



*Predictions for LHC physics
from extra-dimensional gauge-Higgs unification*

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Higgs particles

To be discovered at LHC.

symmetry breaking

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

$$m_H^2 \sim 2\lambda v^2$$

Yukawa couplings $g_j \phi_H \bar{\psi}_j \psi_j$

→ fermion masses $m_j \sim g_j v$

many “ g_j ” , λ

No principle

Mass hierarchy

Ugly.

Gauge-Higgs unification

Fairlie 1979, Manton 1979
Hosotani 1983, 1989

In higher-dimensional gauge theory

$$A_M = (A_\mu , A_{y_j})$$

extra-dimensional components

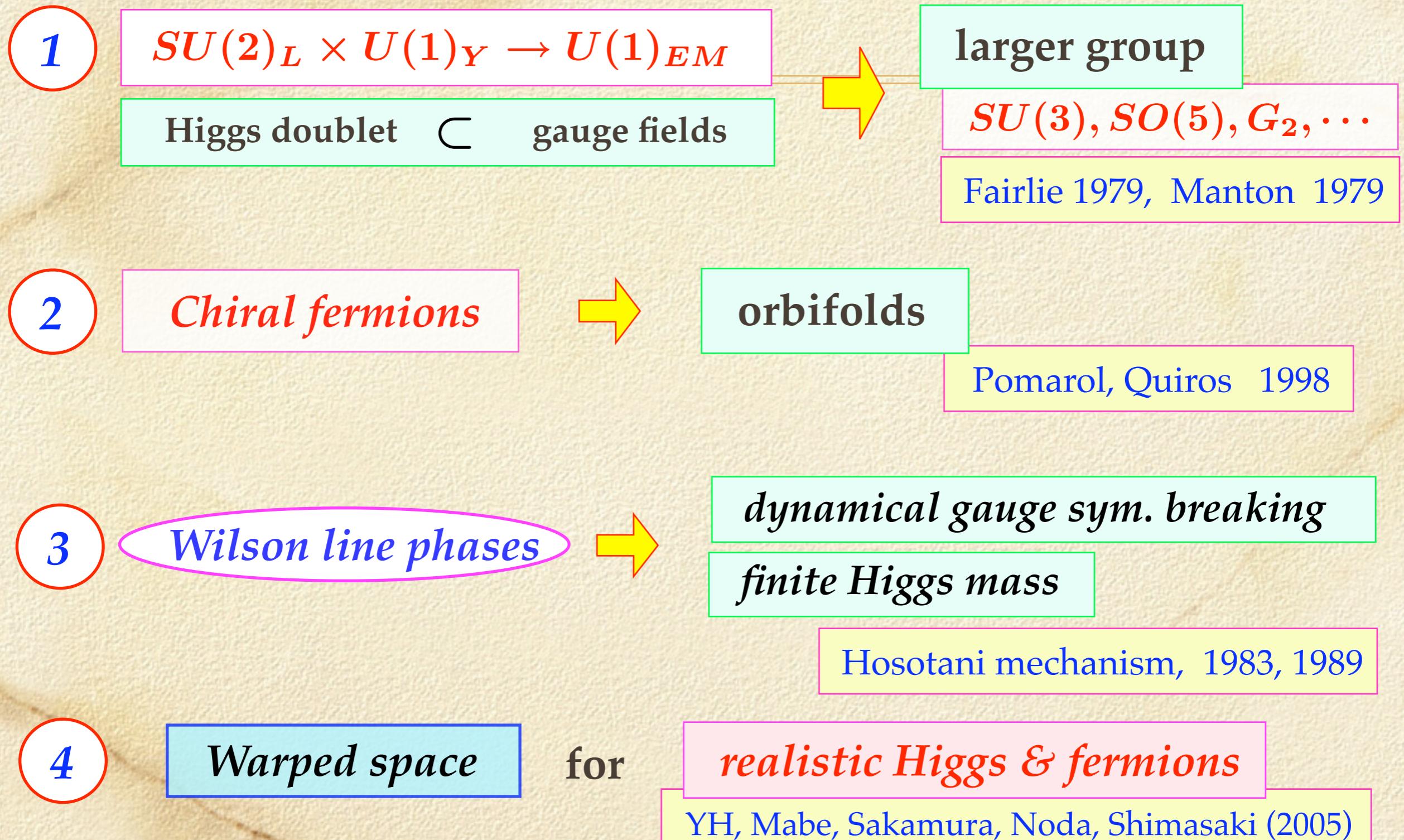
A_{y_j} serve as 4-d Higgs fields.

