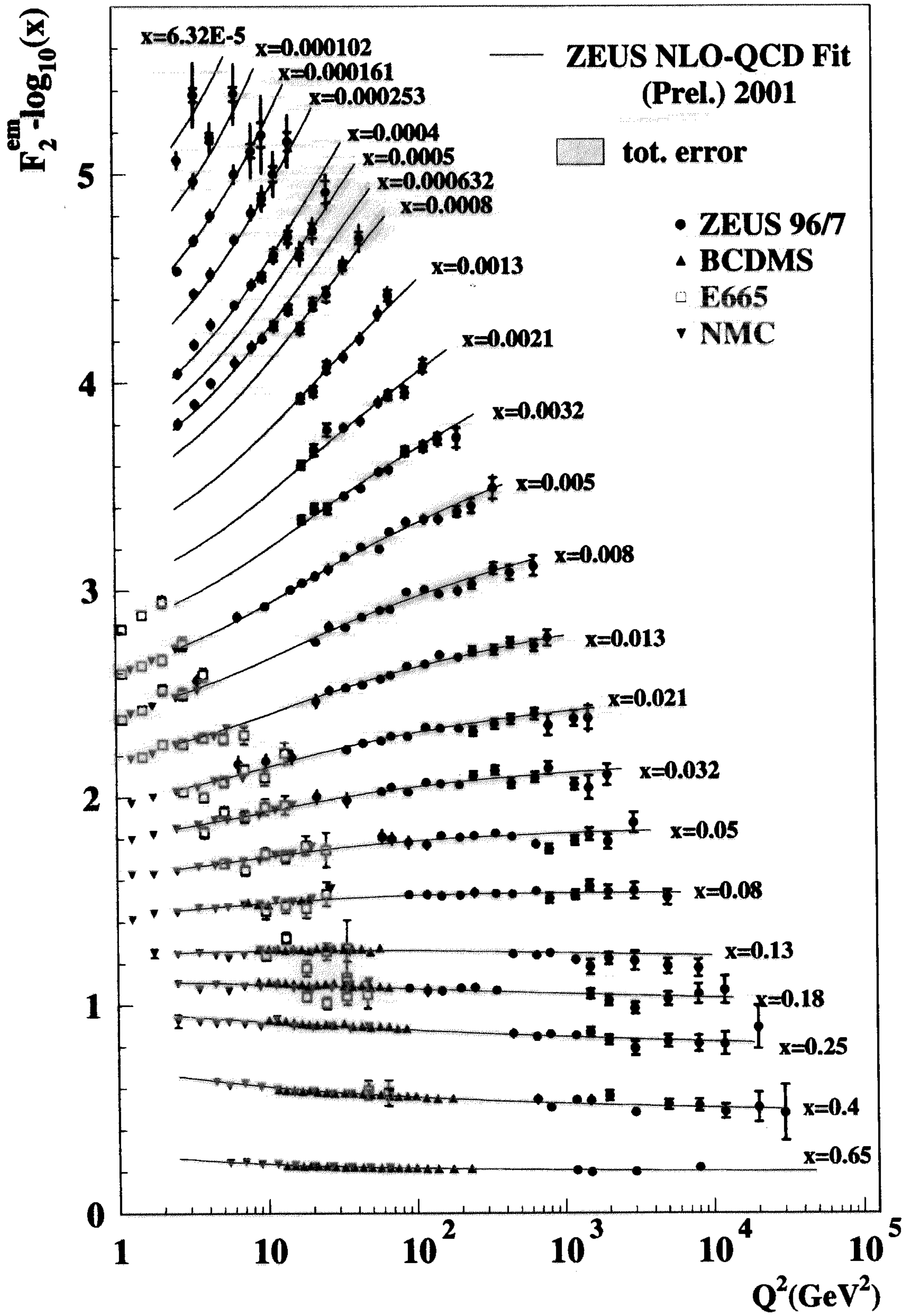
$$q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \{ P(\frac{x}{\xi}) \ln \frac{\mu^2}{\kappa^2} + C(\frac{x}{\xi}) - C'(\frac{x}{\xi}) \}$$

$$\Rightarrow F_2(x,Q^2) = x\Sigma_{q,\bar{q}} \int_x^1 \frac{d\xi}{\xi} q(\xi,\mu^2) \{ \delta(1-\frac{x}{\xi}) + \frac{\alpha_s}{2\pi} P(\frac{x}{\xi}) \ln \frac{Q^2}{\mu^2} + ... \}$$

- q_0: 'bare' PDF, C: calculable QCD correction term.
- $\ln(k^2_{\text{max}}) = \ln(Q^2) + \ln(1/x)$,
  $\ln(1/x)$ was absorbed in $C(x)$ in the above eqs.
- C': arbitrary choices on 'how much' of calculable parts are to be absorbed in bare PDF: factorisation schemes.
  - $\overline{\text{MS}}$ scheme: only divergent term in PDF (C'=C)
  - DIS scheme: all QCD correction terms in PDF (C'=0) $\rightarrow$ LO F_2 formula ($F_2 = x\Sigma(q+\bar{q})$) holds.

- ⋆ F_2 and PDFs: depend logarithmically on Q^2
- ⋆ PDFs: factorisation dependent ($\overline{\text{MS}}$ is commonly used)

F_2^{em} : "scaling violation"



- Data show the scaling violation, which is well reproduced by the NLO-QCD.

Determination of PDFs

- Once the input PDFs (x-dependence) at Q_0^2 are given, **DGLAP** evolution describes μ^2 dependence of $q(x,\mu^2)$:

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix}\Sigma \\ xg\end{pmatrix} = \alpha_s \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix} \otimes \begin{pmatrix}\Sigma \\ xg\end{pmatrix},$$

