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Dynamical Electroweak Symmetry Breaking from Extra Dimensions*

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Abstract

We study the dynamical electroweak symmetry breaking (DEWSB) in the $D(=6,8,\cdots)$-dimensional bulk with compactified extra dimensions. We identify the critical binding strength for triggering the DEWSB, based on the ladder Schwinger-Dyson equation. In the top mode standard model with extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in the bulk, we analyze the most attractive channel (MAC) by using renormalization group equations (RGEs) of (dimensionless) bulk gauge couplings and determine the effective cutoff where the MAC coupling exceeds the critical value. We then find that the top-condensation can take place for $D=8$. Combining RGEs of top-Yukawa and Higgs-quartic couplings with compositeness conditions, we predict the top mass, $m_t = 173-180$ GeV, and the Higgs mass, $m_H = 181-211$ GeV, for $D=8$, where we took the universal compactification scale $1/R = 1-100$ TeV.

1 Introduction

The origin of mass, or the electroweak symmetry breaking (EWSB) is one of the most urgent problems of the particle physics today. There are some dynamical approaches toward this problem such as Technicolor [1], top-quark condensate [2, 3], etc. [4]. In the top-quark condensate or the "top mode standard model" (TMSM), the scalar bound state of $\bar{t}t$ plays the role of the Higgs boson in the SM and the top quark naturally acquires the dynamical mass of the order of the EWSB scale. However, the original version of the TMSM [2] has some problems: The 4-fermion interaction of the top-quark was introduced by hand in order to trigger the EWSB. The top-quark mass m_t was predicted somewhat larger than the experimental value, $m_t \gtrsim 200$GeV, even if we take the cutoff to the Planck or the GUT scale. [2, 5, 6] Such a huge cutoff also causes a serious fine-tuning problem.

Recently, Arkani-Hamed, Cheng, Dobrescu, and Hall (ACDH) [7] proposed an interesting version of TMSM in extra dimensions, where the SM gauge bosons and some generations of quarks and leptons are embedded in $D(=6,8,\cdots)$ dimensions. Since bulk gauge interactions become naively non-perturbative in the high-energy region, the bulk QCD may trigger the top-condensation without introducing ad hoc 4-fermion interactions. Moreover, all condensates of Kaluza-Klein (KK) modes of the top quark contribute to the EWSB, thereby suppressing the predicted value of m_t.

However, we have found that the bulk QCD coupling has an upper bound. [8] It is thus nontrivial whether the EWSB dynamically takes place due to the bulk QCD or not. We thus studied the dynamics of bulk gauge theories, based on the ladder Schwinger-Dyson (SD) equation. [8, 9] By this we estimated the critical value of the binding strength $\kappa_D^{\rm crit}$, which particularly disfavored the top-quark condensate in the $D=6$ case of the ACDH version. (See Sec. 2.)

Now, in order for the ACDH version to work phenomenologically, the top-condensate should be the most attractive channel (MAC) [10] and its binding strength κ_t at the cutoff Λ should exceed $\kappa_D^{\rm crit}$, whereas those of other bound states such as the tau-condensate (κ_τ) should not:

$$\kappa_t(\Lambda) > \kappa_D^{\rm crit} > \kappa_\tau(\Lambda), \cdots . \tag{1}$$

Comparing our estimations of $\kappa_D^{\rm crit}$ with the binding strengths obeying the renormalization group equations (RGEs) of the gauge couplings, we determine the cutoff Λ satisfying Eq. (1) and thereby predict the top mass as well as the Higgs mass. [11] In fact we can obtain a certain region of the effective cutoff Λ satisfying Eq. (1) for $D = 8$, while the top-condensation is unlikely to occur for $D = 6$. (See Sec. 3.) This is in sharp contrast to the earlier approaches of ACDH [7] and Kobakhidze [12] where the cutoff Λ is treated as an adjustable parameter.

In Sec. 4, we predict masses of the top quark and the Higgs boson in the formulation á la Bardeen, Hill, and Lindner (BHL) [5] based on RGEs and compositeness conditions. The value of m_t around the universal compactification scale $1/R$ is governed by the quasi infrared-fixed point (IR-FP) for the top-Yukawa coupling y_* [13]. The behavior of y_* is approximately given by $y_*^2/g_3^2 = C_F(6+\delta)/(2^{\delta/2}N_c + 3/2)$, where $C_F = (N_c^2 - 1)/(2N_c)$, $N_c = 3$, $\delta \equiv D - 4$, and g_3 is the conventional QCD coupling. Thus, the prediction of m_t can be suppressed as δ increases. We predict numerically the top mass, $m_t = 173 - 180$ GeV, and the Higgs mass, $m_H = 181 - 211$ GeV, for $D = 8$, where we took $1/R = 1 - 100$ TeV.