Higgs fields are unified with gauge fields.

on $M^4 \times S^1$
$$A_y(x, y) = \sum_{n=-\infty}^{\infty} A_y^{(n)}(x) e^{iny/R}$$

$A_y^{(0)}(x)$ Higgs fields in the adjoint rep.

Dynamical gauge-Higgs unification



Orbifolds

... extra dimensional space

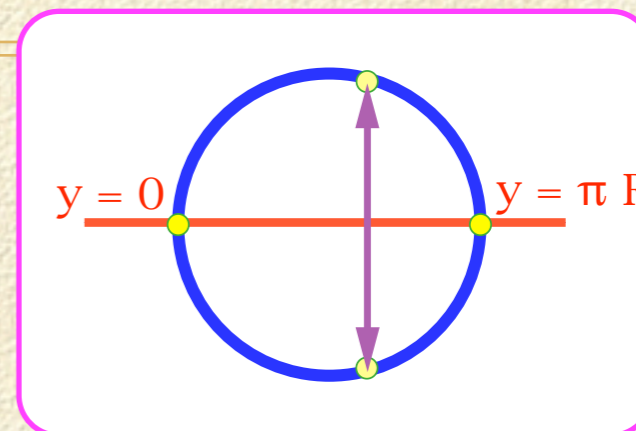
Pomarol, Quiros 1998

$$M^4 \times S^1$$



$$M^4 \times (S^1 / Z_2)$$

$$\begin{aligned} (x^\mu, +y) &\sim (x^\mu, -y) \\ (x^\mu, \pi R + y) &\sim (x^\mu, \pi R - y) \end{aligned}$$



Restriction - boundary conditions

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, -y) = P_0 \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y) P_0^\dagger \quad P_0 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & +1 \end{pmatrix}$$



Low energy gauge symmetry
Chiral fermions

Low energy theory

$$A_\mu = \left(\begin{array}{c} \square \\ \square \end{array} \right) \Rightarrow \frac{1}{\sqrt{\pi R}} \left\{ W_\mu(x), Z_\mu(x), A_\mu^{EM}(x) \right\}$$

$$A_y = \left(\begin{array}{c} \square \\ \square \end{array} \right) \Rightarrow \frac{1}{\sqrt{\pi R}} \Phi(x) \quad (\text{Higgs})$$

Chiral quarks and leptons

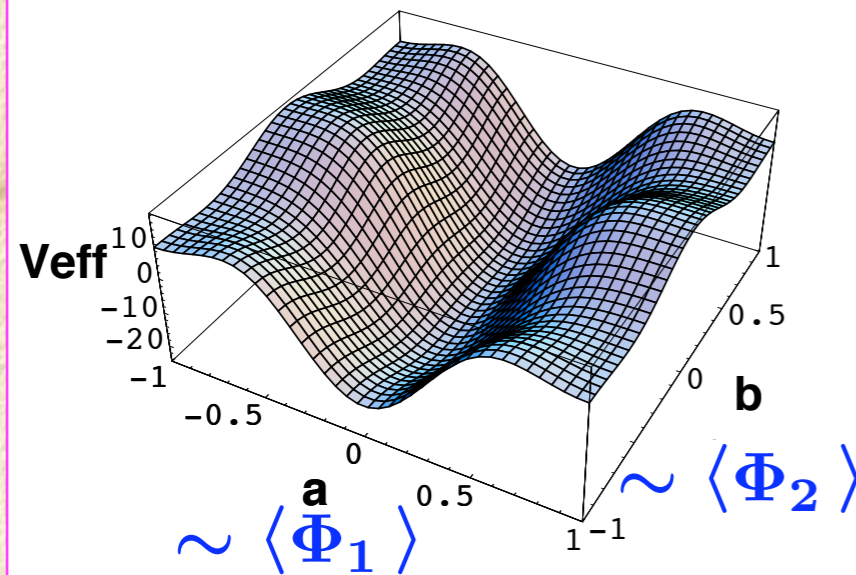
$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} e_R \\ u_R, d_R \end{matrix}$$

Example in *flat* space

$U(3)_S \times U(3)_W$ model

Antoniadis, Benakli, Quiros, NJP (2001)
 Hosotani, Noda, Takenaga, PLB (2005)

$$N_{Ad} = 1, N_F = 3$$



Dynamical gauge-Higgs unification is achieved.