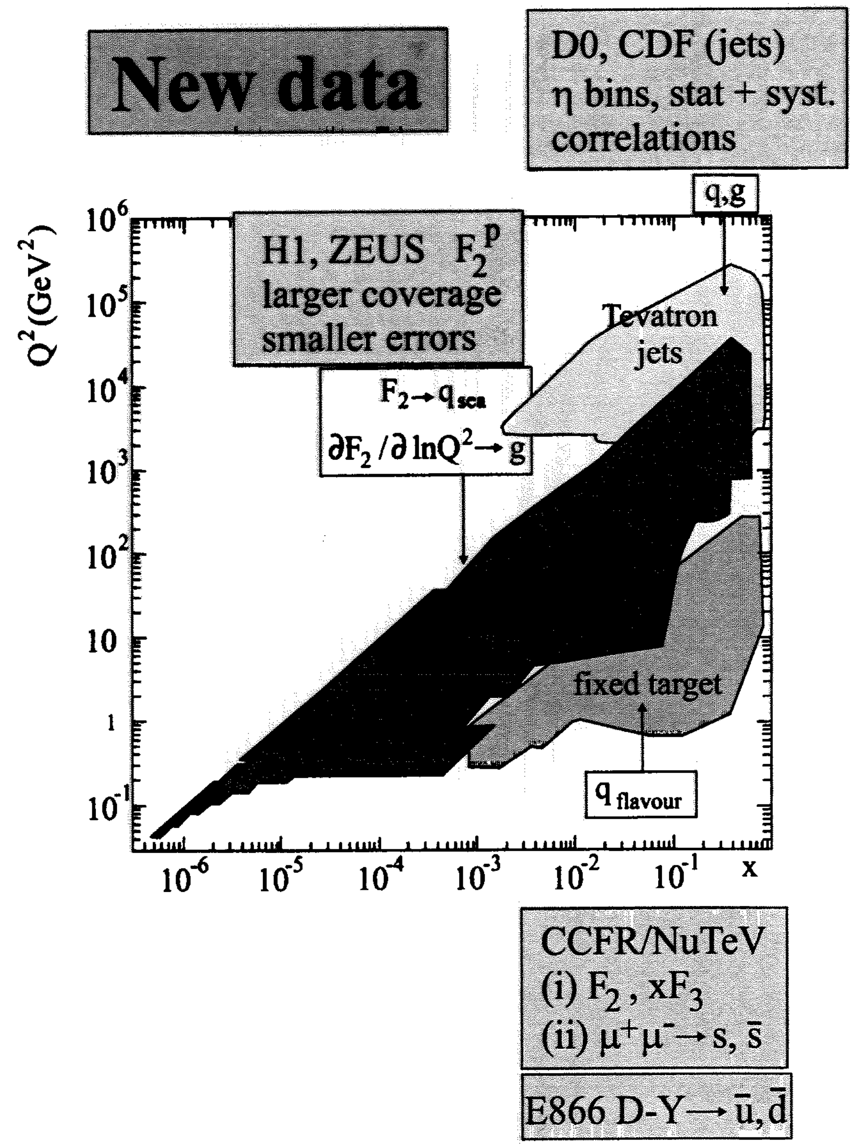
$$\frac{\partial}{\partial \ln \mu^2} q_{ns} = \alpha_s P_{qq} \otimes q_{ns}$$

(Σ: singlet dist., g: gluon dist., q_{ns}: non-singlet dist. P: splitting functions, $\otimes$: Mellen convolution)

$*$ $\ln Q^2$ re-summation of soft gluon emission

- PDFs are not observable.(but F_2 are!)
  $\rightarrow$ universality should be checked in various processes
  $\rightarrow$ a global analysis.

$\Rightarrow$ Input PDFs at Q_0^2 are determined by a global fit to experimental data.



(A.D.Martin, DIS2001 conference)

Global PDF analyses

TABLE III. Processes studied in the global analysis (* indicates data fitted).

Process/ Experiment	Leading order subprocess	Parton behaviour probed
DIS ($\mu N \to \mu X$) $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)*	$\gamma^* q \to q$	Four structure functions → $u+\bar{u}$, $d+\bar{d}$, $\bar{u}+\bar{d}$, s (assumed $= \bar{s}$), but only $\int xg(x,Q_0^2)dx \simeq 0.35$ and $\int(\bar{d}-\bar{u})dx \simeq 0.1$
DIS ($\nu N \to \mu X$) $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)*	$W^* q \to q'$	
DIS (small x) F_2^{ep} (H1, ZEUS)*	$\gamma^*(Z^*)q \to q$	λ ($x\bar{q} \sim x^{-\lambda_S}$, $xg \sim x^{-\lambda_g}$)
DIS (F_L) NMC, HERA	$\gamma^* g \to q\bar{q}$	g
$\ell N \to c\bar{c}X$ F_2^c (EMC; H1, ZEUS)*	$\gamma^* c \to c$	c ($x \gtrsim 0.01$; $x \lesssim 0.01$)
$\nu N \to \mu^+\mu^- X$ (CCFR)*	$W^* s \to c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{4}(\bar{u}+\bar{d})$
$pN \to \gamma X$ (WA70*, UA6, E706, ...)	$qg \to \gamma q$	g at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2 - 0.6$
$pN \to \mu^+\mu^- X$ (E605, E772)*	$q\bar{q} \to \gamma^*$	$\bar{q} = ...(1-x)^{\eta_S}$
$pp, pn \to \mu^+\mu^- X$ (E866, NA51)*	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{u}-\bar{d}$ ($0.04 \lesssim x \lesssim 0.3$)
$ep, en \to e\pi X$ (HERMES)	$\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$	$\bar{u}-\bar{d}$ ($0.04 \lesssim x \lesssim 0.2$)
$p\bar{p} \to WX(ZX)$ (UA1, UA2; CDF, D0)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13$; 0.05
$\to \ell^\pm$ asym (CDF)*		slope of u/d at $x \approx 0.05 - 0.1$
$p\bar{p} \to t\bar{t}X$ (CDF, D0)	$q\bar{q}, gg \to t\bar{t}$	q, g at $x \gtrsim 2m_t/\sqrt{s} \simeq 0.2$
$p\bar{p} \to$ jet $+ X$ (CDF, D0)	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05 - 0.5$

33

(MRST, *Eur.Phy.J.* **C4** (1998) 463.)

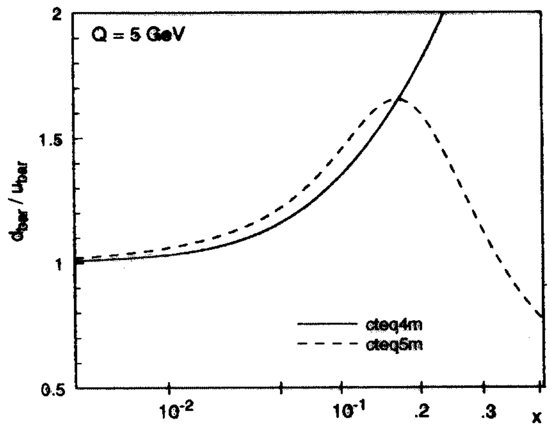
Lepton-lepton, lepton-hadron and hadron-hadron collisions probe complementary aspects of pQCD:

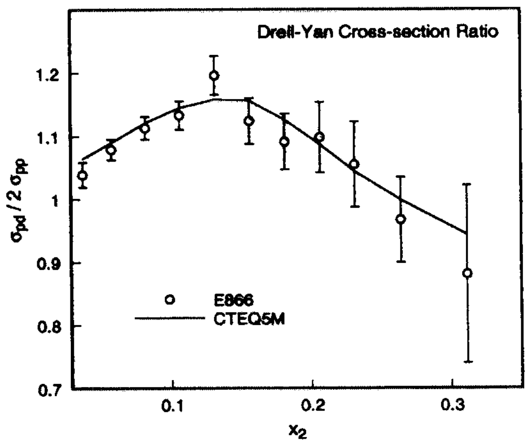
- *Lepton-lepton provides* $\alpha_s(Q^2)$ *and frag. functions*
- *Lepton-hadron (DIS;* F_2*,* F_3*) along with hadron-hadron (DY) provide info on quark distributions*
- *At LO,* $g(x,Q)$ *enters directly in hadron-hadron production of photons and jets.*

(J. Huston, "Physics in Collision" 1999)

Parameterisation

- flavour decomposition: xu_V, xd_V, xg, xq_{sea}, $x(\overline{d} - \overline{u})$
  - number sum-rule: $\int_0^1 u_V$=2, $\int_0^1 d_V$=1
  - momentum sum-rule: $\int_0^1(xu_V + xd_V + xg + xq_{sea})$=1
  - s-quark: $s = \frac{1}{4}(\overline{u} + \overline{d})$ from CCFR measurement
  - $x(\overline{d} - \overline{u})$: Gottfried sum-rule.





