THE EXTRA-TALK ON

SEARCH FOR QUANTUM GRAVITY EFFECTS IN EXTRA-DIMENSIONS AT e^+e^- COLLIDERS

PRESENT AND FUTURE

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NEW THEORY,

WHICH MIGHT BE A NEW
PARADIGM OR
JUST A GARBAGE,

SHOULD BE TESTED EXPERIMENTALLY.

THE NEW THEORY, WHICH
I AM GOING TO EXPLAIN,
IS A <u>TESTABLE</u> THEORY.

SOME ANALYSES HAVE
BEEN DONE USING OPAL
LEP2 DATA.

A FEW COMMENTS FOR e^+e^-
LINEAR COLLIDER ANALYSIS.

MOTIVATION OF THE THEORY.

Hierarchy Problem

Gravitational Interaction is very weak compared to the strength of the EW interaction.

$$G_N = \frac{1}{M_{Pl}^2} = 6.707 \times 10^{-39} \hbar c \, (GeV^2/c^2)^{-1}$$

$$M_{Pl} \sim 10^{18-19} \text{ GeV}$$

$$M_{EW} \sim 10^{2\sim 3} \text{ GeV}$$

$$\frac{M_{Pl}}{M_{EW}} \sim 10^{16}$$

WHY THERE ARE SUCH DIFFERENT 2 FUNDAMENTAL SCALES IN THE NATURE ?

EW interaction has been proben at the distance of $m_{EW}^{-1} \sim 10^{-17}$ cm

Gravity has not been proben at the distance of $(M_{Pl})^{-1} \sim 10^{-32}$ cm

It is only proben down to $\lesssim 1$ cm.

Extrapolating the gravity interaction from 1 cm to 10^{-32} cm is a very strong assumption.

New theory_ Arkani-Hamed, Dimopoulos, Dvali

M_{EW} IS THE ONLY FUNDAMENTAL SHORT DISTANCE SCALE.

GRAVITY IS AS STRONG AS EW INTERACTION IN HIGHER DIMENSION SPACE. (4+n dim.)

IN 4+n DIMENSION SPACE PLANCK MASS ≈ EW SCALE

NEWTONIAN FORCE IN 1+3 DIM. SPACE

$$F = G_N \frac{m_1 m_2}{r^2} = \boxed{\frac{1}{M_{pl}^2}} \frac{m_1 m_2}{r^2}$$

EXTRA n DIMENSION SPACE IS COMPACTIFIED WITH RADIUS R

NEWTONIAN FORCE FOR $r \ll R$ IN $4+n$ DIMENSION IS

$$F = \frac{1}{M_D^{n+2}} \frac{m_1 m_2}{r^{n+2}} \Leftarrow \text{1+3+n DIM. GAUSS'S LAW}$$

$M_D \equiv M_{pl(n+4)} \approx M_{EW}$

FOR LARGER r $\quad r > R$

r IS FROZEN TO R FOR THE EXTRA-DIMENSIONS

$$F = \boxed{\frac{1}{M_D^{n+2}} \frac{1}{R^n}} \frac{m_1 m_2}{r^2}$$

$F \sim \frac{1}{r^2}$ SHOULD BE RESTORED

$$\boxed{M_{pl}^2 = M_D^{n+2} R^n}$$

$M_D \equiv M_{pl(n+4)}$

$$M_{Pl}^2 = M_D^{2+n} \cdot R^n \qquad n = 1, 2, \cdots$$

$$M_D \equiv M_{Pl(4+n)} \sim M_{EW}$$

$$\Rightarrow R = (M_{Pl})^{\frac{2}{n}} (M_D)^{-(\frac{2}{n}+1)}$$

$$\sim 10^{\frac{30}{n}-17} \left(\frac{1\,TeV}{M_{EW}}\right)^{1+\frac{2}{n}} \; [cm]$$

$n=1 \qquad R \sim 10^{13}\,cm \qquad$ TOO LARGE

$\qquad\qquad \gg 1\,cm \qquad$ EXCLUDED

$n=2 \qquad R \sim 1\,mm$

$n=7 \qquad R \sim 1\,fm \qquad$ (Superstring M-theory $4+n \leq 11$)

COMPACTIFICASION SCALE

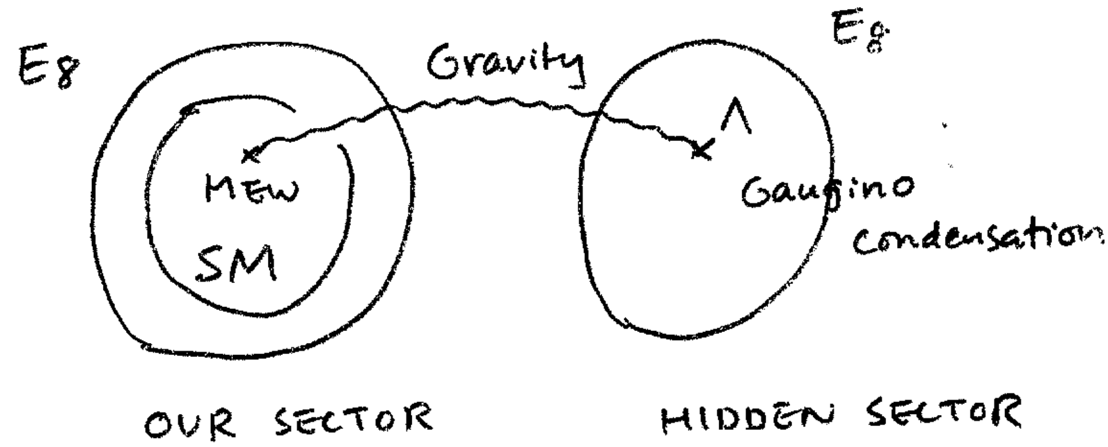
$$R \gg \frac{1}{M_{Pl}} \sim 10^{-32}\,cm$$

LARGE VALUE OF M_{Pl} IN OUR 1+3 DIM SPACE IS DUE TO THE LARGE SIZE OF R.

criticism

why R is so large ?

c.f. SUPERGRAVITY



$$M_{EW} \sim \frac{\Lambda^2}{M_{Pl}}$$

For $\Lambda \sim 10^{11}$ GeV

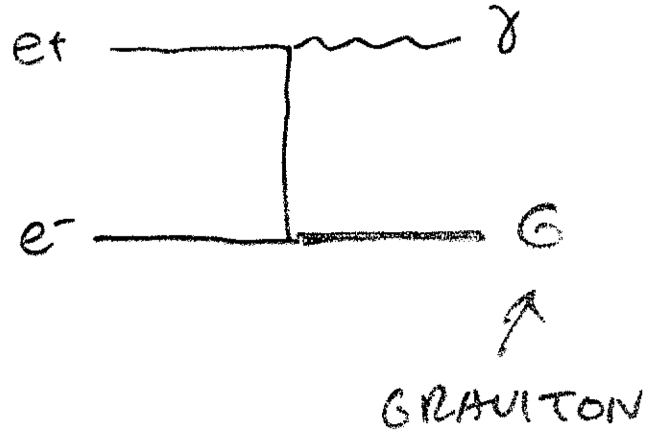
$\Rightarrow M_{EW} \sim 1$ TeV

Mpl IS THE FUNDAMENTAL MASS SCALE.