W

$$m_W = 0.135 \times \frac{1}{R}$$

too small $M_{KK} = \frac{1}{R} \sim 10 m_W$

Higgs

$$m_H = 0.871 \times \sqrt{\frac{g_4^2}{4\pi}} m_W$$

too small $m_H \sim \sqrt{\frac{g_4^2}{4\pi}} m_W$

4

Warped space

for

realistic Higgs & fermions

YH, Mabe, Sakamura, Noda, Shimasaki (2005)

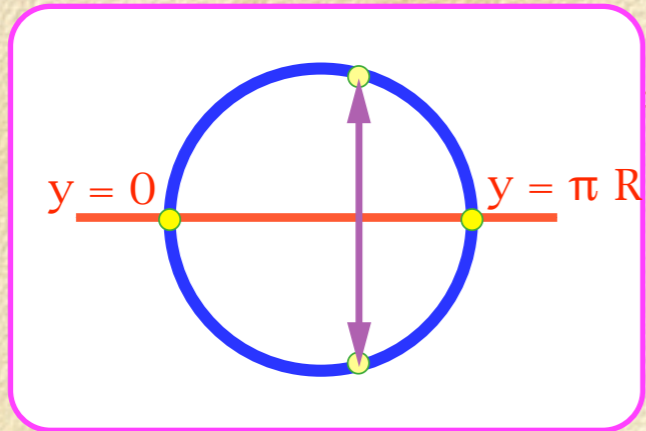
**The situation drastically changes for the better,
if the extra-dimensional space is curved and warped.**

Electroweak unification in warped space

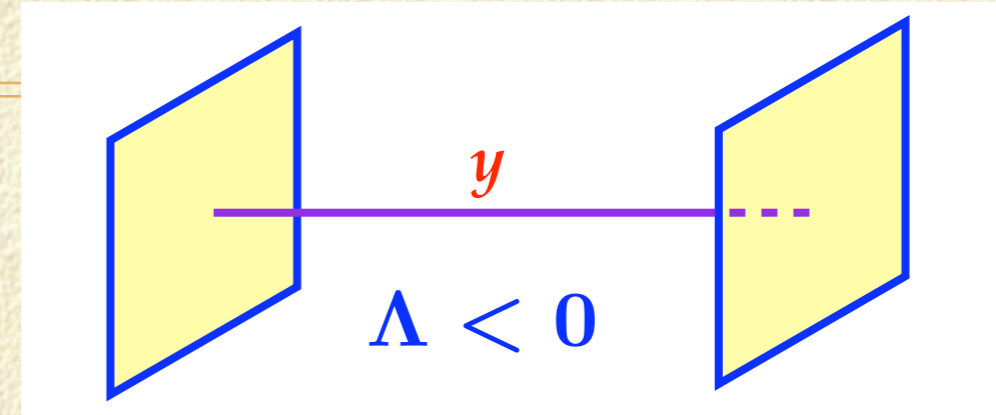
YH, Mabe (2005)

YH, Noda, Sakamura, Shimasaki (2005)

$$M^4 \times (S^1/Z_2)$$



Randall-Sundrum



$$ds^2 = dx_\mu dx^\mu + dy^2$$



$$ds^2 = e^{-2k|y|} dx_\mu dx^\mu + dy^2$$

$$M_{KK} \sim \frac{1}{R}$$



?

$$M_{KK} \sim 10 m_W$$

too small!



?

$$m_H \sim \sqrt{\alpha_w} m_W$$

too small!



?

KK mass scale

$$m_n \sim M_{KK} n \quad \text{for } m_n \gg k$$

$$M_{KK} \sim \frac{\pi k}{e^{\pi k R} - 1} = \begin{cases} 1/R & \text{as } k \rightarrow 0 \\ \pi k e^{-\pi k R} & \text{for } kR > 2 \end{cases}$$

Wilson line phase

($F_{MN} = 0$, but nontrivial)

$$A_w = \begin{pmatrix} & a \\ a & \end{pmatrix}: \quad 2g \int_{w_0}^{w_1} dw A_w \Rightarrow \theta_W = 2ga(w_1 - w_0)$$

$$(w = e^{2ky} = z^2, \quad w_0 = 1, \quad w_1 = e^{2\pi kR})$$

W-boson mass

$$\int d^4x dw (-2k) \text{Tr} F_{\mu w} F^{\mu}_w \Rightarrow m_W = \sqrt{\frac{2k}{\pi R}} e^{-\pi kR} \sin \frac{\theta_W}{2}$$

$$m_W, \theta_W = (0.2 \sim 0.4)\pi$$

$$k \sim M_{Pl}$$



$$kR \sim 12$$

Higgs field

$$\theta_W \leftrightarrow 2gA_w(w_1 - w_0) \Rightarrow g_4 \sqrt{\frac{\pi R(w_1 - w_0)}{2k}} (v + \phi)$$