- functional form: all global analyses use a generic form for the parameterisation of PDFs:
  $f(x) = A_0 x^{A_1}(1-x)^{A_2} S(x; A_4, ...)$
  - A_0: normalisation.
  - A_1: associated with small-x Regge behaviour
  - A_2: associated with large-x quark counting rule
  - S: "smoothing function":
    MRS uses $1 + A_3\sqrt{x} + A_4 x$, CTEQ uses $1 + A_3 x^{A_4}$

PDFs on market

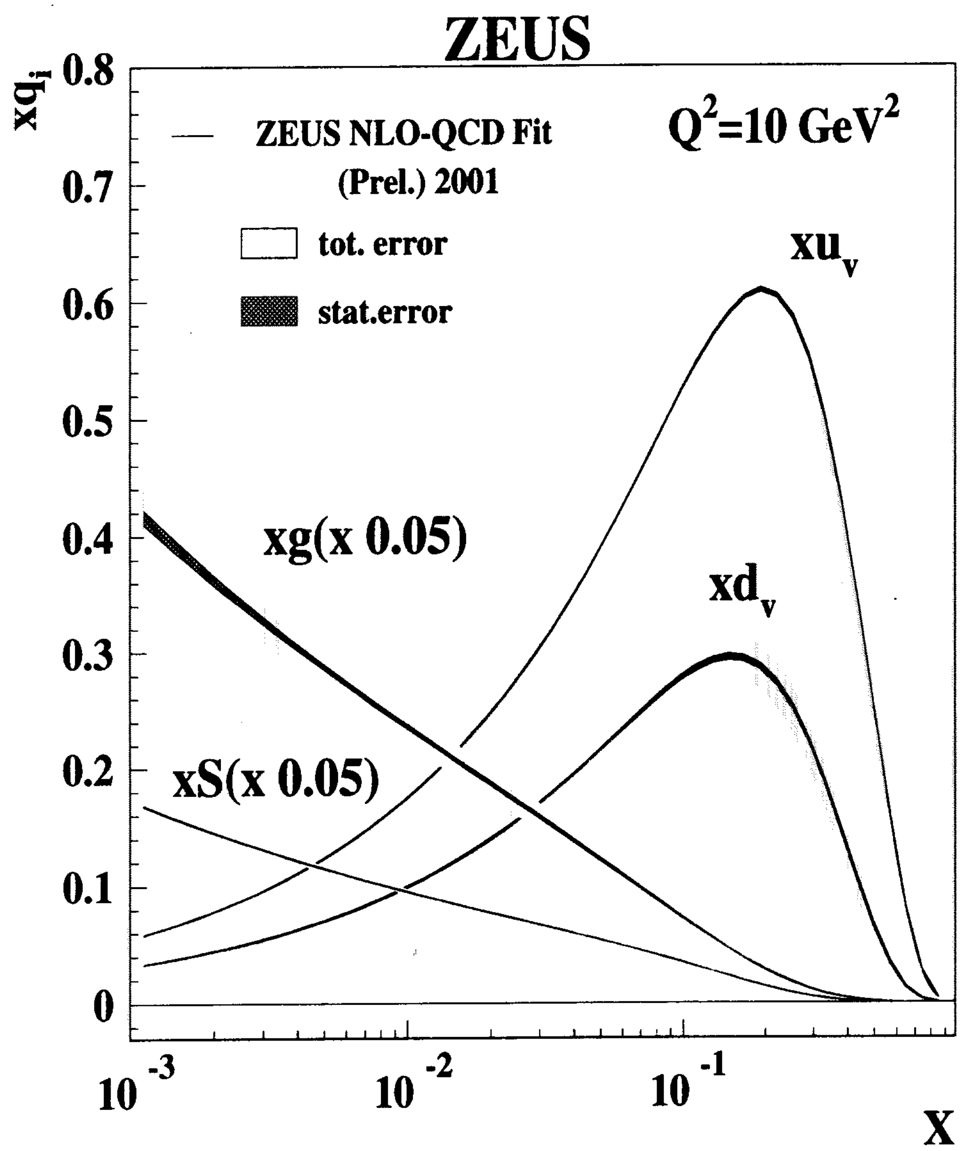
- Fitters:
  - Two major groups, MRS and CTEQ:
    $\longrightarrow$ really a global fit on DIS, DY, jets etc.
    - MRS(T) = A.D. Martin, R.G. Roberts, W.J. Stirling, R.Thorne
    - CTEQ = Coordinated Theoretical-Experimental project on QCD
  - HERA experiments, H1 and ZEUS:
    $\longrightarrow$ QCD fit on their own + other DIS data.
- Semi regular updates driven by new data and theory
  - "MRST 2001" (hep-ph/0110215,submitted to *Eur.Phy.J.*)
  - "CTEQ5" (*Eur.Phy.J.* **C12** (2000) 375)