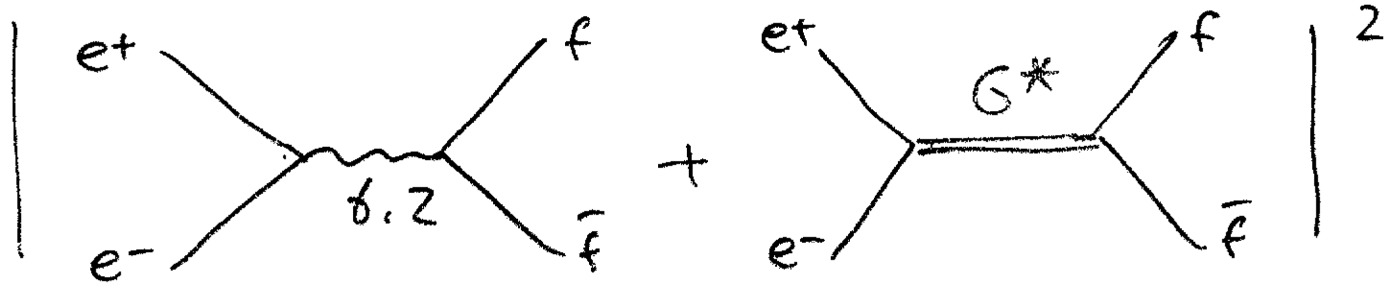
SMALLNESS OF THE EW SCALE IS DUE TO "SEE-SAW MECANISM."

SINCE Λ IS IN THE HIDDEN SECTOR, YOU CAN ASSUME ITS SIZE AS YOU LIKE.

NORMALLY (in 1+3 DIM), GRAVITATONAL INTERACTION IS TINY.



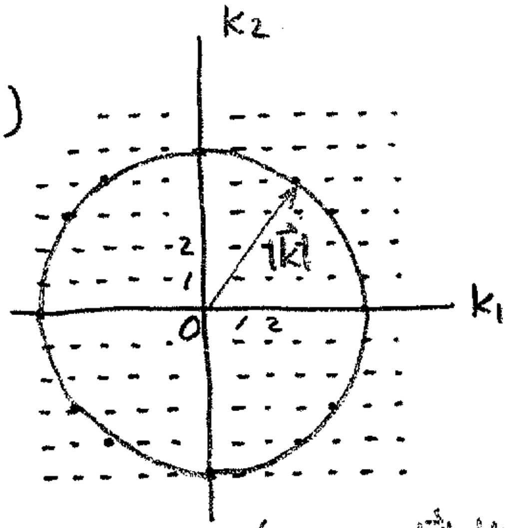
$$\sigma \propto \alpha G_N$$
$$\approx 10^{-66}\ \mathrm{cm}^{-2}$$
$$= 10^{-30}\ \mathrm{pb}$$



Interference $\sim \alpha G_N$

WHY THESE σ'S CAN BE OBSERVABLE IN THE NEW THEORY.

$$m^2 = q_1^2 + \cdots + q_n^2$$
$$= \frac{1}{R^2}(k_1^2 + \cdots + k_n^2)$$
$$= \frac{|\vec{k}|^2}{R^2}$$



Number of states in $|\vec{k}| \sim |\vec{k}| + dk$

$$\boxed{dN = S_{n-1} |\vec{k}|^{n-1} dk}$$

($dN = 2\pi |\vec{k}| dk$ for $n=2$)

$$\left(\begin{array}{l} S_{n-1} = 2\pi^{n/2} / \Gamma(n/2) \\ |\vec{k}| = R \cdot m \quad \to dk = R\,dm \end{array} \right)$$

$$\Rightarrow \boxed{\begin{aligned} dN &= S_{n-1} \, R^n \, m^{n-1} \, dm \\ &= S_{n-1} \frac{M_{Pl}^2}{M_D^{n+2}} m^{n-1} \, dm \end{aligned}} \Leftarrow \quad M_{Pl}^2 = R^n M_D^{n+2}$$

FOR HIGH ENERGY GRAVITATIONAL PROCESSE ONE HAVE TO INTEGRATE OVER ALL THE GRAVITON STATES

$$\boxed{\frac{d\sigma}{dz} = \int_0^{m_{max}} S_{n-1} \frac{M_{Pl}^2}{M_D^{n+2}} m^{n-1} \underbrace{\left[\frac{d^2\sigma}{dm\,dz}\right]}_{\sim \frac{1}{M_{Pl}^2}} dm}$$

← very small

For $e^+e^- \to G\gamma$ $\quad m_{max} = \sqrt{s}$

for $e^+e^- \to G^* \to f\bar{f}$ $\quad m_{max} = \Lambda_{cut\ off} \sim M_D$

PROBLEMS IN COSMOLOGY

1) INFLATION IS HARD TO WORK.

2) UNIVERSE CAN BE OVERCLOSED.

⋮

THE THEORY CAN BE TESTED AT ANY HIGH ENERGY COLLIDERS.
($\sqrt{s}$ > EW SCALE)

e^+e^- COLLIDERS

1) DIRECT PROCESSES

$$e^+e^- \rightarrow \gamma\, G$$

INVISIBLE WITH $m < \sqrt{s}$

$$e^+e^- \rightarrow Z\, G$$

$m < \sqrt{s} - M_Z$

σ's DEPEND ON THE NUMBERS OF EXTRA-DIMENSIONS n.

2) INDIRECT PROCESSES

$$e^+e^- \rightarrow G^* \rightarrow \ell^+\ell^-,\ q\bar{q},\ \gamma\gamma,\ gg, \dots$$

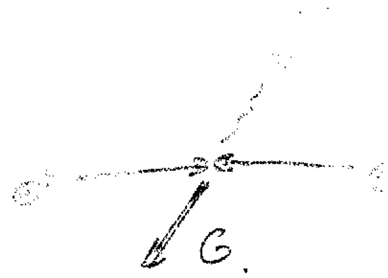
CUT OFF $\approx$ EW SCALE

σ's ALMOST INDEPENDENT OF n.

Physics Processes

1) GRAVITON PRODUCTION

$e^+e^- \rightarrow \gamma G$



Giudice et al CERN
Mirabelli et al. SLAC

ALTHOUGH γG IS TWO BODY FINAL STATE, THE DIFFERENTIAL CROSS SECTION DEPENDS ON THE TWO KINEMATIC VARIABLES, DUE TO $\approx$ CONTINUOUS DISTRIBUTION OF GRAVITON MASS

Spin average

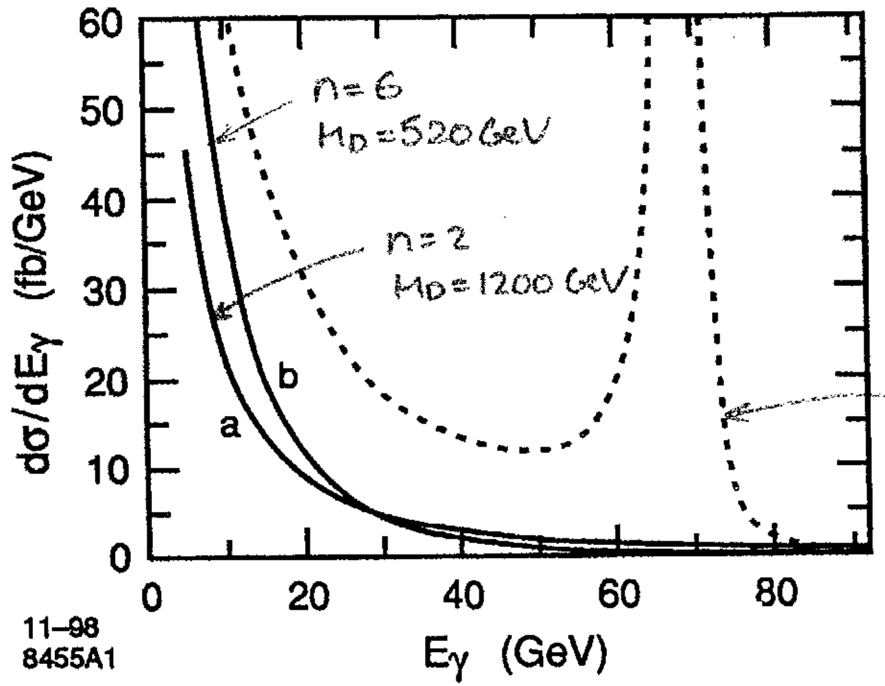
$$\frac{d^2\sigma}{dz\,dm^2} = \frac{\Omega_n}{8\pi \cancel{G_N}} \frac{1}{M_D^{n+2}} (m^2)^{\frac{n-2}{2}} \times \frac{1}{2} \qquad \Omega_n = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)}$$