... *fluctuations in θ_W*

$$V_{\text{eff}}(\theta_W) = \text{const} + \frac{1}{2}m_H^2\phi^2 + \frac{1}{3}\eta\phi^3 + \frac{1}{4}\lambda\phi^4 + \dots$$

| | <i>flat</i> R | <i>Randall-Sundrum</i> R, kR |
|-----------|--------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| M_{KK} | $2 \frac{\pi}{\theta_W} m_W$ | $\frac{\pi}{\sin(\theta_W/2)} \sqrt{\frac{\pi k R}{2}} m_W$ |
| m_H | $\sqrt{\frac{3}{32\pi} f^{(2)}(\theta_W)} \sqrt{\alpha_W} \frac{m_W}{\frac{1}{2}\theta_W}$ | $\sqrt{\frac{3}{32\pi} f^{(2)}(\theta_W)} \sqrt{\alpha_W} \frac{\pi k R}{2} \frac{m_W}{\sin \frac{1}{2}\theta_W}$ |
| λ | $\frac{\alpha_W^2}{16} f^{(4)}(\bar{\theta}_W)$ | $\frac{\alpha_W^2}{16} f^{(4)}(\bar{\theta}_W) \left(\frac{\pi k R}{2}\right)^2$ |

| | <i>flat</i> <i>R</i> | <i>Randall-Sundrum</i> <i>R</i> , $kR = 12$ |
|-----------------------------------------|----------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| M_{KK} | $2 \frac{\pi}{\theta_W} m_W$ 400 ~ 800 GeV | $\frac{\pi}{\sin(\theta_W/2)} \sqrt{\frac{\pi kR}{2}} m_W$ 1.8 ~ 3.5 TeV |
| $\theta_W^{min} : 0.2 \pi \sim 0.4 \pi$ | | |
| m_H | $\sqrt{\frac{3}{32\pi} f^{(2)}(\theta_W)} \sqrt{\alpha_W} \frac{m_W}{\frac{1}{2}\theta_W}$ 7.5 GeV ~ 15 GeV | $\sqrt{\frac{3}{32\pi} f^{(2)}(\theta_W)} \sqrt{\alpha_W} \frac{\pi kR}{2} \frac{m_W}{\sin \frac{1}{2}\theta_W}$ 150 ~ 285 GeV |
| λ | $\frac{\alpha_W^2}{16} f^{(4)}(\bar{\theta}_W)$ 0.0008 | $\frac{\alpha_W^2}{16} f^{(4)}(\bar{\theta}_W) \left(\frac{\pi kR}{2}\right)^2$ 0.3 |

Quarks and leptons

$$\mathcal{L} = -\bar{\psi} i \Gamma^a e_a^M \left\{ \partial_M + \frac{1}{8} \omega_{bcM} [\Gamma^b, \Gamma^c] - ig A_M \right\} \psi - ic \sigma' \bar{\psi} \psi$$

bulk (kink) mass
Gherghetta-Pomarol (2000)

Zero (massless) modes exist only for 'quarks/leptons'.

$$\begin{pmatrix} \nu_L \\ e_L \\ \tilde{e}_L \end{pmatrix} \begin{pmatrix} \tilde{\nu}_R \\ \tilde{e}_R \\ e_R \end{pmatrix} \begin{pmatrix} b_R^c \\ t_R^c \\ \tilde{t}_R^c \end{pmatrix} \begin{pmatrix} \tilde{b}_L^c \\ \tilde{t}_L^c \\ t_L^c \end{pmatrix}$$

