

(余剰次元モデルでの)

電弱対称性の力学的破れ

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1. Technicolor models
 - (2. Extended technicolor (ETC) & FCNC)
 3. Walking technicolor
 4. Peskin-Takeuchi parameters (S, T, U)
 5. Anomalous Triple Gauge Vertex (TGV)
 6. Top quark condensation
 7. Top-color model (Top-color assisted TC: TC^2)
 8. Top seesaw model
 9. Extra dimensions
- Hashimoto, M.T., Yamawaki

1. Technicolor models

* Higgs field in the Standard Model (SM)



$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad v \simeq 250 \text{ GeV}$$

↑ Origin of masses

$$M_W^2 = \frac{g_2^2}{4} v^2, \quad M_Z^2 = \frac{g_Z^2}{4} v^2, \dots$$

$v \ll M_{\text{pl}}, M_{\text{GUT}}$ Why? (Hierarchy problem)

* What happens in the SM without Higgs?

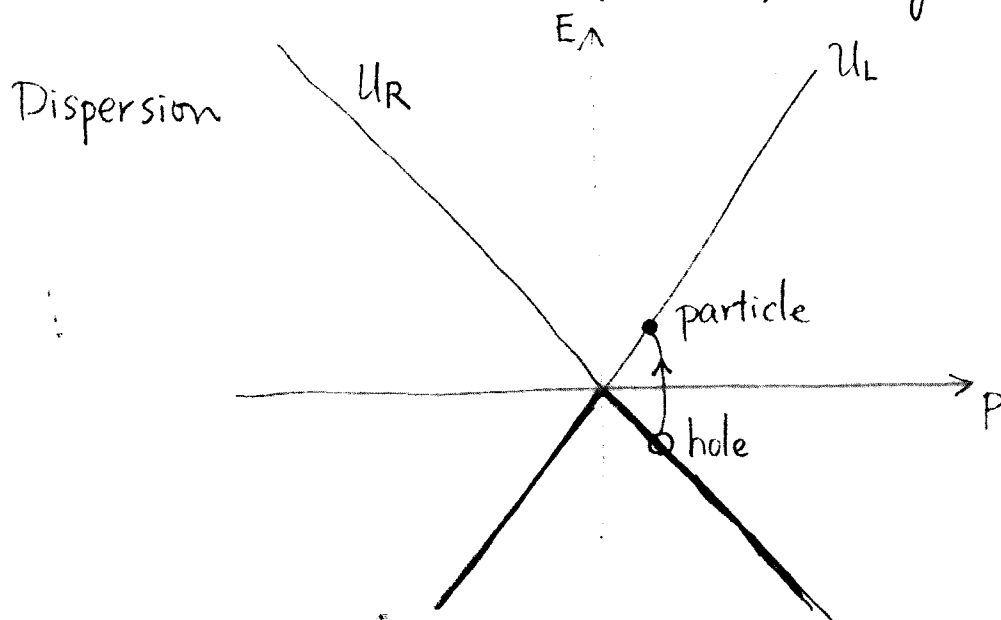
W and Z remain massless exactly ($M_W = M_Z = 0$)?

No! Non-perturbative dynamics of QCD ($\sim 1 \text{ GeV}$) feed non-zero $M_W, M_Z \sim O(10 \text{ MeV})$

* Dynamical chiral symmetry breaking (DχSB) in QCD

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$G = SU(2)_L \times SU(2)_R \quad \text{chiral symmetry}$$



Energy of particle-hole bound state

$$2|p| - E_{\text{binding}} < 0 : \text{unstable vacuum} \\ \int_{\Lambda_{\text{QCD}}}^S \Rightarrow \langle \bar{\psi}\psi \rangle \neq 0$$


$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \neq 0.$$

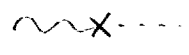
Dynamical chiral symmetry breaking

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

Pions ($\pi^+ \sim \bar{d}i\gamma_5 u$, $\pi^- \sim \bar{u}i\gamma_5 d$, $\pi^0 \sim \bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d$)
are NG bosons of D χ SB