$$\times \frac{\pi\alpha \cancel{G_N}}{1-\frac{m^2}{s}} \Big[(1+z^2)\left(1+\left(\frac{m^2}{s}\right)^4\right)$$

$$+ \left(\frac{1-3z^2+4z^4}{1-z^2}\right)\frac{m^2}{s}\left(1+\left(\frac{m^2}{s}\right)^2\right) + 6z^2\left(\frac{m^2}{s}\right)^2 \Big]$$

$$m^2 = s\,(1-x_\gamma)$$

E. A. Mirabelli, M. Perelson, M. E. Peskin hep-ph/9811337



$\sqrt{s} = 183$ GeV

$|\cos\theta| < 0.95$

SM background

F. A. Berends et al. NP B301 ('88) 583

183 GeV γ + MISSING RESULTS AT LEP2

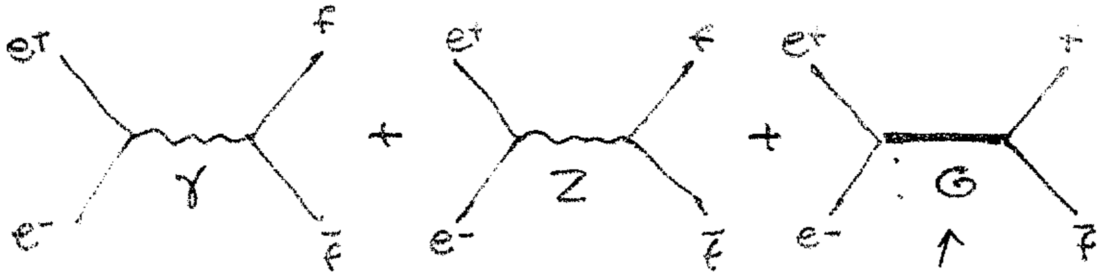
OPAL CERN-EP/98-143

ALEPH PL B429 ('98) 201

$\Rightarrow$ $n = 2$ (THE MOST OPTIMISTIC CASE)

$M_D > 1200$ GeV, $R < 0.48$ mm @ 95% CL

2) $e^+e^- \to f\bar{f}$ $\qquad z \equiv \cos\theta$



$$\frac{d\sigma}{dz} = N_c \frac{\pi \alpha^2}{2s} \Big\{ \mathrm{SM} \qquad \sum_{k_1, \cdots k_n} \frac{1}{s - m^2}$$

$$- \frac{s^2}{4\pi\alpha} \Big[2z^3 Q_e Q_f$$

$$+ \left(2z^3 v_e v_f - (1 - 3z^2) a_e a_f\right) \frac{s(s - m_Z^2)}{(s - m_Z^2) + m_Z^2 \Gamma_Z^2} \Big] \frac{\lambda}{M_s^4}$$

$$+ \frac{s^4}{16\pi^2 \alpha^2} \left[1 - 3z^2 + 4z^4\right] \left(\frac{\lambda}{M_s^4}\right)^2 \Big\}$$

$$\equiv A + B \frac{\lambda}{M_s^4} + C \left(\frac{\lambda}{M_s^4}\right)^2$$

J. L. Hewett.

- checked with independent calculation by Giudice et al.

- $C > 0, \; B^2 - 4AC < 0 \quad \Leftarrow \quad \frac{d\sigma}{dz} > 0 \left(\forall \frac{\lambda}{M_s^4}\right)$

$$\frac{\lambda}{M_s^4} \sim c_1 \left(\frac{\Lambda}{M_D}\right)^{n-2} \frac{1}{M_D^4}$$

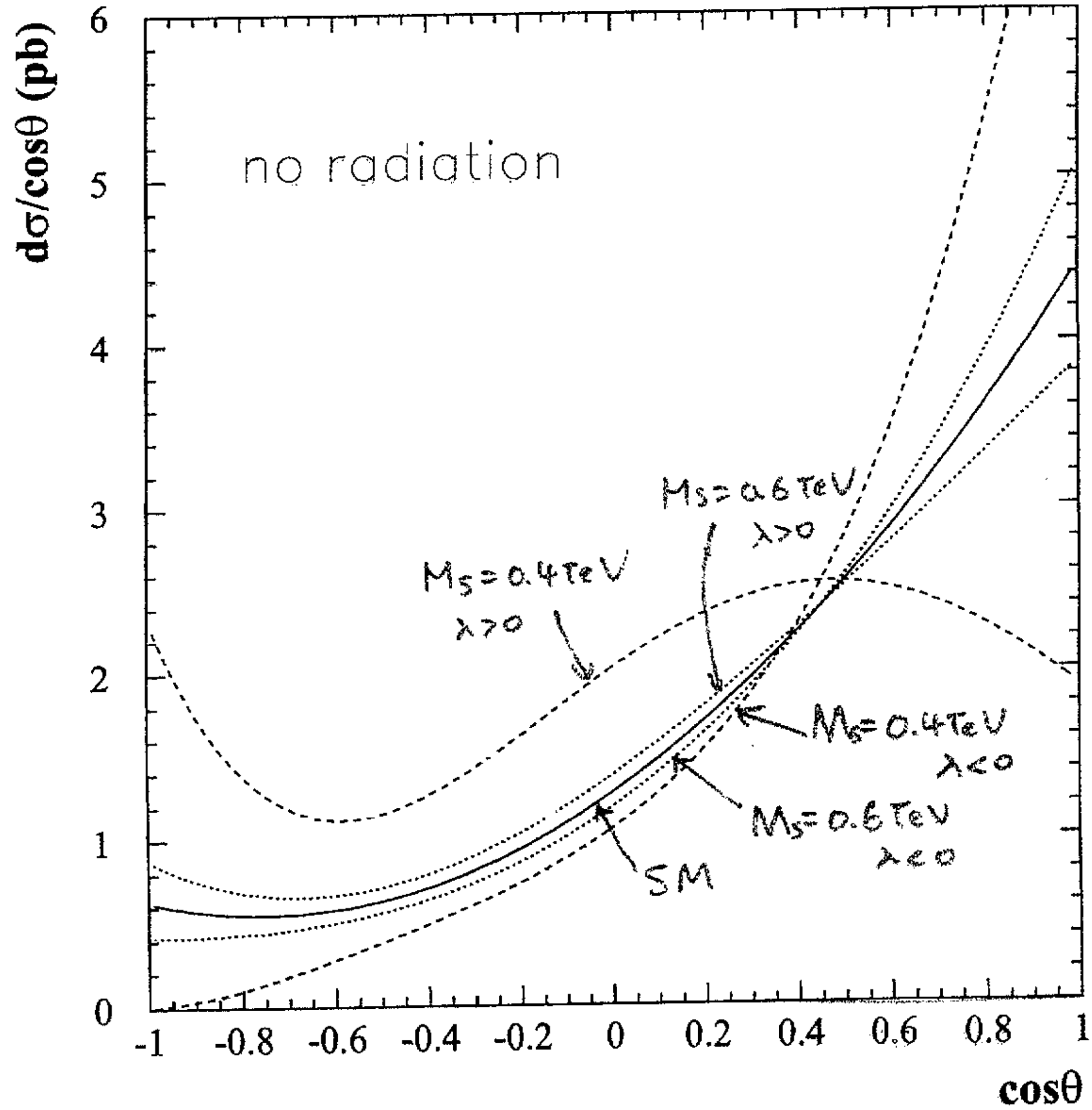
almost indep. of n

$\Lambda \sim M_D$

depends on

[illegible]