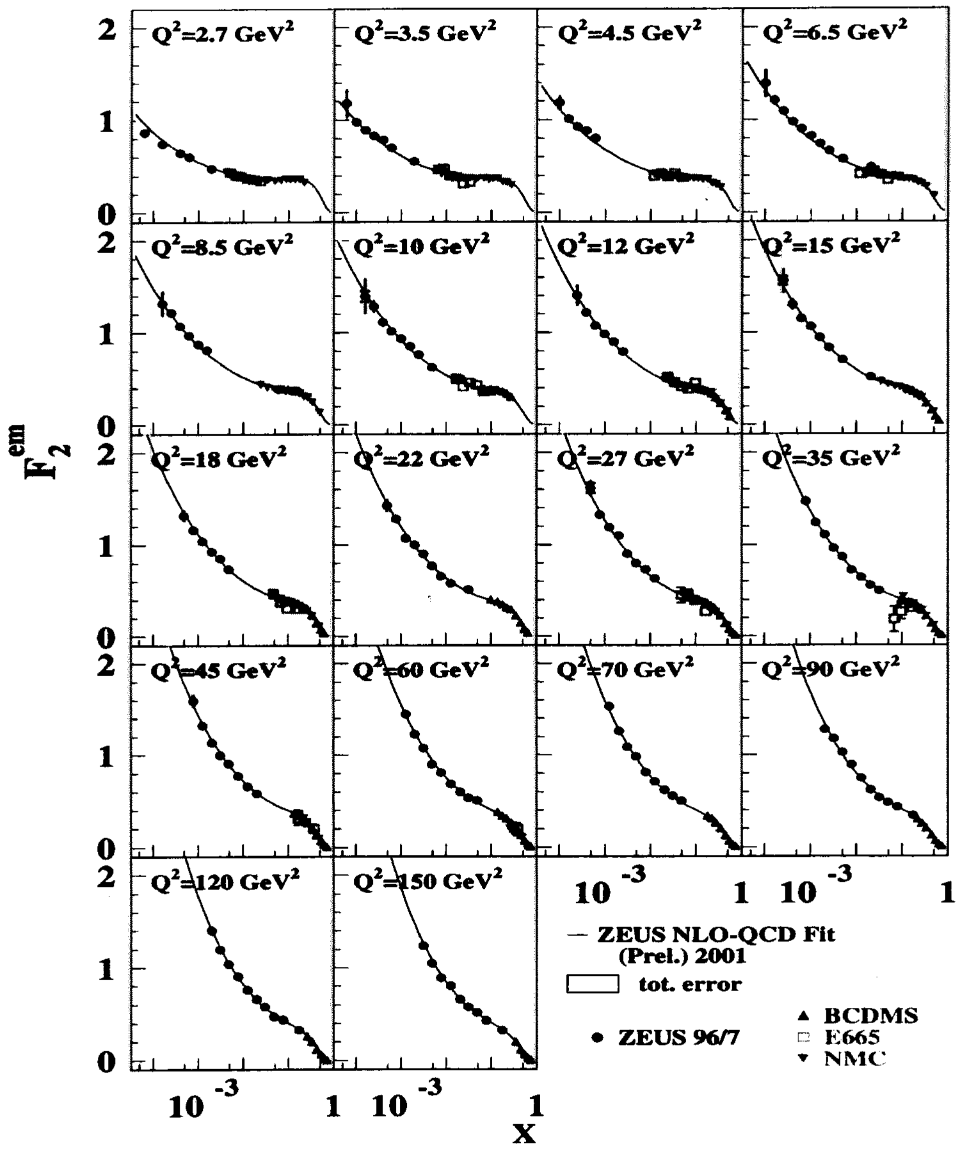
MRS .vs. CTEQ

In a word, 'similar'. Largest difference is (was?) xg at large-x.

- MRST: prompt-γ to constraint xg at large-x
- MRST2001:
  - TEVATRON jet for xg at large-x
  - New HERA F_2 with full accounting of correlated errors
  - NNLO
  - heavy-quark mass scheme: RT-VFN
  - $\Delta\sigma(W)/\sigma(W)$ at LHC: 2%.(just data errors)
- CTEQ5: TEVATRON jet to constraint xg at large-x
  - heavy-quark mass scheme: ZM-VFN
  - $\Delta\sigma(W)/\sigma(W)$ at LHC: 8%.(just data errors)

PDFs: status





- NLO-QCD gives good description ($\chi^2 \sim 1$)

High Q^2 DIS cross-section

High-Q^2 $\Rightarrow$ EW contribution to NC, CC can be observed

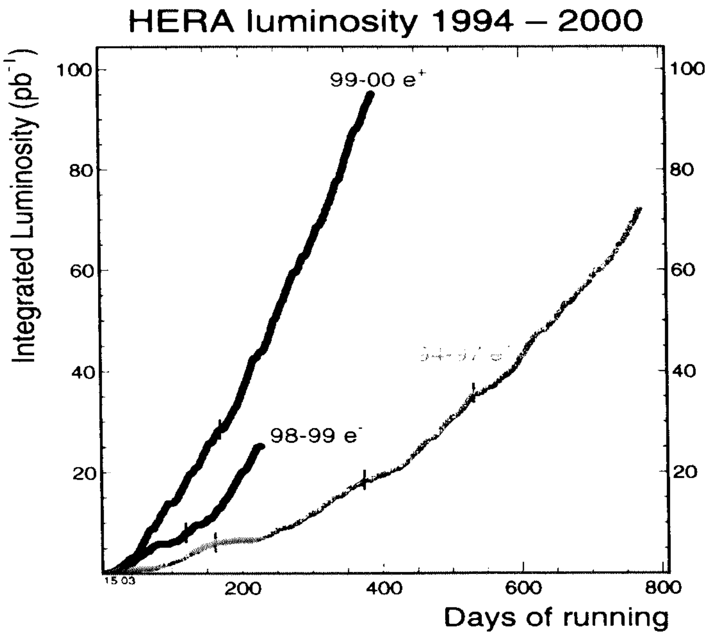
- NC: $\frac{d^2\sigma(e^\pm p)}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4} \times \{Y_+ F_2^{NC} \mp Y_- xF_3^{NC} - y^2 F_L^{NC}\}$
  - xF_3 is a parity violating term. Sign changes in e^+/e^-.
- CC ($ep \longrightarrow \nu X$)

$$e^+p: \frac{d^2\sigma}{dxdQ^2} = \frac{{G_F}^2}{2\pi}\left(\frac{{M_W}^2}{{M_W}^2+Q^2}\right)^2 \times \sum_{i=1,2}[\overline{u_i} + (1-y)^2 d_i]$$

$$e^-p: \frac{d^2\sigma}{dxdQ^2} = \frac{{G_F}^2}{2\pi}\left(\frac{{M_W}^2}{{M_W}^2+Q^2}\right)^2 \times \sum_{i=1,2}[u_i + (1-y)^2 \overline{d_i}]$$

  - flavour selecting; $d(u)$-quark contributes only to $e^+(e^-)$.

High-Q^2 $\Rightarrow$ needs large $\mathcal{L}$

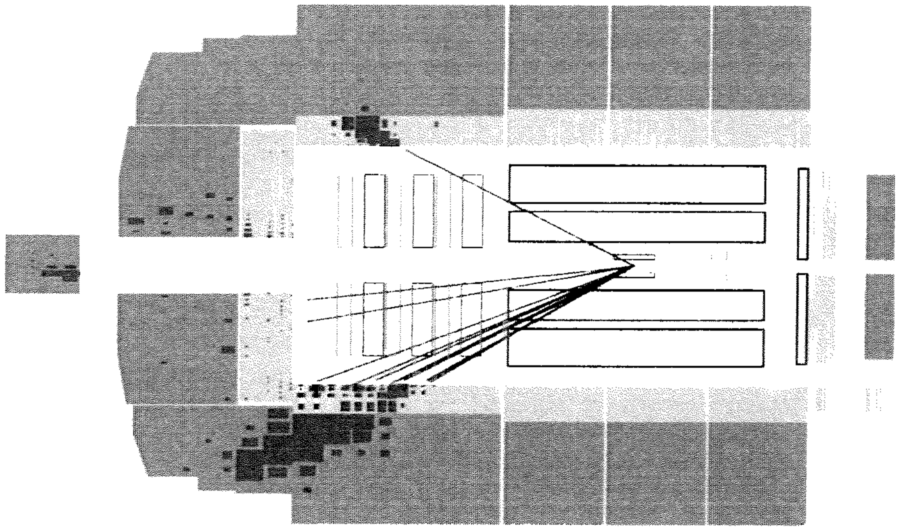


- '94–'97 e^+: 47 pb^{-1}
  - NC $Q^2 > 2.5$ GeV^2
  - CC $Q^2 > 200$ GeV^2
- '98–'99 e^-: 16 pb^{-1}
  - NC/CC prelim. $Q^2 > 200$ GeV^2

