String Landscape and Distribution of Vacua

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♣ Moduli Problem

Consider string theory/Calabi-Yau mfd

Calabi-Yau mfds;
$SU(3)$ holonomy $\Longrightarrow \mathcal{N}=1$ SUSY

CY mfds have moduli

Ricci flatness condition

$$R_{IJ}(g)=0, \quad I,J=1,2,\cdots,6$$
$$\Longrightarrow R_{IJ}(g+\delta g)=0$$

Two types of deformations; $i,j=1,2,3$

δg_{ij} : complex structure deformation
$\delta g_{ij} \Longrightarrow \delta g_{ij} g^{j\bar{k}} \bar{\Omega}_{\bar{k}\bar{\ell}\bar{m}} = \delta g_{i,\bar{k}\bar{m}}$, (1,2)-type
(Ω_{ijk} ; holomorphic 3-form)

$\delta g_{i\bar{j}}$: Kähler deformation, (1,1)-type

These degrees of freedom appear as massless scalar fields in 4 dimensions in string compactification.

♣ Moduli Stabilization

Existence of massless scalars is in direct conflict with phenomenology. One has to generates a potential V for moduli fields so that moduli are fixed at its extremum. Moduli fields acquire masses proportional to the 2nd derivatives of V.

How can we generate potential for moduli fields preserving SUSY?

$\mathcal{N} = 1$ local SUSY gives

$$V = e^K (g^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3W\bar{W})$$

where

W : superpotential

K : Kähler potential over moduli space $\mathcal{M}$

$D_a = \partial_a + \partial_a K$

Possible mechanism of generating superpotentials:

In type IIB theory

1. complex structure moduli $(z_a,\ a = 1, \ldots, h_{1,2})$

When RR or NS fluxes are turned on, superpotentials become generated for complex structure moduli

$$H_{RR} \equiv H_1 = dB_1,\ H_{NSNS} \equiv H_2 = dB_2$$

$$W(z_a) = \int_M (H_1 - \tau H_2) \wedge \Omega$$

$$= \sum N_I X_I - \sum M_I \frac{\partial F}{\partial X_I}$$

$$M_I = \int_{A_I} (H_1 - \tau H_2),\ N_I = \int_{B_I} (H_1 - \tau H_2)$$

$$X_I = \int_{A_I} \Omega,\ \frac{\partial F}{\partial X_I} = \int_{B_I} \Omega,\ I = 1, \ldots, h_{1,2} + 1$$

Gukov-Vafa-Witten

$$\frac{\partial W}{\partial z_a} = 0 \Longrightarrow \{z_a\} \text{ all fixed}$$

Giddings-Kachru-Polchinski

These are the flux vacua.

2. Kähler moduli $(t_i, i = 1, \ldots, h_{1,1})$

Instanton amplitudes of D3-branes wrapping divisors of CY

$$\text{Amplitude} \approx e^{-t_i}$$

t_i denotes the size of the divisor D_i. Amplitude does no vanish when $\chi(D_i) = 1$.

Witten

This condition is modified in the presence of flux.

♣ String Landscape

Enormous Number of vacua in string theory

- Fix CY mfd M
- typical CY has 3-cyles: 100∼200
- Upper bound on fluxes:

$$\int H_1 \wedge H_2 \leq \text{const. (depending on geometry of M)}$$

$$\approx 100 - 1000, \text{ tadopole condition}$$

Suppose $|H_1|, |H_2| \leq 10$

- possible choice of fluxes:

$$10^{100} \sim 10^{200}$$

- There may be several vacua for each flux congifuration.

Altogether there exist an enormous number of string vacua $\mathcal{O}(10^{100})$

$\Downarrow$

Discretuum

♣ Statistical treatment of Vacua

Douglas, Ashok, Denef, ...