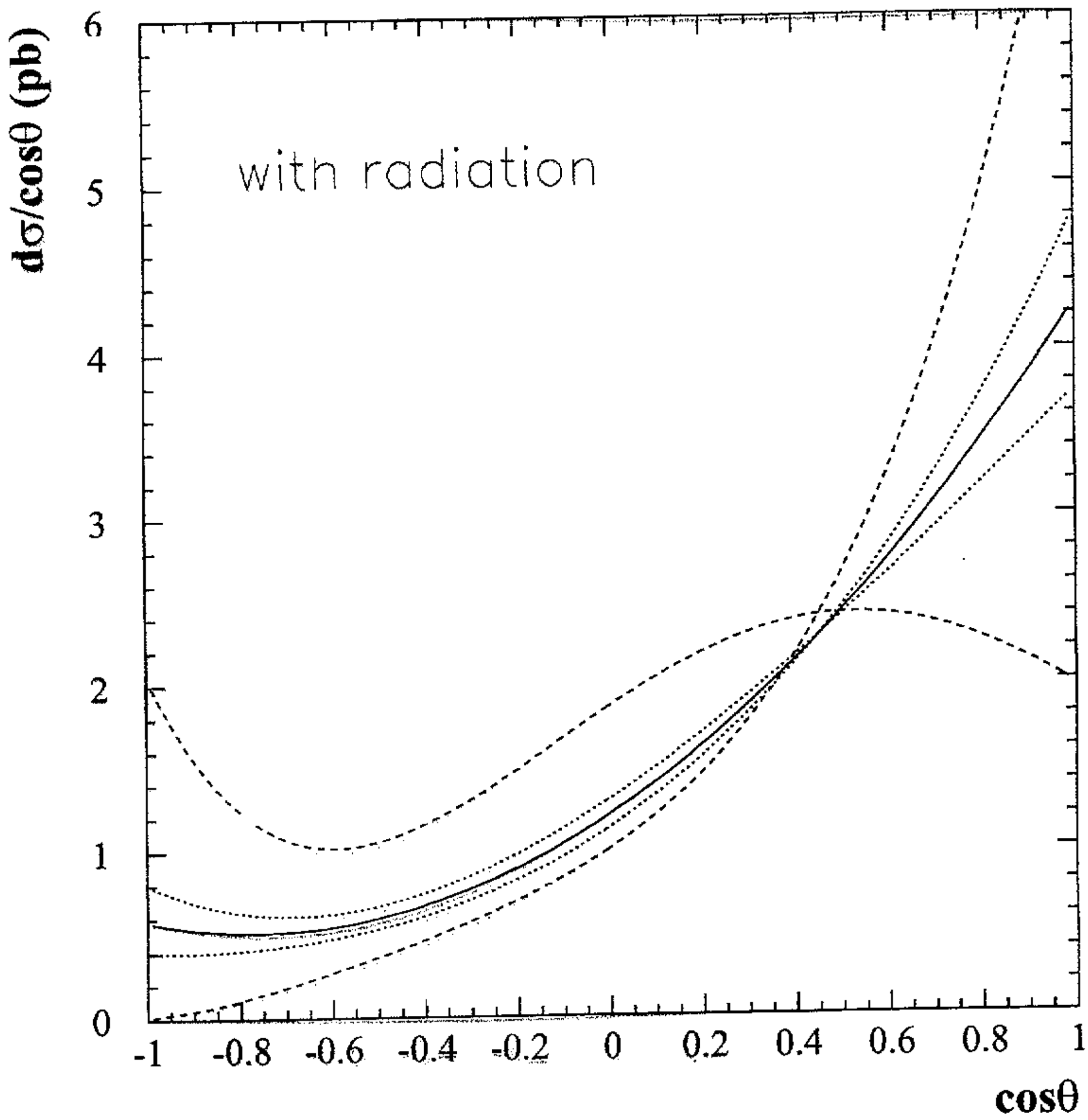
$\frac{d\sigma}{d\cos\theta}$ for $\mu^+\mu^-$ or $\tau^+\tau^-$ at 189 GeV



$$A + B\frac{\lambda}{M_s^4} + C\left(\frac{\lambda}{M_s^4}\right)^2$$

ZFITTER FOR

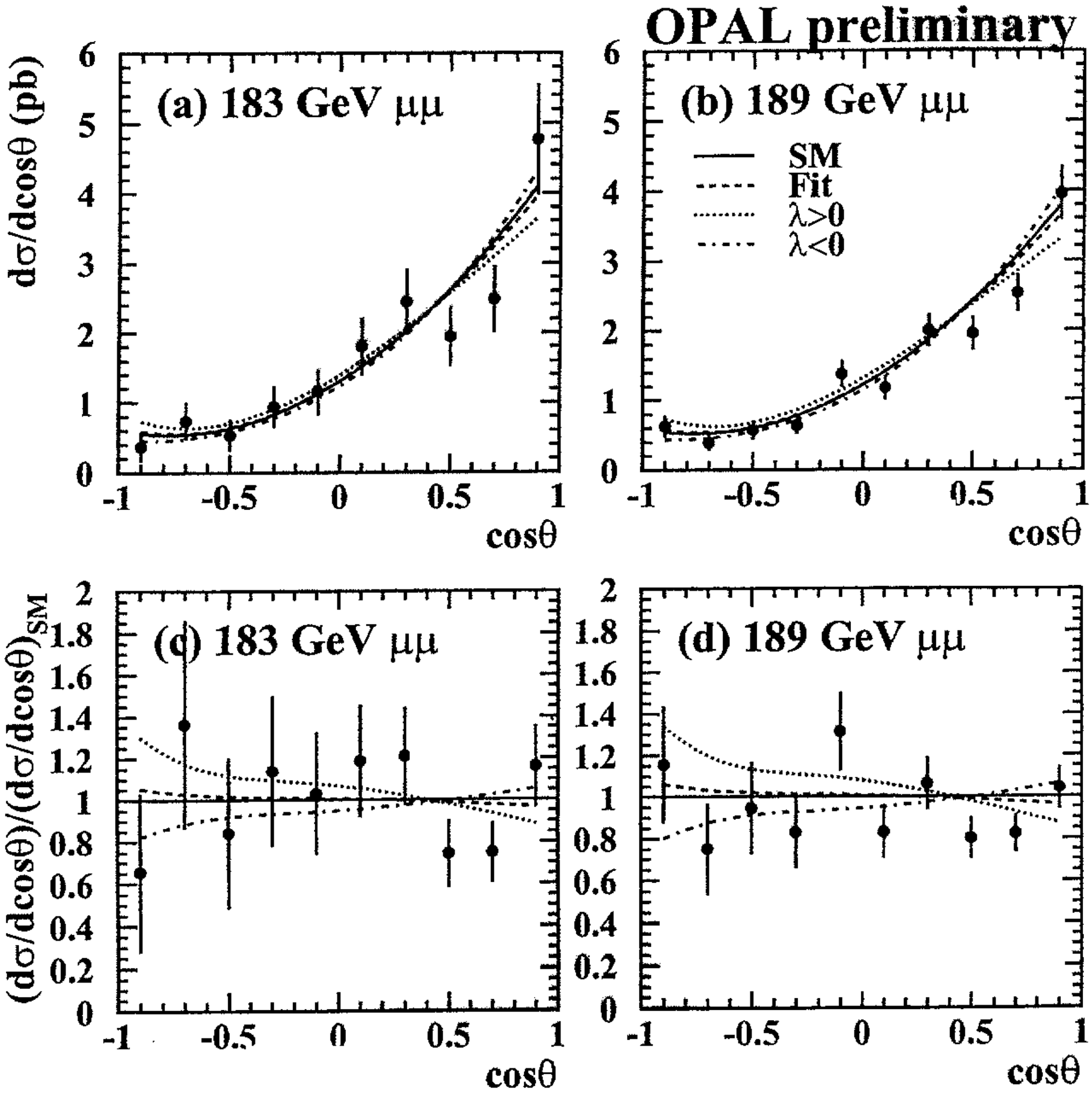
RAD CORR + HI[illegible]