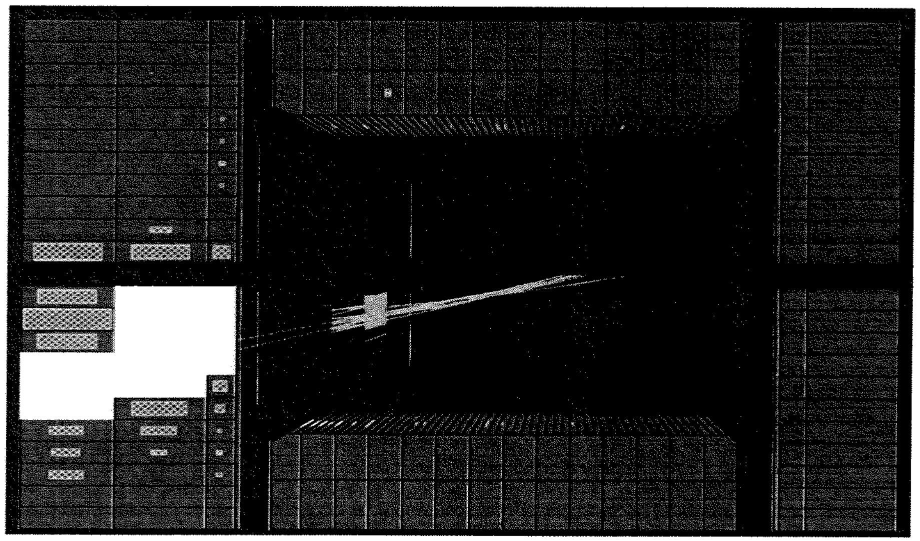
- Remind: only NC e^+ $Q^2 > 2.5$ GeV^{2}was used in the fit.

DIS cross-section measurement

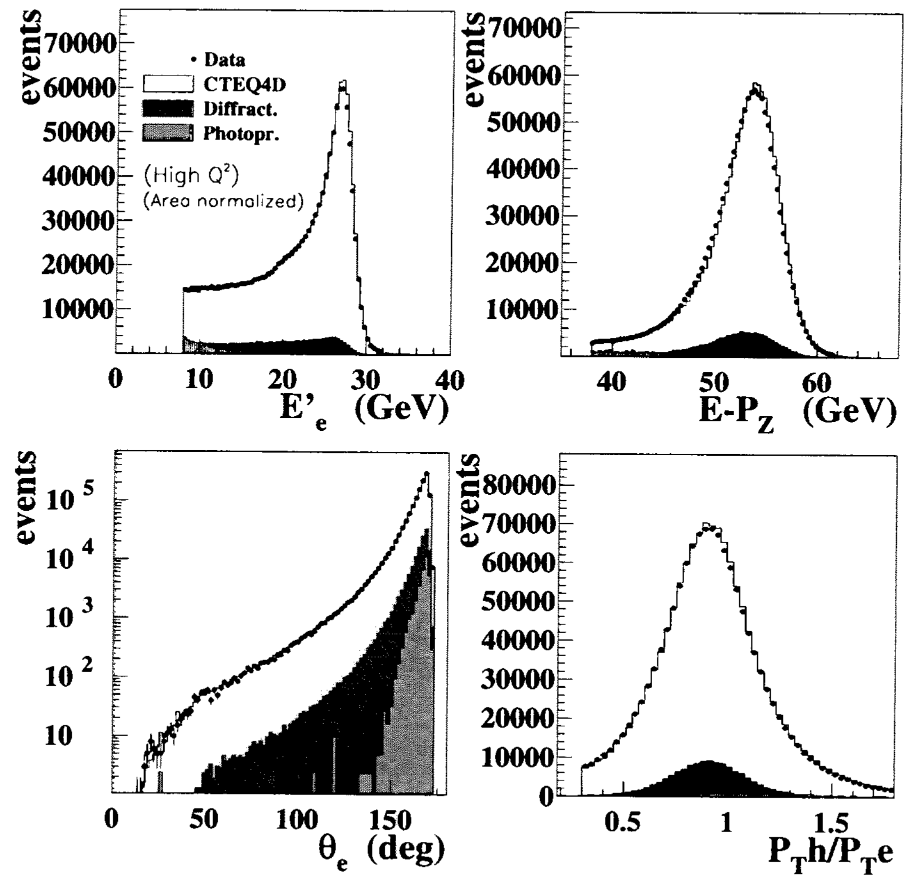
NC signature: a large E_T scattered-e.



CC signature: a large $\not{P}_T$ (due to escaped-ν).