Vacua distribution function on moduli space $\mathcal{M}$

$$\rho(z) = \delta(D_a W)\delta(D_{\bar{b}} W^*) \times \left| \det \begin{pmatrix} \partial_a D_b W & \partial_a D_{\bar{b}} W^* \\ \partial_{\bar{a}} D_b W & \partial_{\bar{a}} D_{\bar{b}} W^* \end{pmatrix} \right|$$

Simplify

$\Downarrow$

$$\tilde{\rho}(z) = \delta(D_a W)\delta(D_{\bar{b}} W^*) \times \det \begin{pmatrix} \partial_a D_b W & \partial_a D_{\bar{b}} W^* \\ \partial_{\bar{a}} D_b W & \partial_{\bar{a}} D_{\bar{b}} W^* \end{pmatrix}$$

Simplifying assumption:
Fluxes obey Gaussian distribution $\Longrightarrow$ W itself obeys Gaussian distribution.

$$\langle W(w)W(z)\rangle = \langle W^*(w)W^*(z)\rangle = 0$$
$$\langle W(w)W(z)^*\rangle \sim Im \sum w_a \left(\frac{\partial F}{\partial z_a}\right)^* = e^{-K(w,\bar{z})}$$

It follows

$$\tilde{\rho}(z) \prod dz^a \wedge d\bar{z}^{\bar{a}}$$
$$= \det \frac{1}{2\pi} \underbrace{\left(R^a_b + \delta^a_b \omega\right)}$$

curvature and Kähler form on M

Ashok-Douglas

Here we consider unbroken SUSY, $D_i W = 0$ and hence

$$V = e^K(|D_i W|^2 - 3|W|^2) = -3e^K|W|^2 < 0$$

Thus we obtain negative cosmological constant and AdS spaces. In order to construct vacua with a positive cosmological constant one has to break SUSY to generate positive vacuum energy (KKLTmodel).

Most of flux vacua have properties which are far removed from our own universe. It there any principle to select acceptable vacua?

- enough amount of inflation

- sufficiently high gauge symmetry
- chiral matter content
- electro-weak symmetry breaking scale $\approx$ 1Tev
- low energy SUSY breaking scale ?

Anthropic principle: Parameters in Nature are tuned to special set of values so that the life with high inteligence can exist.
Example: if the value of Λ is positive and large, expansion of the universe is too fast and formation of galaxies becomes impossible. If Λ is negative and large, universe will collapse very quickly.

It is difficult to make a distinction between those which should be explained by a fundamental theory and those which are more accidental and do not have basic significance.
Example

- Size of orbits of solar planets are in (approximately) integral ratios.
- Orbit of each planet is an ellipse.

♣ Singular loci in Calabi-Yau moduli space

String vacua are not distributed uniformly but concentrated around singular points of $\mathcal{M}$.

Singular CY maniflods:

- conifold points $\longrightarrow$ massless matter
- ADE singularities over P^1 $\longrightarrow$ non-Abelian gauge symmetry
- Argyres-Douglas points $\longrightarrow$ conformal inv. theory
- Complex structure limit

We study the distribution of vacua around singular loci in CY moduli space.