$$\frac{d\sigma}{dz} = \left(1 + \delta(x_{min})\right)\frac{d\sigma}{dz}$$

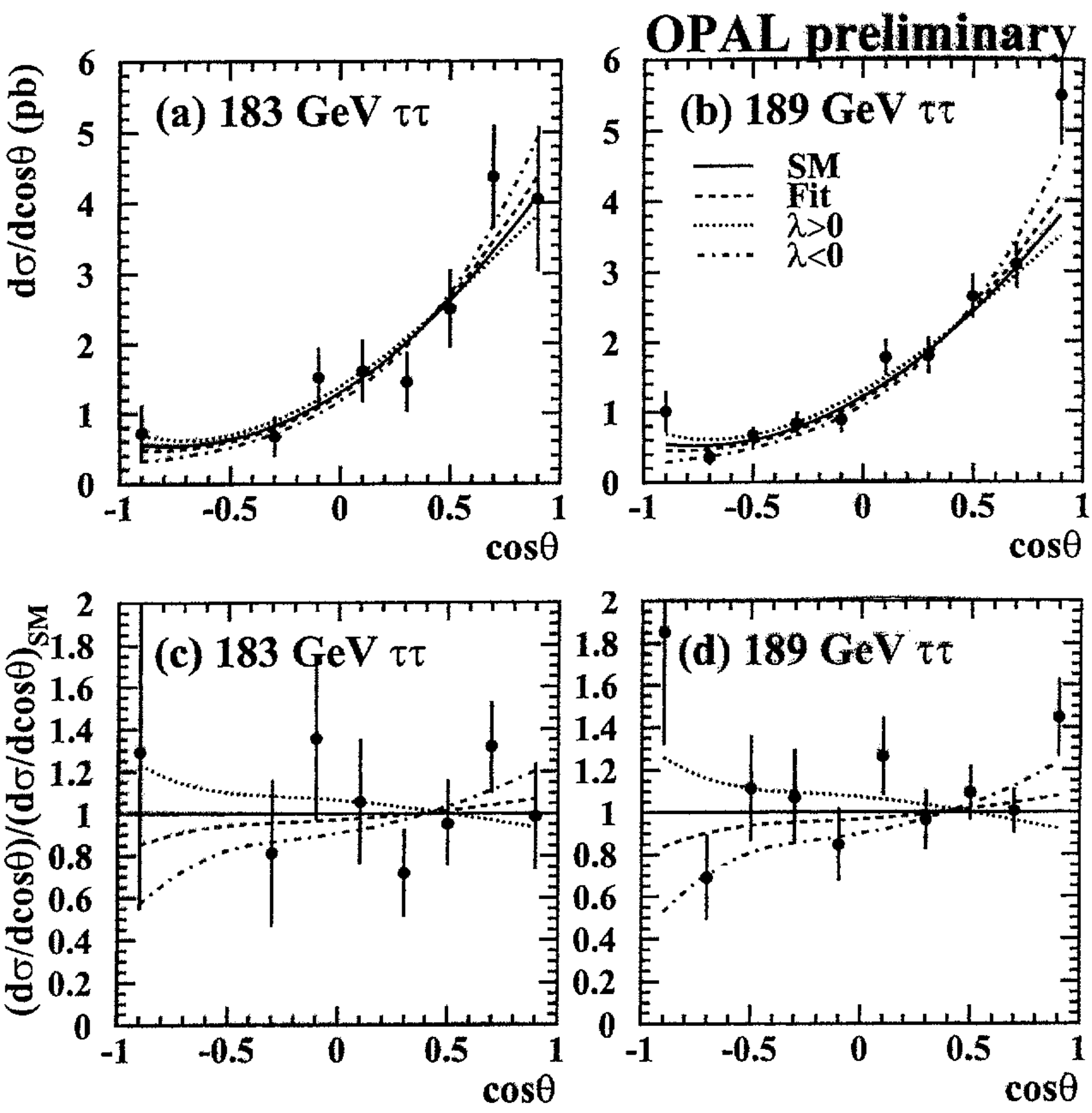
$$+ \left[\frac{\alpha}{\pi}\ln\frac{s}{m_e^2} - 1\right] * \int_{x_{min}}^{x_{max}} \left[\frac{d\sigma}{dz^*}\frac{dz^*}{dz}\right]\overbrace{(s(1-x))}^{s'} \quad \frac{1+(1-x)^2}{x}\,dx$$