4. Dynamical Higgs Mechanism

 : W propagator $\sim \frac{1}{g^2}$

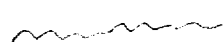
 : Pion decay constant $\frac{g_2}{2} f_\pi g^\mu$

$$\begin{aligned} \text{wavy line with two crosses} &= \frac{1}{g^2} \cdot \frac{g_2}{2} f_\pi g^\mu \cdot \frac{1}{g^2} \cdot \frac{g_2}{2} f_\pi g_\mu \cdot \frac{1}{g^2} \\ &= \frac{g_2^2}{4} f_\pi^2 \frac{1}{g^4} \end{aligned}$$

$$\text{wavy line} + \text{wavy line with two crosses} + \text{wavy line with four crosses} + \dots$$

$$= \frac{1}{g^2} + \frac{g_2^2}{4} f_\pi^2 \frac{1}{g^4} + \left(\frac{g_2^2}{4} f_\pi^2 \right)^2 \frac{1}{g^6} + \dots$$

$$= \frac{1}{g^2 - \frac{g_2^2}{4} f_\pi^2}$$



W boson becomes massive $\sim O(30 \text{ MeV})$

* Hierarchy problem is resolved in this scheme.

Running of QCD gauge coupling

$$\alpha_3(\mu) = \frac{2\pi}{b_3 \ln(\mu/\Lambda_{\text{QCD}})}$$

$$b_3 = -11 + \frac{4}{3}N_f$$

\Downarrow

$$\Lambda_{\text{QCD}}^2 = \mu^2 \exp\left(-\frac{4\pi}{b_3 \alpha_3(\mu)}\right)$$



Essential singularity scaling

* Technicolor models

Introduce new gauge interaction (technicolor)

Decay constant of TC NG boson

$$f_{TC} = v \simeq 246 \text{ GeV} \sim 2600 f_\pi$$

- Minimal model (one doublet technicolor)

- $SU(N_{TC})$ technicolor
- Techni-fermions

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \quad \begin{matrix} U_R \\ D_R \end{matrix}$$

	$SU(N_{TC})$	$SU(2)_W$	$U(1)_Y$
Q_L	$\underline{N_{TC}}$	$\underline{2}$	0
U_R	$\underline{N_{TC}}$	$\underline{1}$	$1/2$
D_R	$\underline{N_{TC}}$	$\underline{1}$	$-1/2$

$$\Lambda_{TC} \sim \Lambda_{QCD} \sqrt{\frac{3}{N_{TC}}} \frac{f_{TC}}{f_\pi}$$

Mass of techni- ρ resonance

$$M_{T-\rho} \simeq \sqrt{\frac{3}{N_{TC}}} \cdot 2 \text{ TeV}$$

- Multiscale technicolor

Technifermion	T_1	T_2
TC repr.	R_1	R_2

Analysis of gap equation

- Λ_1 : scale of $\bar{T}_1 T_1$ condensate

$$C_2(R_1) \alpha_{TC}(\mu = \Lambda_1) \simeq \frac{\pi}{3}$$

- Λ_2 : scale of $\bar{T}_2 T_2$ condensate

$$C_2(R_2) \alpha_{TC}(\mu = \Lambda_2) \simeq \frac{\pi}{3}$$

$$\text{For } C_2(R_2) > C_2(R_1)$$

$$\Lambda_1 \ll \Lambda_2 \quad (\text{two scale TC})$$

if $\beta \simeq 0$ between Λ_1 and Λ_2 (walking TC)

Particle spectrum below TeV

$$\left. \begin{array}{l} \text{light-} \\ \text{scale} \\ \text{resonances} \end{array} \right\} \begin{array}{l} \cdot \text{Pseudo NG bosons} \\ \cdot \bar{T}_1 T_1 \text{ bound states, e.g. techni-}\rho \\ \quad \text{at } \Lambda_1 \\ \cdot \bar{T}_2 T_2 \text{ bound states,} \end{array} \left\{ \Rightarrow \text{CDF bound} \right.$$

4. Peskin-Takeuchi parameters S, T, U

Chiral perturbation of $D\chi SB$ in QCD

$$O(E^2) \quad \mathcal{L}_2 = \frac{f^2}{4} \text{tr} (D_\mu U^\dagger D^\mu U) \quad f \simeq 90 \text{ MeV}$$

$$O(E^4) \quad \mathcal{L}_4 = L_{10} \text{tr} \left(W_{\mu\nu}^a \frac{\tau^a}{2} U B^{\mu\nu} \frac{\tau^3}{2} U^\dagger \right) \\ + i L_9 \text{tr} \left(W_{\mu\nu}^a \frac{\tau^a}{2} D^\mu U D^\nu U^\dagger + D^\nu U B_{\mu\nu} \frac{\tau^3}{2} D^\mu U^\dagger \right)$$

$$U = \exp \left(\frac{i \tau^a \pi^a}{f_\pi} + \dots \right)$$

⋮

Observed value

$$\left(\begin{array}{l} L_{10}(\mu = M_\gamma) = (-5.6 \pm 0.3) \times 10^{-3} \\ \quad (\pi \rightarrow e \nu \gamma) \\ L_9(\mu = M_\gamma) = (7.1 \pm 0.3) \times 10^{-3} \\ \quad (\pi \text{ charge radius}) \end{array} \right)$$