2 Analysis of the ladder SD equation

We consider that the SM gauge group and the third generation of quarks and leptons are put in the $D(= 6, 8, \cdots)$-dimensional bulk, while the first and second generations are confined in 3-brane (4-dimensions). We assume that four of D-dimensions are the usual Minkowski spacetime and extra δ spatial dimensions are compactified at a universal scale $1/R \sim \mathcal{O}(\text{TeV})$.

Before analyzing the ladder SD equation, we study running effects of dimensionless bulk gauge couplings $\hat{g}_i$ ($i = 3, 2, Y$). Above the compactification scale $1/R$, we should take into account contributions of Kaluza-Klein (KK) modes. We find approximately the total number of KK modes $N_{\rm KK}(\mu)$ below the renormalization point μ,

$$N_{\rm KK}(\mu) = \frac{1}{2^n}\frac{\pi^{\delta/2}}{\Gamma(1+\delta/2)}(\mu R)^\delta, \quad (\mu \gg 1/R) \tag{2}$$

with the orbifold compactification T^δ/Z_2^n, where we take Z_2 and $Z_2 \times Z_2'$ projections for $D = 6$ and $D = 8$, respectively. The dimensionfull bulk gauge coupling g_D is related to the conventional 4-dimensional gauge coupling g as $g_D^2 = g^2 \cdot (2\pi R)^\delta/2^n$. Combining

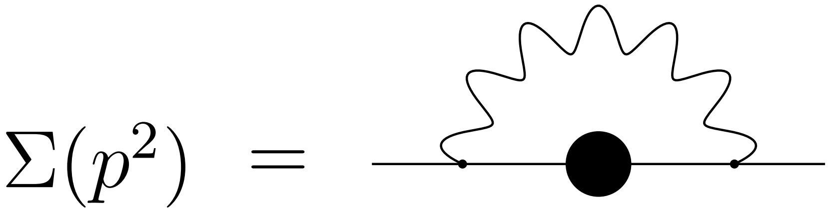

Figure 1: The ladder SD equation. The solid and wavy lines with and without the shaded blob represent the full propagator of the fermion and the bare one of the gauge boson, respectively. The mass function of the fermion is written as $\Sigma(p^2)$ with the external (Euclidean) momentum p.

the definition of $\hat{g}(\equiv g_D \mu^{\delta/2})$ with RGEs for g_i and Eq. (2), we obtain approximately RGEs for $\hat{g}_i$,

$$\mu \frac{d}{d\mu}\hat{g}_i = \frac{\delta}{2}\hat{g}_i + (1+\delta/2)\Omega_{\rm NDA} b'_i \,\hat{g}_i^3, \quad (\mu \gg 1/R) \tag{3}$$

with $\Omega_{\rm NDA} \equiv [(4\pi)^{D/2}\Gamma(D/2)]^{-1}$. RGE coefficients b'_3 and b'_Y are given by

$$b'_3 = -11 + \frac{\delta}{2} + \frac{4}{3}\cdot 2^{\delta/2}\cdot n_g, \qquad b'_Y = \frac{20}{9}\cdot 2^{\delta/2}\cdot n_g + \frac{1}{6} n_H, \tag{4}$$

where n_g (n_H) denotes number of generations (composite Higgs bosons) in the bulk. Hereafter, we assume $n_g = 1, n_H = 1$. We note $b'_3 < 0$ for $D = 6, n_g = 1, 2, 3$ and $D = 8, n_g = 1$. We find that the bulk QCD coupling with $b'_3 < 0$ has the ultraviolet fixed-point (UV-FP) g_{3*},

$$g_{3*}^2 \Omega_{\rm NDA} = \frac{1}{-(1+2/\delta)\, b'_3} \tag{5}$$

by using Eq. (3). We can also show that the UV-FP is the upper bound of $\hat{g}_3^2$. We thus need to investigate whether the bulk QCD coupling can be sufficiently large to trigger the EWSB or not.