$$z^* = \frac{1 \pm \beta z}{\beta \pm z}$$



COMBINED FIT

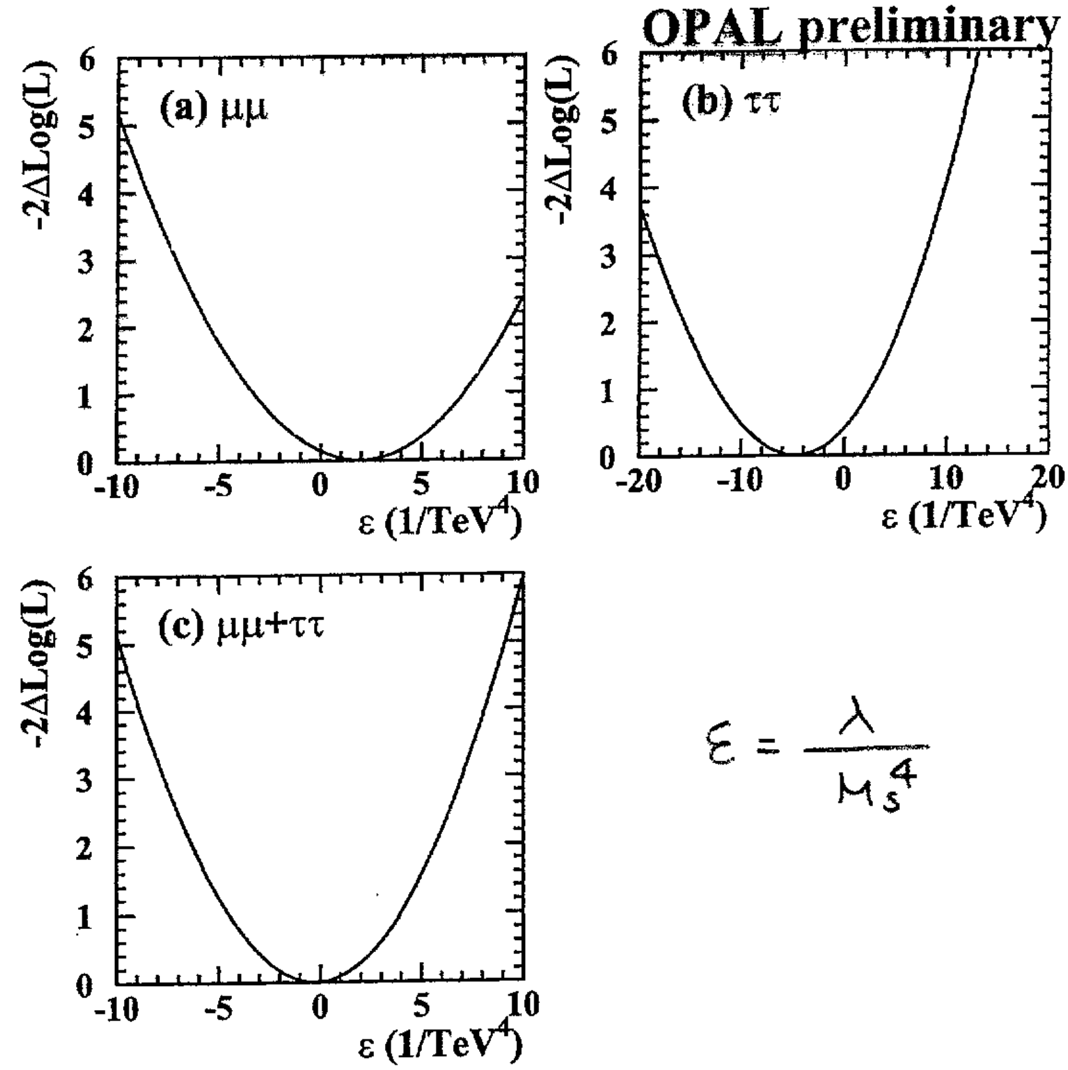
183 GeV + 189 GeV



COMBINED FIT

183 GeV + 189 GeV

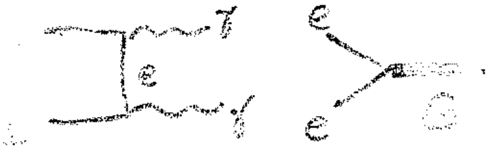
RESULTS OF BINNED LOG-LIKELIHOOD FIT.



$$\varepsilon = \frac{\lambda}{M_s^4}$$

$\mu^+\mu^-$	$M_s > 576$ GeV	$\lambda = +1$	95% C.L.
	$M_s > 651$ GeV	$\lambda = -1$	
$\tau^+\tau^-$	$M_s > 626$ GeV	$\lambda = +1$	
	$M_s > 504$ GeV	$\lambda = -1$	
COMBINED	$M_s > 649$ GeV	$\lambda = +1$	
	$M_s > 635$ GeV	$\lambda = -1$	

3) $e^+e^- \to \gamma\gamma$



$$\frac{d\sigma}{dz} = \frac{2\pi\alpha^2}{s}\left[\frac{1+z^2}{1-z^2}\right] + \frac{2\alpha s}{\pi}(1+z^2)\left[\frac{\lambda}{M_S^4}\right] + \frac{s^3}{32\pi}(1-z^4)\left[\frac{\lambda}{M_S^4}\right]^2$$

$z \equiv \cos\theta$

$0 \le z < 1$

G.F. Giudice, R. Rattazzi, J.D. Wells
hep-ph/9811291 ← FACTOR 2 DIFFERENT $-1<z<1$

QED TEST.

$$\frac{d\sigma}{dz} = \frac{2\pi\alpha^2}{s}\left[\frac{1+z^2}{1-z^2}\right] \pm \pi\alpha^2 s(1+z^2)\left[\frac{1}{\Lambda_\pm^4}\right]$$

NEGLECTING $\left[\frac{\lambda}{M_S^4}\right]^2$ TERM.

TWO EQUATIONS HAVE THE SAME FORM.

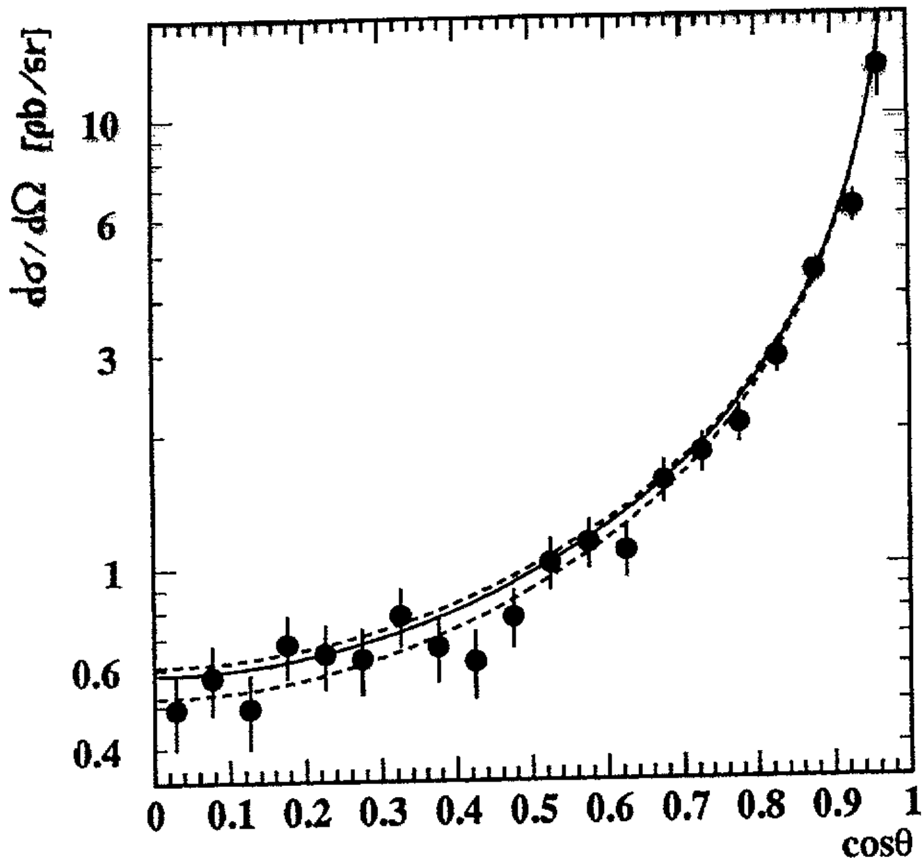
$\Lambda_+ > 345$ GeV
$\Lambda_- > 278$ GeV (95%CL)

OPAL PN368 (LEPC)

$M_s = 2.13 * \Lambda$

$$\Rightarrow \begin{cases} M_s > 735\ \text{GeV} & (\lambda = +1) \\ M_s > 592\ \text{GeV} & (\lambda = -1) \end{cases}$$