Note: μ -dependence (renormalization scale)

from pion loop

$$\mu \frac{d}{d\mu} L_{10}(\mu) = + \frac{1}{6(4\pi)^2}$$

$$\mu \frac{d}{d\mu} L_9(\mu) = - \frac{1}{6(4\pi)^2}$$

$\times 2600$ scale up \Rightarrow Technicolor model.

Peskin-Takeuchi S-parameter

(kinetic mixing of W_μ^3 and B_μ)

$$S = -16\pi L_{10} \simeq 0.3 \frac{N_{TC}}{3}$$

$$\left(\begin{array}{l} \text{c.f.: Free fermion loop} \\ S = \frac{N}{6\pi} \left[1 - Y \ln \frac{m_U^2}{m_D^2} \right] \end{array} \right)$$

Naive QCD-like TC is excluded from S-parameter

$$S = -0.16 \pm 0.11$$

($M_{H_1} = 300 \text{ GeV}$ in
T-H)

Open Question:

Reliable estimation of S-parameter
in walking TC

5. Anomalous Triple Gauge Vertex (TGV)

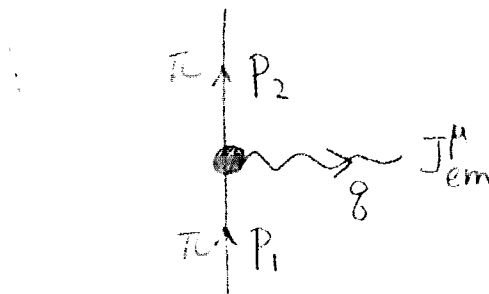
QCD:

composite NG boson (pion)

Hellom
Longhitano
Appelquist-Wu

\Rightarrow non-zero charge radius $\langle r^2 \rangle_\pi \neq 0$

$$\langle \pi^+(p_2) | J_{em}^\mu | \pi^+(p_1) \rangle = F_V(g^2) (p_1 + p_2)^\mu$$



$$F_V(g^2) = 1 + \frac{1}{6} \langle r^2 \rangle_\pi g^2 + \dots$$

Chiral perturbation

$$F_V(g^2) = 1 + \frac{2L_9}{f_\pi^2} g^2 + \mathcal{O}(g^4)$$

$$\boxed{\langle r^2 \rangle_\pi = \frac{12L_9}{f_\pi^2}}$$

Chiral coefficient L_9 is proportional to the NG boson charge radius.

Electroweak chiral perturbation

$L_9 \propto$ radius of the would-be NG boson //

Unitary gauge $U=1$

$L_9 \Rightarrow$ Anomalous TGV

$$g_1^Z = 1 - \frac{1}{2} \frac{e^2}{s^2 c^2} L_9$$

$$K_Z = 1 + \frac{1}{2} \frac{c^2 - s^2}{s^2 c^2} e^2 L_9$$

$$K_\gamma = 1 - \frac{e^2}{s^2} L_9$$

$\sim \mathcal{O}(10^{-3} \sim 10^{-2})$ for QCD-like TC

$$\text{c.f. } \Delta K_\gamma = 0.08 \pm 0.17 \quad (\text{PDG})$$

6. Top quark condensation

$$m_t \simeq 175 \text{ GeV} \iff v \simeq 250 \text{ GeV}$$

relation?

Idea: TC top condensate
 techni-fermion \rightarrow top quark
 technicolor int. \rightarrow NJL type

$$\langle \bar{T}T \rangle \neq 0 \rightarrow \langle \bar{t}t \rangle \neq 0$$

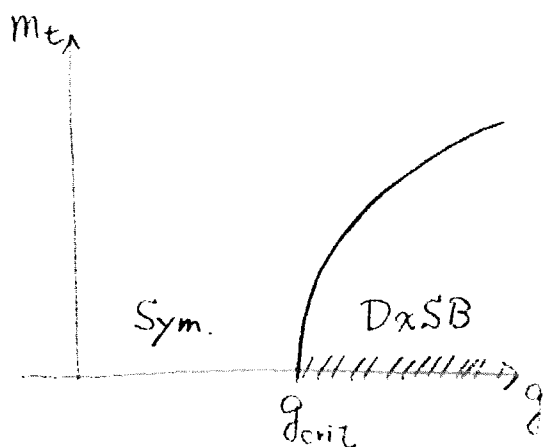
NJL-type interaction

$$\mathcal{L}_{\text{NJL}} = \frac{G}{N_c} \bar{\psi}_L t_R \bar{t}_R \psi_L$$

$$\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$G = \frac{g}{\Lambda^2}$$