ZEUS Preliminary 1996-97

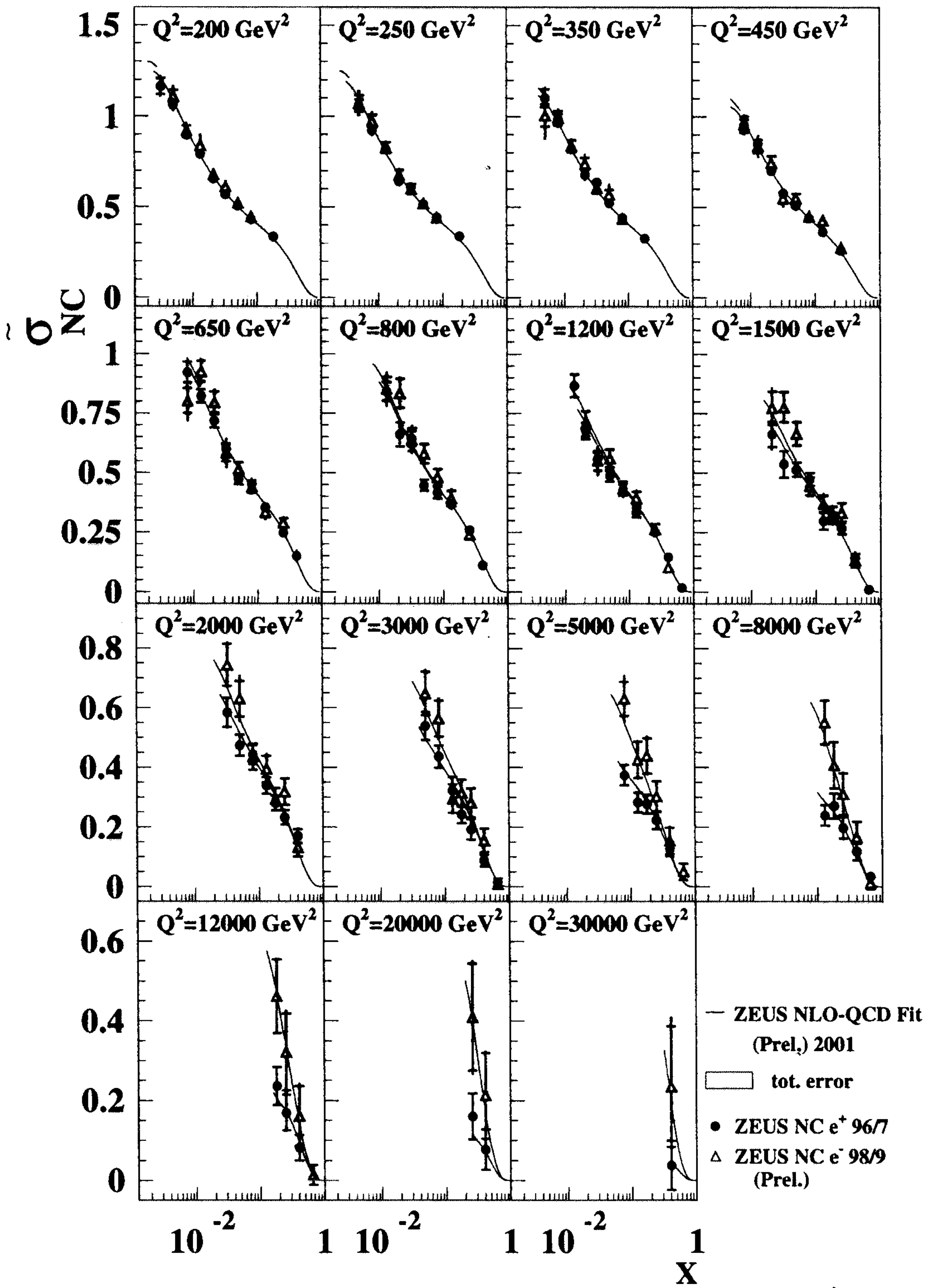


- NC: Both E'_e and θ_e are well reproduced.
- CC: Hadronic energy measurement is crucial. Well understood and checked with $p_T h / p_T e$ in NC data.

High-Q^2 NC reduced $\tilde{\sigma}_{NC}$

- Note: e^-p is NOT used in the fit.

$$\tilde{\sigma}_{NC}(e^{\pm}p) \equiv \left(\frac{Q^2 x}{2\pi\alpha^2 Y_+}\right)\frac{d^2\sigma}{dxdQ^2} = F_2^{NC} \mp \frac{Y_-}{Y_+} xF_3^{NC} - \frac{y^2}{Y_+} F_L^{NC}$$



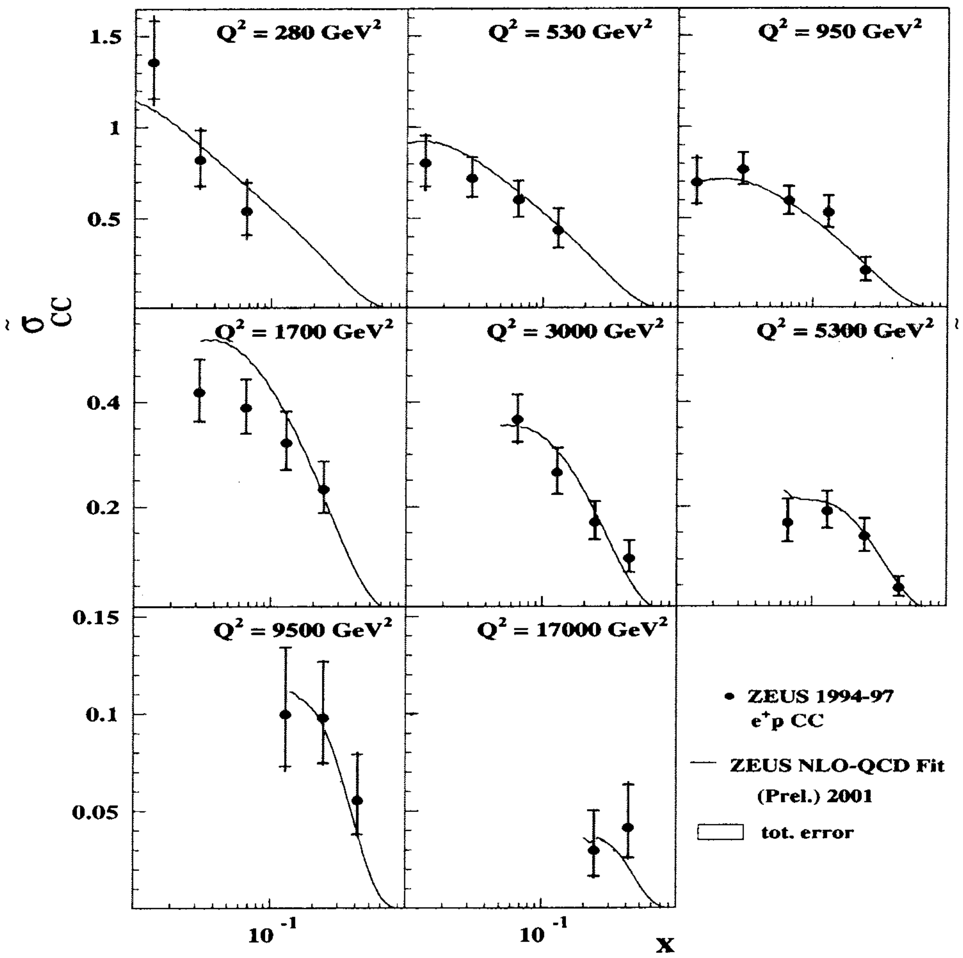
- NLO-QCD gives good description both for e^+p and e^-p .
  $\Rightarrow xF_3^{NC}$ component (i.e. Z^0 effect coupling to valence PDFs).