$e^+e^- \to \gamma\gamma$



FIT BY KIRSTEN SACHS

$M_s > 749$ GeV $\lambda > 0$

$M_s > 597$ GeV $\lambda < 0$

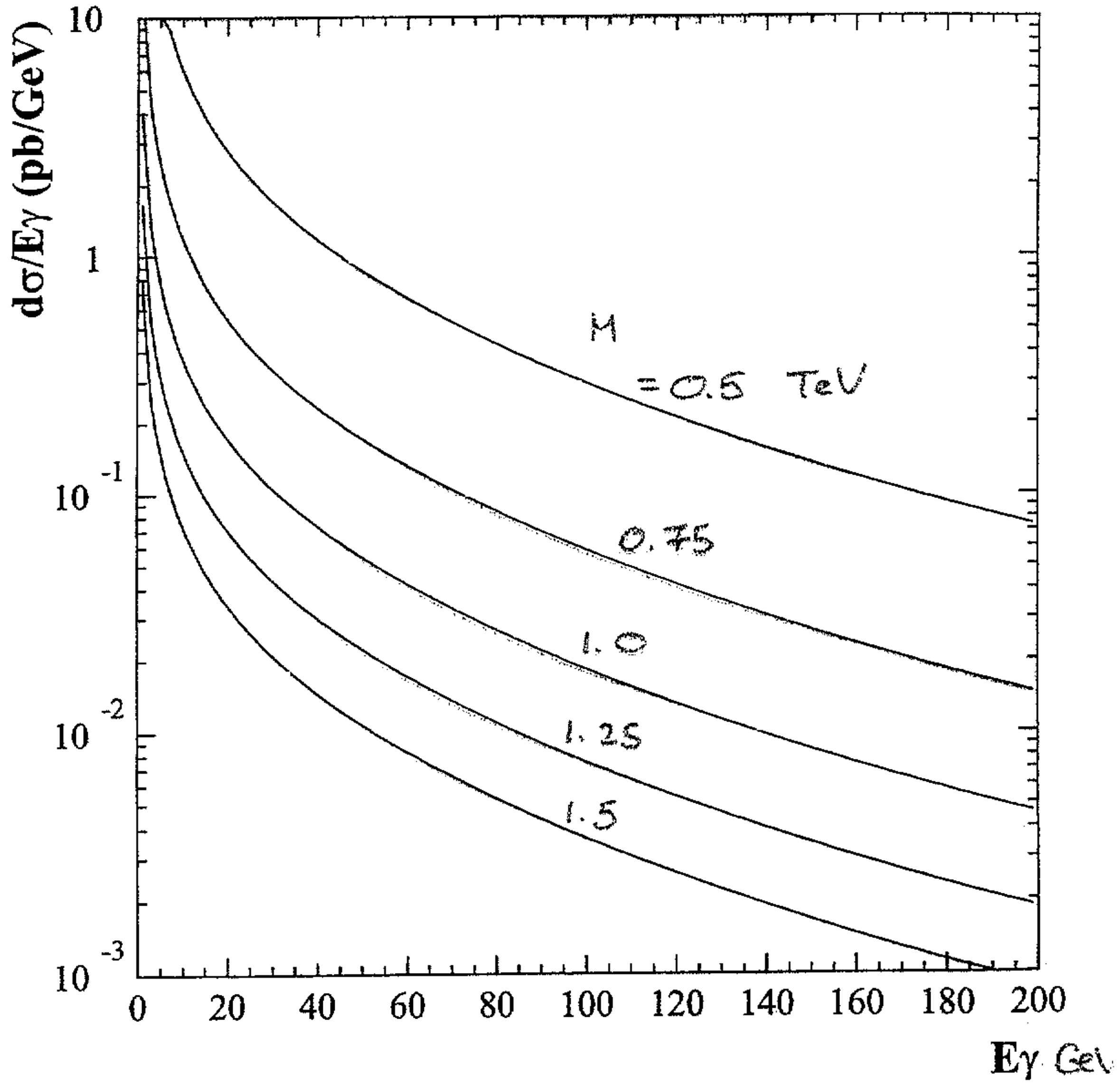
(95% CL)

$\sqrt{s} = 189$ GeV

$|\cos\theta| < 0.9$ (FIT REGION)