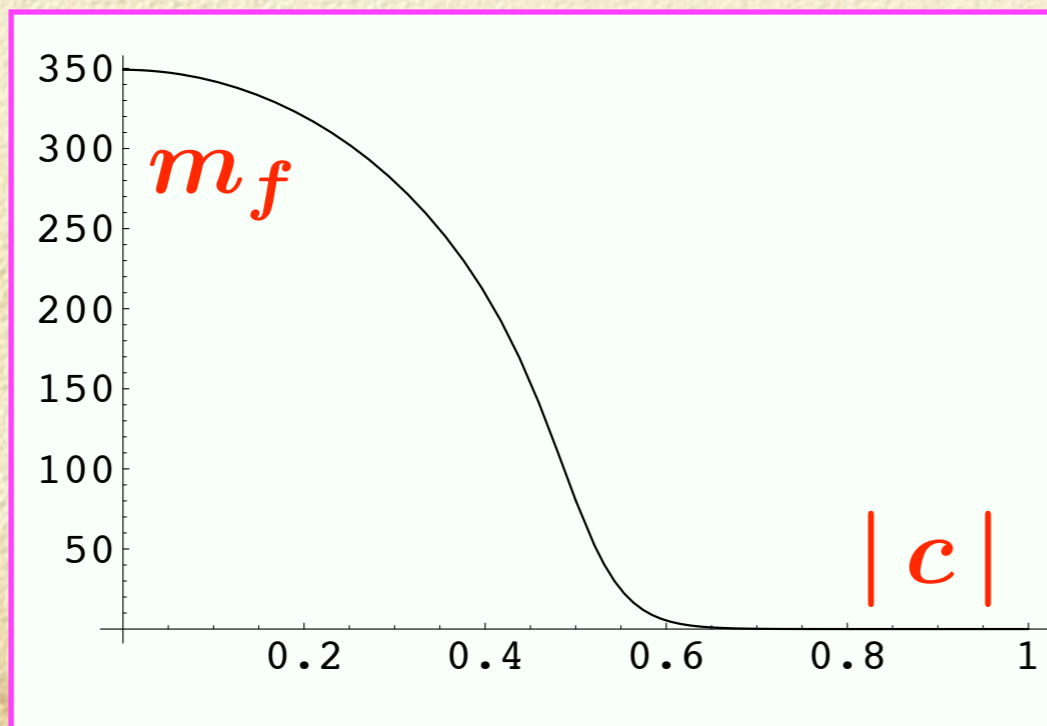
$$f_0^{(\pm)}(z) = \sqrt{\frac{1 \mp 2c}{e^{(1 \mp 2c)\pi k R} - 1}} z^{\mp c}$$

Fermion masses

Gauge interactions $g \bar{\psi} \Gamma^5 e_5^z \langle A_z \rangle \psi$



$$m_f = \sqrt{\frac{\pi k R}{2} \cdot \frac{(1 - 4c^2)(z_1^2 - 1)}{(z_1^{1-2c} - 1)(z_1^{1+2c} - 1)}} m_W$$



| | $ c $ |
|----------|-------|
| m_e | 0.87 |
| m_μ | 0.71 |
| m_τ | 0.63 |
| m_u | 0.81 |
| m_c | 0.64 |
| m_t | 0.43 |

Explains the hierarchy.

Non-universality of weak interactions

Gauge interactions $g \bar{\psi} \Gamma^a e_a^\mu A_\mu \psi(x, z)$



$$g_4(\theta_W, c) \bar{e}_L(x) \gamma^\mu \nu_L(x) W_\mu(x)$$

$$\frac{g_4(\theta_W, c)}{g_4(\theta_W, c_e)}$$

| θ_W | μ | τ | t |
|------------|---------|----------|-------|
| 0 | 1 | 1 | 1 |
| π | 1.00000 | 0.999988 | 0.951 |

When $SU(2)_L$ is broken,
weak interactions are **not universal** in 4-d effective theory.

Summary

Dynamical gauge-Higgs unification is promising.

| | |
|-----------|---------------|
| M_{KK} | 1.8 ~ 3.5 TeV |
| m_H | 150 ~ 285 GeV |
| λ | 0.3 |

$$\frac{m_H^2}{2v^2} = (1.9 \sim 7.6) \lambda$$

Higgs predicted at the LHC energies !!

Natural fermion-mass hierarchy

| | | |
|-------------|-------|-------|
| <i>mass</i> | m_e | m_t |
| <i>c</i> | 0.87 | 0.43 |

Non-universality

| | | | |
|------------|---------|----------|-------|
| | e | τ | t |
| g_{weak} | 1.00000 | 0.999988 | 0.951 |

A man wearing a tan bucket hat, glasses, a tan jacket, and a backpack is sitting on a rock by a lake. He is smiling and looking towards the camera. The background features rugged mountains with patches of snow under a clear blue sky. The lake is calm and reflects the surrounding landscape.

We might be able to see the EXTRA dimensions !