Let us investigate the condition for the EWSB triggered by the bulk QCD. The D-dimensional ladder SD equation is given by Fig. 1:

$$\Sigma(p^2) = \int \frac{d^D q}{(2\pi)^D} \frac{\Sigma(q^2)}{q^2 + \Sigma(q^2)} \frac{(D-1) C_F g_D^2}{(p-q)^2} \tag{6}$$

with the mass function Σ and Euclidean momenta p, q, where we took the Landau gauge in order to make the wave function renormalization identically unity. [8] For simplicity, we incorporate running effects of the bulk QCD as $\kappa_D(\mu) \equiv C_F \hat{g}_3^2(\mu)\Omega_{\rm NDA} = \text{const.}$, which is closely related to the MAC coupling, just on the UV-FP. This simplification obviously makes the critical point lower. We then obtain the critical binding strength $\kappa_D^{\rm crit}$:

$$\kappa_6^{\rm crit} \simeq 0.122, \quad \kappa_8^{\rm crit} \simeq 0.146 \tag{7}$$

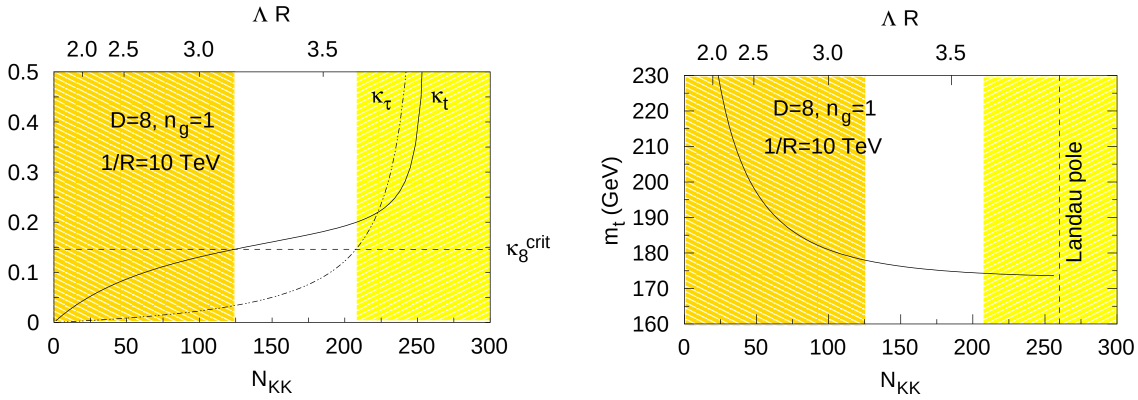

Figure 2: Effective cutoff Λ for the top-condensation in the bulk and prediction of the top-quark mass m_t (GeV) in $D = 8$. The unshaded regions satisfy $\kappa_t(\Lambda) > \kappa_D^{\text{crit}} > \kappa_\tau(\Lambda)$. The L.H.S. and R.H.S. graphs show behaviors of $\kappa_{t,\tau}$ and m_t for various cutoffs Λ, respectively, where we took $1/R = 10$ TeV. The top and bottom lines represent ΛR and $N_{\text{KK}}(\Lambda)$, respectively, where we used Eq. (2) in the estimation of $N_{\text{KK}}(\Lambda)$.

for $D = 6$ and $D = 8$, respectively. [8] These are minimal values within uncertainties of the ladder SD equation. [8, 9] The critical points are unlikely to be smaller than the above values, even if we take into account the effect that the cutoff Λ is not so large in fact as compared with the compactification scale $1/R$. [8, 9] We use most conservatively the values of Eq. (7) in the following analysis.

The upper bounds of the bulk QCD coupling in the ACDH scenario are found as $\kappa_{6,8} = 0.091, 0.242$ for $D = 6, 8$. [8] Thus, the top-condensation is unlikely to occur for $D = 6$.

3 Conditions for the top-condensation in the bulk

We analyze the MAC at the cutoff Λ by using RGEs of bulk gauge couplings. In the one-gauge-boson-exchange approximation, we can easily obtain $\kappa_{t,\tau}$:

$$\kappa_t(\mu) = C_F \hat{g}_3^2(\mu)\Omega_{\text{NDA}} + \frac{1}{9}\hat{g}_Y^2(\mu)\Omega_{\text{NDA}}, \tag{8}$$

$$\kappa_\tau(\mu) = \frac{1}{2}\hat{g}_Y^2(\mu)\Omega_{\text{NDA}}. \tag{9}$$

The MAC is the top (tau)-condensate, when the bulk QCD (hypercharge) is dominant.

Now, we are ready to compare $\kappa_t(\Lambda)$ and $\kappa_\tau(\Lambda)$ with the critical values Eq. (7). Unless the MAC coupling exceeds at least κ_D^{crit} estimated in Eq. (7), any condensates cannot be generated in the bulk. When Eq. (1) is satisfied, the top-quark acquires the large dynamical mass, whereas the tau-lepton still remains massless. We show our results in Fig. 2. Actually, Eq. (1) can be satisfied for $D = 8$. We can confirm that the top-condensation is not favored for $D = 6$. We also note that behaviors of $\kappa_{t,\tau}$ are almost unchanged for $1/R = 1 - 100$ TeV.