Λ : cutoff



Gap eq

$$m_t^2 \ln \frac{\Lambda^2}{m_t^2} = \Lambda^2 \left(1 - \frac{g_{\text{crit}}}{g} \right)$$

Method to predict m_t (Compositeness condition)

Bardeen-Hill-Lindner

* Cutoff scale

$$\mathcal{L}_{NJL} = \frac{G}{N_c} \bar{\delta}_L t_R \bar{t}_R \delta_L$$

Auxiliary field H

$$\mathcal{L}_{aux} = -\frac{G}{N_c} \left[\bar{\delta}_L t_R + \frac{N_c}{G} H^\dagger \right] \left[\bar{t}_R \delta_L + \frac{N_c}{G} H \right]$$

$$\mathcal{L}_{NJL} + \mathcal{L}_{aux} = -\frac{N_c}{G} H^\dagger H - \bar{\delta}_L H t_R - \bar{t}_R H^\dagger \delta_L$$

* Low energy \downarrow RG.

$$\mathcal{L}_{Yukawa} = Z_H \partial_\mu H^\dagger \partial^\mu H - y \bar{\delta}_L H t_R - y \bar{t}_R H^\dagger \delta_L - m_H^2 H^\dagger H$$

field redefinition

$$H_r \equiv Z_H^{1/2} H$$

$$y_r \equiv \frac{y}{\sqrt{Z_H}}$$

$$\mathcal{L}_{Yukawa} = \partial_\mu H_r^\dagger \partial^\mu H_r - y_r \bar{\delta}_L H_r t_R + \dots$$

Compositeness condition

$$Z_H(\mu = \Lambda) = 0 \quad \text{or} \quad y_r(\mu = \Lambda) = \infty$$

Predictions for m_t, m_H		Bardeen-Hill-Lindner		
Λ [GeV]	10^{19}	10^{15}	10^{10}	10^4
m_t [GeV]	220	231	260	460
m_H [GeV]	241	258	300	610

Problems

- * Too heavy top quark $m_t \gtrsim 220 \text{ GeV} > 175 \text{ GeV}$
- * Origin of NJL int. ?
- * Severe fine tuning for $\Lambda \gg m_t$

7. Top-color model

H!!

A model to provide effective NJL interactions for top condensate.

Gauge group

$$SU(3)_1 \times SU(3)_2 \times SU(2)_W \times U(1)_1 \times U(1)_2$$

$q_L = (t_L, b_L)$	3	1	2	1/6	0
t_R	3	1	1	2/3	0
b_R	3	1	1	-1/3	0
(ν_L, τ_L)	1	1	2	-1/2	0
τ_R	1	1	1	-1	0
(u_L, d_L)	1	3	2	0	1/6
u_R	1	3	1	0	2/3
d_R	1	3	1	0	-1/3
(ν_e, e_L)	1	1	2	0	-1/2
e_R	1	1	1	0	-1

* Assume spontaneous breakdown of top-color

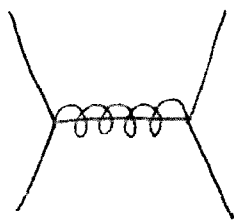
$$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_{\text{color}}$$

$$\frac{g_s}{S_\theta} \quad \frac{g_s}{C_\theta} \quad g_s$$

$$S_\theta \ll 1$$

Color octet massive vector boson : "coloron"

Effective four-fermion int from "coloron exchange"



$$= \frac{-1}{2M^2} \frac{C_\theta^2}{S_\theta^2} g_s^2 \left[\bar{q}_L \gamma^\mu T^A q_L + \bar{t}_R \gamma^\mu T^A t_R + \bar{b}_R \gamma^\mu T^A b_R \right]$$

* Assume spontaneous breakdown of top- Z'

$$U(1)_1 \times U(1)_2 \rightarrow U(1)_Y$$

$$\frac{g_Y}{S'_\theta} \quad \frac{g_Y}{C'_\theta} \quad g_Y$$

Effective four-fermion int from Z' exchange

$$= \frac{-1}{2M^2} \left(\frac{C'_\theta}{S'_\theta} \right)^2 g_Y^2 \left[\frac{1}{6} \bar{q}_L \gamma^\mu q_L + \frac{2}{3} \bar{t}_R \gamma^\mu t_R - \frac{1}{3} \bar{b}_R \gamma^\mu b_R \right]^2$$

Fiertz transf.