T.E. and Y. Tachikawa hep-th/0510061

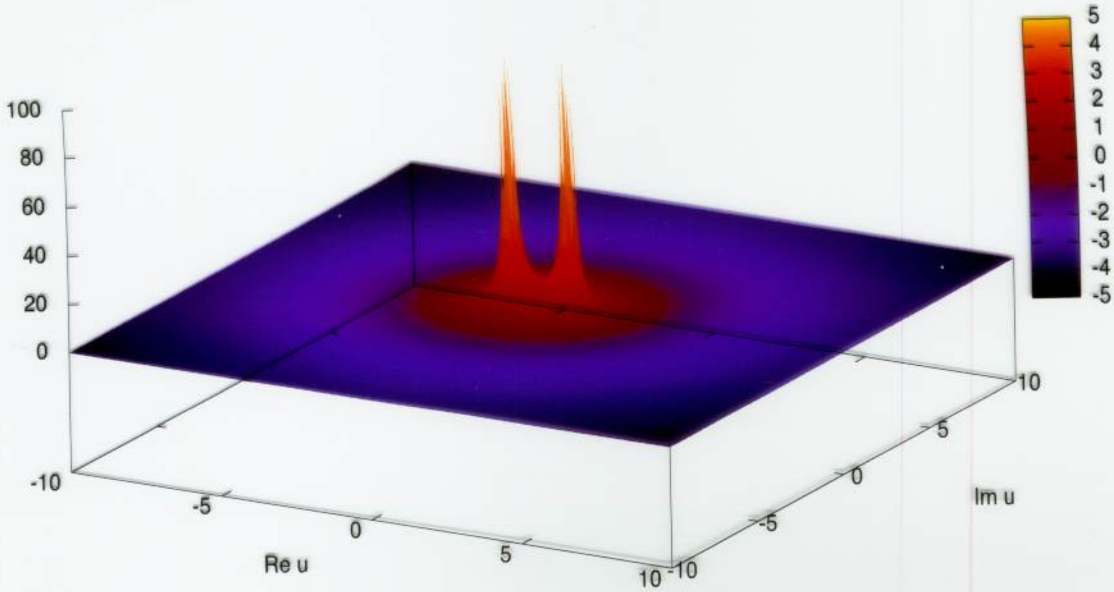
We claim that the vacuum density behaves generically as

$$\text{vacuum density} \approx \frac{dz d\bar{z}}{|z|^2 \log^p |z|}$$

(p; positive integer ≥ 2)

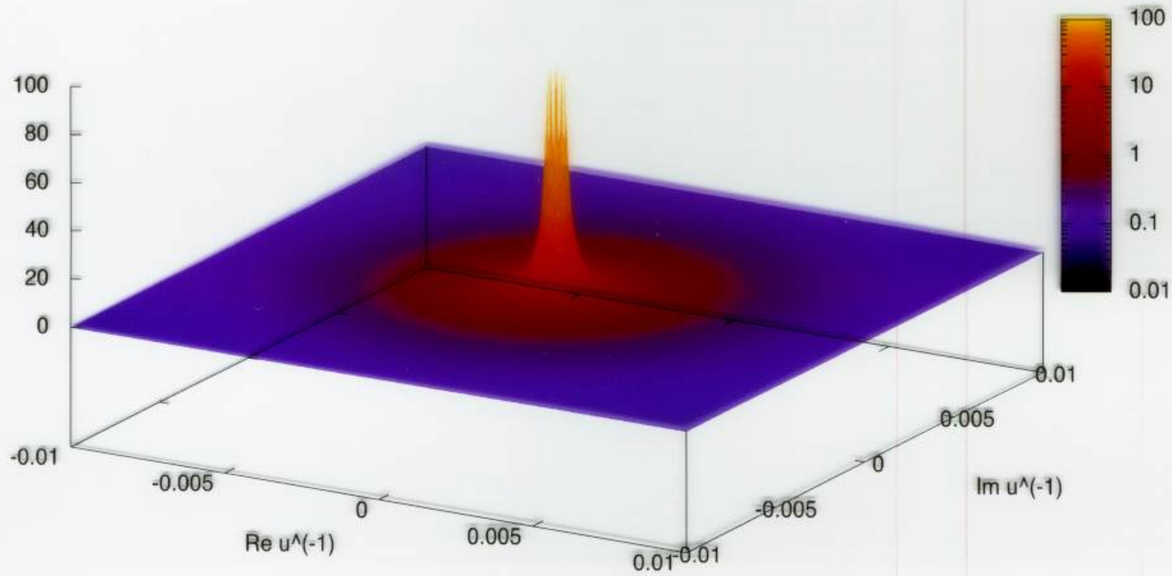
so that there exist a finite number of vacua around each singular locus.

$u \sim 0$



$$\det(R + \omega) \sim \frac{1}{|u-1|^2 (\log |u-1|)^2}$$

$u \sim \infty$



$$\det(R + \omega) \sim \frac{1}{|u|^{-2} (\log |u|)^4}$$