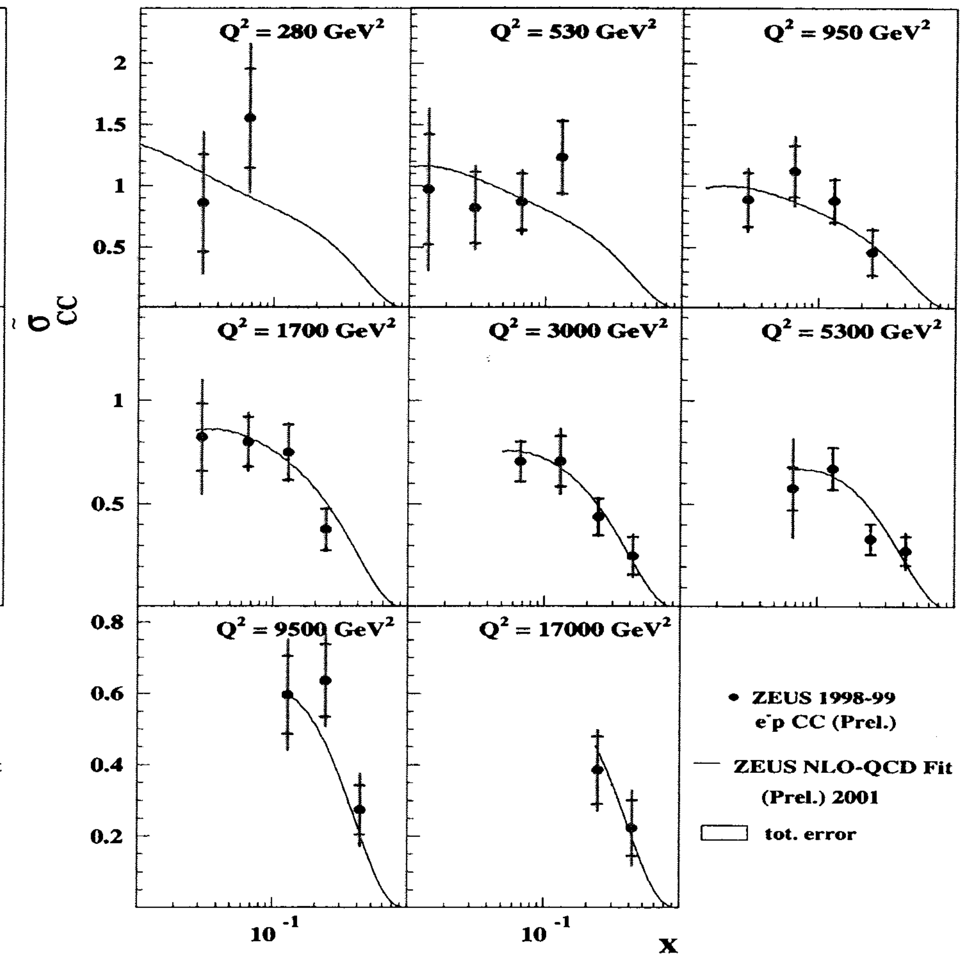
High-Q^2 CC reduced $\tilde{\sigma}_{CC} \equiv \frac{G_F^2}{2\pi x}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2 \frac{d^2\sigma}{dx dQ^2}$

- Note: CC were NOT used in the NLO-QCD fit.

$\tilde{\sigma}_{CC}(e^+p) = x \sum_{i=1,2} [\overline{u_i} + (1-y)^2 d_i]$

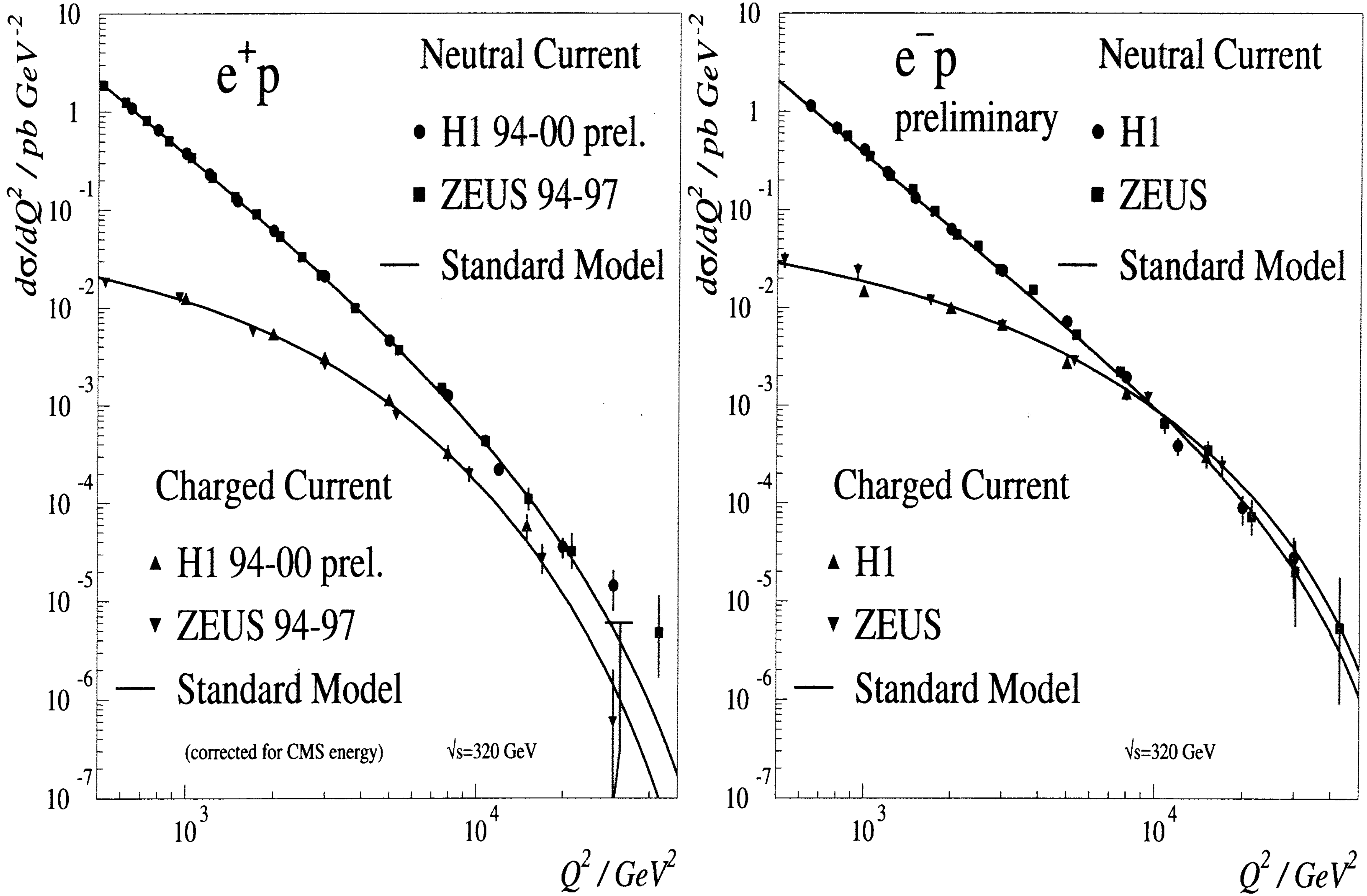
$\tilde{\sigma}_{CC}(e^-p) = x \sum_{i=1,2} [u_i + (1-y)^2 \overline{d_i}]$





- NLO-QCD gives good description both for e^+p and e^-p CC. (flavour specific)

High-Q^2 DIS at HERA: Electro-weak Unification



PDFs: issues, progress, prospects

1. Uncertainties
2. Gluon PDF and α_s
3. PDFs at large-x
4. Heavy-quark PDFs
5. NNLO and $\ln(1/x)$ re-summations

Uncertainties in PDFs

Sources of uncertainty

- data errors:
  - statistical errors
  - systematic uncertainties
    - ∗ often not randomly distributed
    - ∗ point-to-point correlations
- theory errors and assumptions made in the analysis: e.g.
  - higher order QCD
  - factorisation and renormalisation scale uncertainties
  - parameterisation of starting PDFs

Estimating uncertainty...is a nightmare !