For top quark

$$\mathcal{L} = \frac{2}{M^2} \left(C_F \frac{C_\theta^2}{S_\theta^2} g_s^2 + \frac{1}{9} \frac{C_\theta'^2}{S_\theta'^2} g_Y^2 \right) (\bar{\theta}_L t_R) (\bar{t}_R \theta_L)$$

For bottom quark

$$\mathcal{L} = \frac{2}{M^2} \left(C_F \frac{C_\theta^2}{S_\theta^2} g_s^2 - \frac{1}{18} \frac{C_\theta'^2}{S_\theta'^2} g_Y^2 \right) (\bar{\theta}_L b_R) (\bar{b}_R \theta_L)$$

↑
repulsive $\langle \bar{b}b \rangle$ is suppressed.

Note:

$$\# m_t \simeq 175 \text{ GeV} \quad M \simeq 1 \text{ TeV (cutoff)}$$

\Rightarrow Top-pion decay constant

$$f_{t\pi} \simeq \frac{3}{8\pi^2} m_t^2 \ln \frac{M^2}{m_t^2} \simeq 60 \text{ GeV}$$

not enough for Dynamical EWSB $v \simeq 246 \text{ GeV}$

Top-color combined with technicolor

(Top-color assisted technicolor TC^2)

8. Top seesaw model

Chivukula - Dobrescu - Georgi - Hill

Use seesaw mechanism to obtain $m_t \approx 175 \text{ GeV}$
 from EWSB mass $\sim 0.6 \text{ TeV}$

Model

	$SU(3)_1$	$\times SU(3)_2$	$\times SU(2)_W$	$\times U(1)_Y$
$q_L = (t_L, b_L)$	3	1	2	$1/6$
χ_R	3	1	1	$2/3$
t_R	1	3	1	$2/3$
χ_L	1	3	1	$2/3$

Assume spontaneous breakdown of top-color

$$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_{\text{color}}$$

Coloron exchange

$$\mathcal{L}_{\text{NJL}} = \frac{g_{tc}^2}{M^2} (\bar{q}_L \chi_R)(\bar{\chi}_R q_L)$$

Dynamical EWSB

$$\langle \bar{q}_L \chi_R \rangle \neq 0$$

Electroweak singlet mass

$$\mathcal{L}_{\text{singlet}} = -\mu_{\chi\chi} \bar{\chi}_L \chi_R - f'_{\chi t} \bar{\chi}_L t_R$$

Mass matrix

$$(\bar{t}_L \quad \bar{\chi}_L) \begin{pmatrix} 0 & m_{t\chi} \\ f'_{\chi t} & \mu_{\chi\chi} \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix}$$

EWSB mass $\sim 0.6 \text{ TeV}$

mass diagonalization

$$m_t \simeq \frac{m_{t\chi} f'_{\chi t}}{\mu_{\chi\chi}} \quad : \text{seesaw}$$

9. Extra dimensions

Cheng-Dobrescu - Hill

Top seesaw

Extra dimensions

Coloron \longrightarrow gluon KK mode

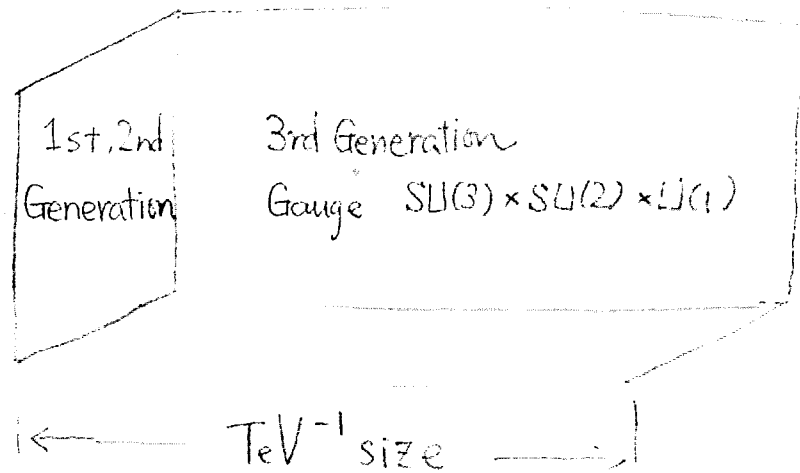
fermions \longrightarrow KK mode of t_R
with vector mass