4 Predictions of m_t and m_H

In the same way as the approach of BHL, we can expect to reproduce the SM in the bulk in the energy scale between $1/R$ and Λ:

$$\mathcal{L}_D = \mathcal{L}_{\rm kin} - y(\bar{q}_L H t_R + {\rm h.c.}) + |D_M H|^2 - m_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \tag{10}$$

with $M = 0, 1, 2, 3, 5, \cdots, D$ and the kinetic term $\mathcal{L}_{\rm kin}$ for the top quark and gauge bosons. Since we find the RGE for the top-Yukawa coupling y,

$$\begin{aligned}(4\pi)^2 \mu \frac{dy}{d\mu} &= N_{\rm KK}(\mu)\, y \left[\left(2^{\delta/2} \cdot N_c + \frac{3}{2}\right) y^2 \right. \\ &\left. -C_F(6+\delta) g_3^2 - \frac{3}{4}(3 - \delta/2) g_2^2 - \frac{(102-\delta)}{72} g_Y^2 \right], \end{aligned}\tag{11}$$

and that for the Higgs-quartic coupling λ,

$$\begin{aligned}(4\pi)^2 \mu \frac{d\lambda}{d\mu} &= N_{\rm KK}(\mu) \left[2^{2+\delta/2} \cdot N_c \left(\lambda y^2 - y^4\right) + 12\lambda^2 \right. \\ &\left. + \frac{3+\delta}{4}(3g_2^4 + 2g_2^2 g_Y^2 + g_Y^4) - 3(3g_2^2 + g_Y^2)\lambda \right], \end{aligned}\tag{12}$$

we can predict m_t and m_H by using compositeness conditions [5],

$$y(\Lambda) \to \infty, \quad \frac{\lambda(\Lambda)}{y(\Lambda)^4} \to 0. \tag{13}$$

Since the running effect of the QCD coupling g_3 is almost negligible around $1/R$, the top-Yukawa coupling in $\mu \sim 1/R$ is attracted toward the quasi IR-FP y_* [13], whose behavior is approximately found as

$$y_* = g_3 \cdot \sqrt{\frac{C_F(6+\delta)}{2^{\delta/2} N_c + \frac{3}{2}}}, \tag{14}$$

neglecting effects of the electroweak interactions. This value obviously decreases as δ increases. As a result, the problem of $m_t \gtrsim 200$ GeV in $D = 4$ is suppressed in our scenario.

Now, we predict m_t and m_H. (See Figs. 2 and 3.) We obtain numerically the top-quark mass m_t as

$$m_t = 173 - 180 \text{ GeV}, \tag{15}$$

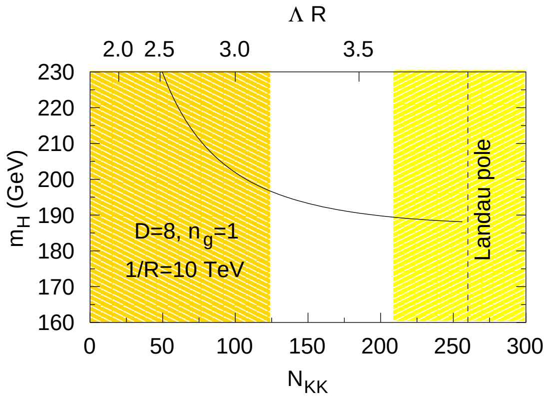

Figure 3: Prediction of the Higgs boson mass m_H for $D = 8, R^{-1} = 10$ TeV.

and the Higgs boson mass m_H as

$$m_H = 181 - 211 \text{ GeV} \tag{16}$$

for $D = 8$, where we took $1/R = 1 - 100$ TeV. For details, see Ref. [11].

5 Summary

We have investigated the idea that the EWSB dynamically occurs due to the top-condensation in the bulk, with emphasis on the dynamics of bulk gauge theories. We estimated the critical binding strength $\kappa_D^{\rm crit}$, based on the ladder SD equation. We also analyzed the MAC by using RGEs for bulk gauge couplings. Combining our MAC analysis with $\kappa_D^{\rm crit}$, we showed that the top-condensation is favored for $D = 8$, while it is not for $D = 6$. We have predicted m_t and m_H in the approach of BHL. We then obtained $m_t = 173 - 180$ GeV, and $m_H = 181 - 211$ GeV for $D = 8$.

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