⋆ Comparing 2 PDFs to estimate uncertainties is not reliable since similar assumptions and data were used in most of the QCD analysis.

⋆ Full error analysis providing error correlation matrix for PDFs is the correct way,but...:

- $\sim$ 20 PDF parameters
- $\sim$ 14 'diverse' experiments $\Rightarrow \sim$ 1500 data point

$\Rightarrow$ Progress: first study by M. Botje (*Eur.Phy.J.* **C14** (2000) 285)

$\Rightarrow$ Progress: H1 and ZEUS NLO-QCD fits (2001), MRST2001: full accounting of correlated systematic errors.

$\rightarrow$ 71 sources taken into account in ZEUS NLO-QCD fit.

Correlated systematic errors

A classical method: "offset method"

- χ^2-minimisation: define χ^2 with stat. errors:
  $\chi^2 \equiv \sum_i^{data} \frac{(t_i(p)-m_i)^2}{(\delta_i^{stat})^2}$
- systematic errors: are evaluated by repeating $\times n_{syst.source}$ times of:
  - shift data points for each systematic source.
  - fit again and take the variation as due to that source.

$\rightarrow$ simple, but, CPU-cost is expensive. (one fit + many fits)

$\Downarrow$

A sophisticated method making use of Hessian matrix (C. Pascaud and F. Zomer pre-print: LAL-95-05)

- modify χ^2 introducing systematic parameter s_l as:
  $\chi^2 \equiv \sum_i^{data} \frac{(t_i(p)/f_i(s)-m_i)^2}{(\delta_i^{stat})^2} + \sum_l^{syst.source}(s_l)^2$

  $f_i = 1 + \sum_l^{syst.source} \beta_{il} s_l,$ (β_{il}: syst. from source l at point i)

- χ^2-minimisation: with stat. errors (i.e. keep s_l=0):
  $\rightarrow$ equivalent to classical method.

- systematic errors are propagated through Hessian matrix:
  $H_{ps} = \frac{\partial^2\chi^2}{\partial p \partial s}$, which can be evaluated after the fit converges.
  $\rightarrow$ a good approximation of classical method.

Correlated systematic errors -cont'd-

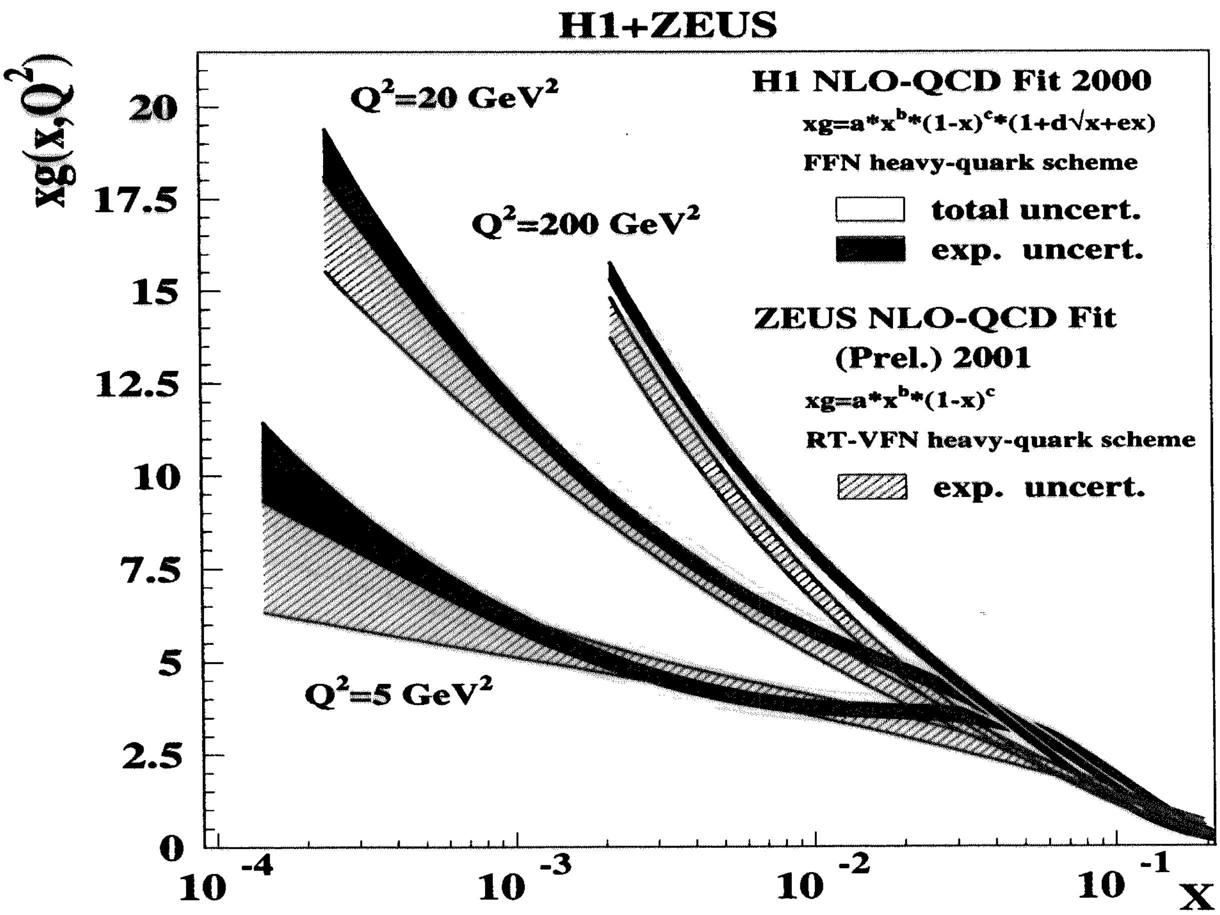
H1: fit also systematic parameters s_l

→ Data central values can be moved by a shift of s_l but with an additional penalty of s_l^2 to χ^2

→ Mathematically correct *if we assume Gaussian distributed nature of systematic errors*

⋆ ZEUS: conservative estimation which is equivalent to the offset method.

⋆ H1: adventurous estimation by fully exploiting the assumption of ideal (Gaussian) behaviour of systematic errors



⋆ Currently, under discussion!