(Top seesaw can be embedded naturally
in models with extra dimensions

Arkani-Hamed, Cheng, Dobrescu, Hall. (ACDH) model.

$$D = 4 + \delta, \quad \delta = 2, 4, \dots$$

Model



Higher dimensional gauge coupling

$$g_D^2 = \frac{\hat{g}^2}{\mu^\delta}$$

has negative mass dimension $-\delta$ and becomes non-perturbative at high energy



Dynamical EWSB in the bulk?

MAC (Most Attractive Channel) Analysis (one gauge boson exchange)

Channel	Binding strength
$\bar{q}_L t_R$	$\frac{4}{3} \hat{g}_3^2 + \frac{1}{9} \hat{g}_Y^2$: MAC
$\bar{q}_L b_R$	$\frac{4}{3} \hat{g}_3^2 - \frac{1}{9} \hat{g}_Y^2$
$\bar{l}_L \tau_R$	$\frac{1}{2} \hat{g}_Y^2$
\vdots	

Top condensate is favored in the MAC analysis

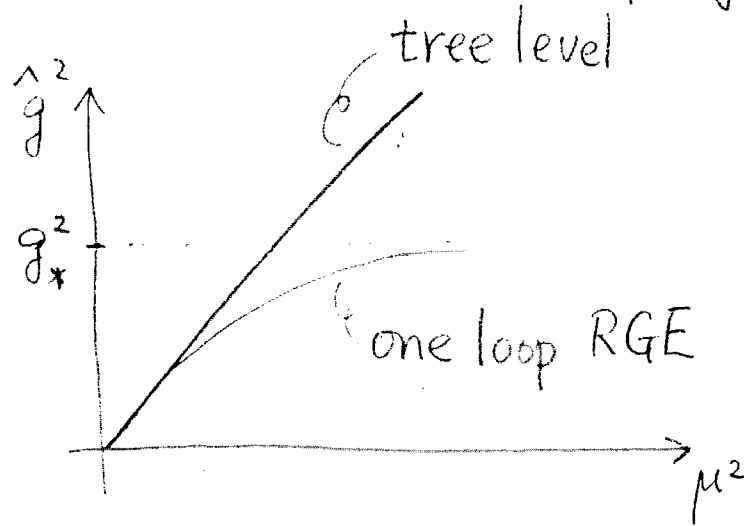
Prediction of m_t and m_H

\Leftrightarrow Compositeness conditions

Problem

Is the bulk gauge coupling strong enough to trigger DEWSB?

Behavior of dimensionless coupling \hat{g}



LIV-FP g_*^2

one-loop estimate of $g_*^2 \Omega_{\text{NDA}}$, $\Omega_{\text{NDA}} = \frac{1}{(4\pi)^{D/2} \Gamma(D/2)}$

$$g_*^2 \Omega_{\text{NDA}} = \frac{1}{-(\frac{2}{\delta} + 1) b'}$$

$$b' = -\frac{26-D}{6} C_G + \frac{\eta}{3} T_R N_f, \quad \eta = \text{tr}_P 1$$

* Naive Dimensional Analysis (NDA)

- condition of dynamical symmetry breaking

$$C_F g_*^2 \Omega_{\text{NDA}} \sim \mathcal{O}(1)$$

\Updownarrow ? compatible

- $D=6$ QCD with 2 flavor

$$C_F g_*^2 \Omega_{\text{NDA}} = 0.09$$

* Gap equation analysis

Dynamical SB occurs only if

$$C_F g_*^2 \Omega_{\text{NDA}} \gtrsim 0.12$$

with $D=6$ dimensions.

ACDH model do not work with $D=6$

We need to modify the model further.

($D=8$ version is OK)

10. Summary (biased?)

Dynamical EWSB models

- Naive Technicolor : S -Parameter, FCNC
- Walking Technicolor : Need reliable calc.
- Top condensate : Fine tuning, m_t
- Top-color : not enough for EWSB
- Top-seesaw : complicated, viable?
- with Extra dimensions : interesting!