n=2

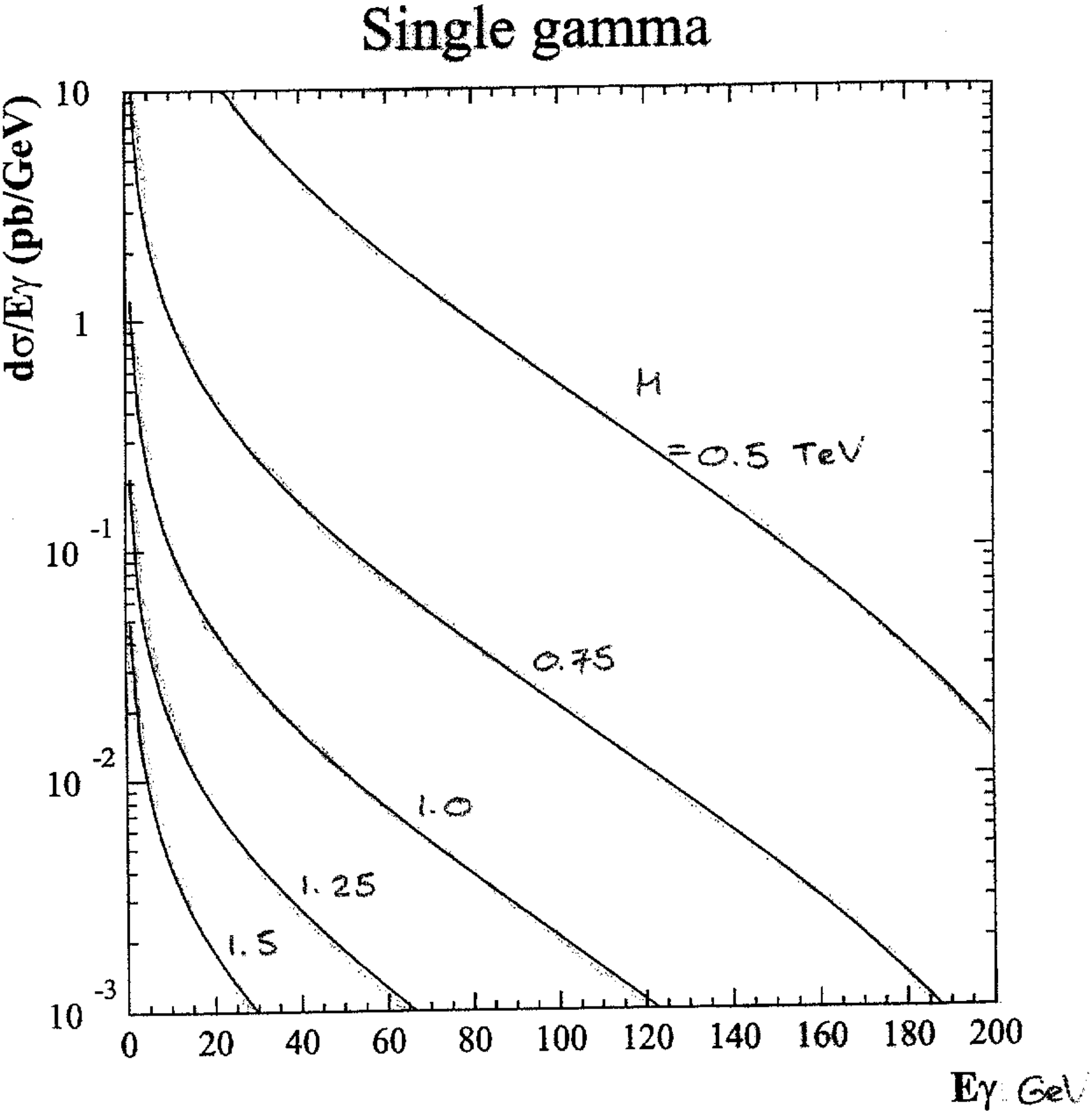
Single gamma



$\sqrt{s} = 500$ GeV

$|\cos\theta_\gamma| < 0.95$

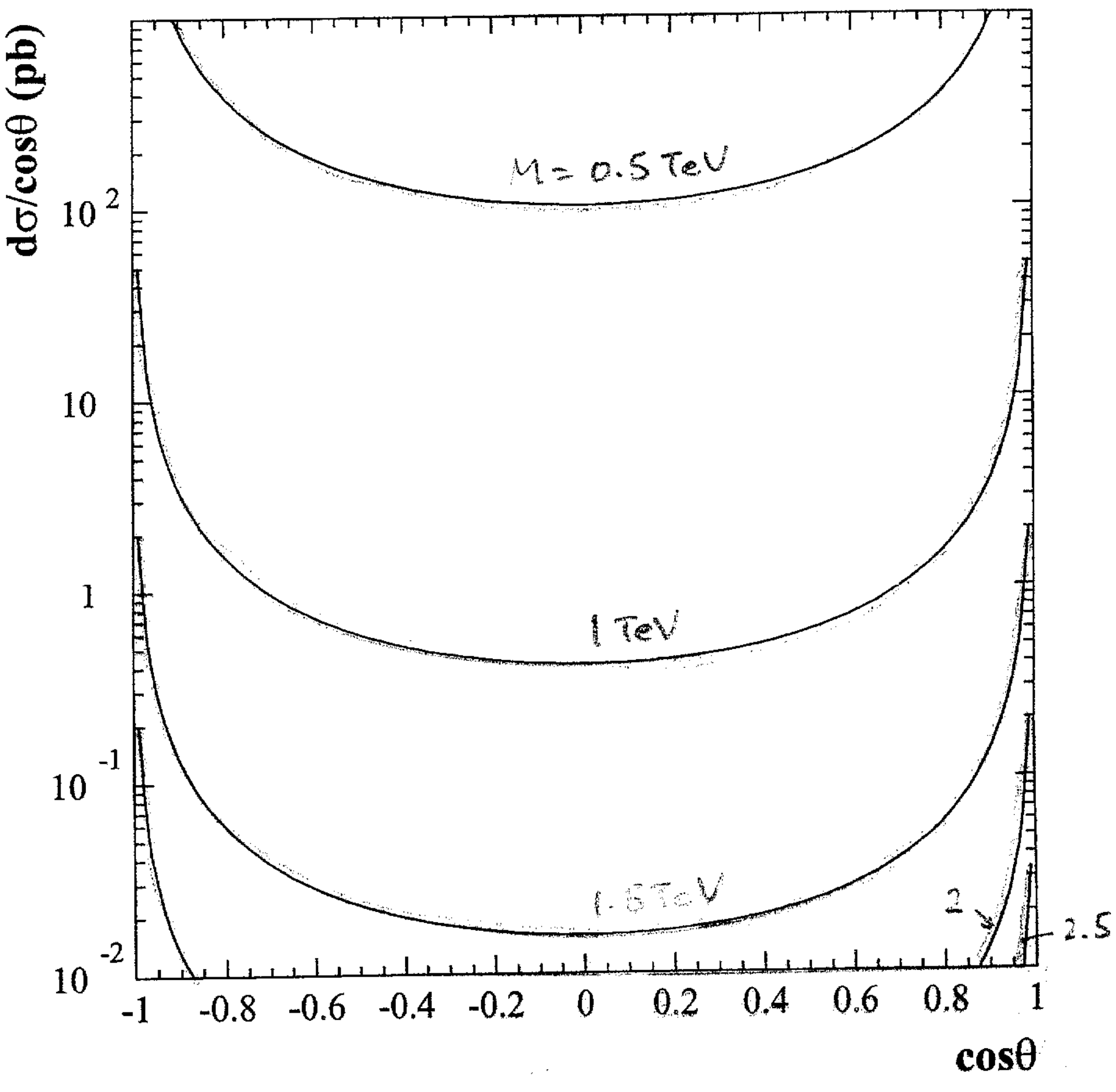
n = 6



$\sqrt{s} = 500$ GeV

$|\cos\theta_\gamma| < 0.95$

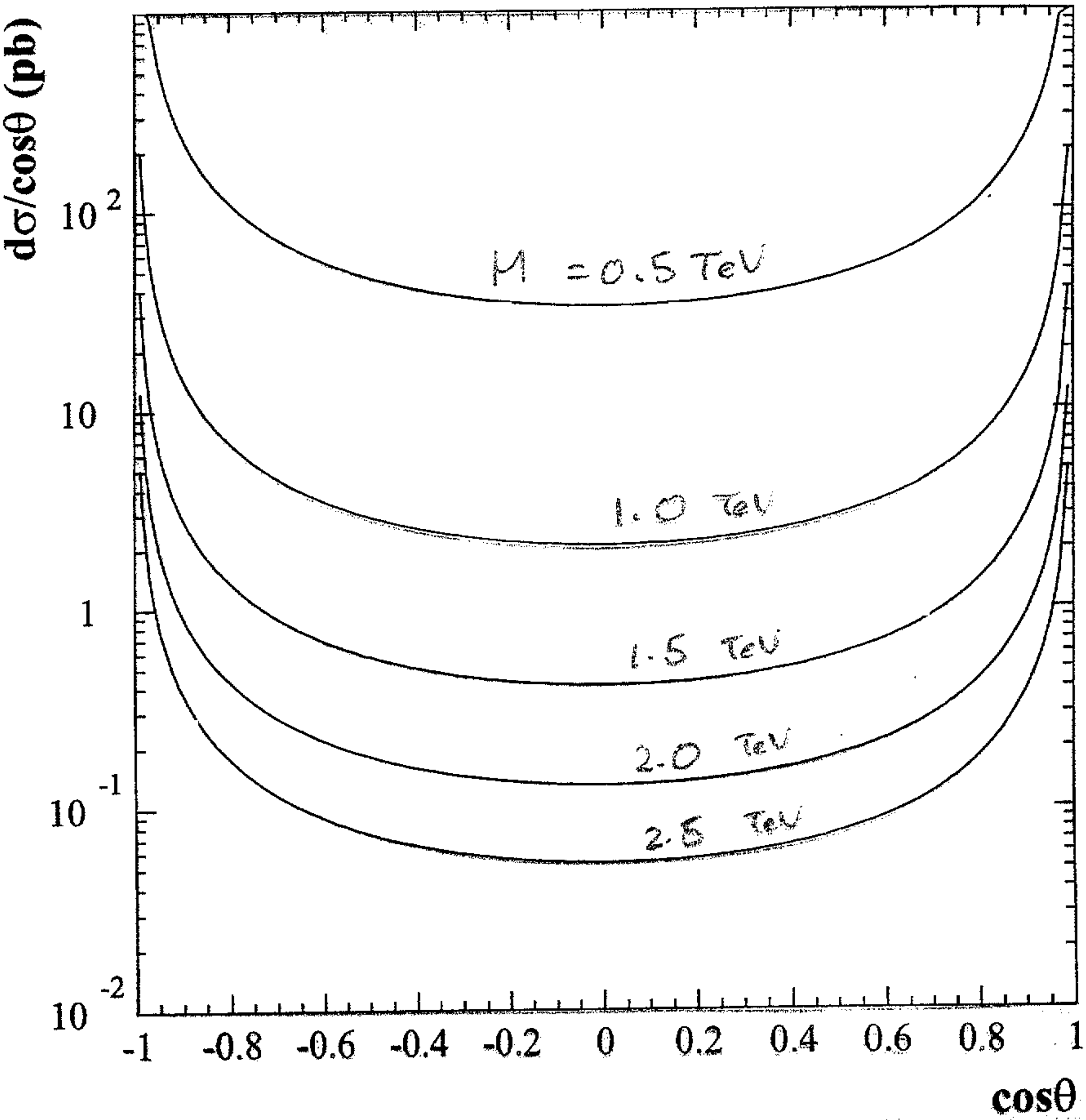
Single gamma



$\sqrt{s} = 500$ GeV

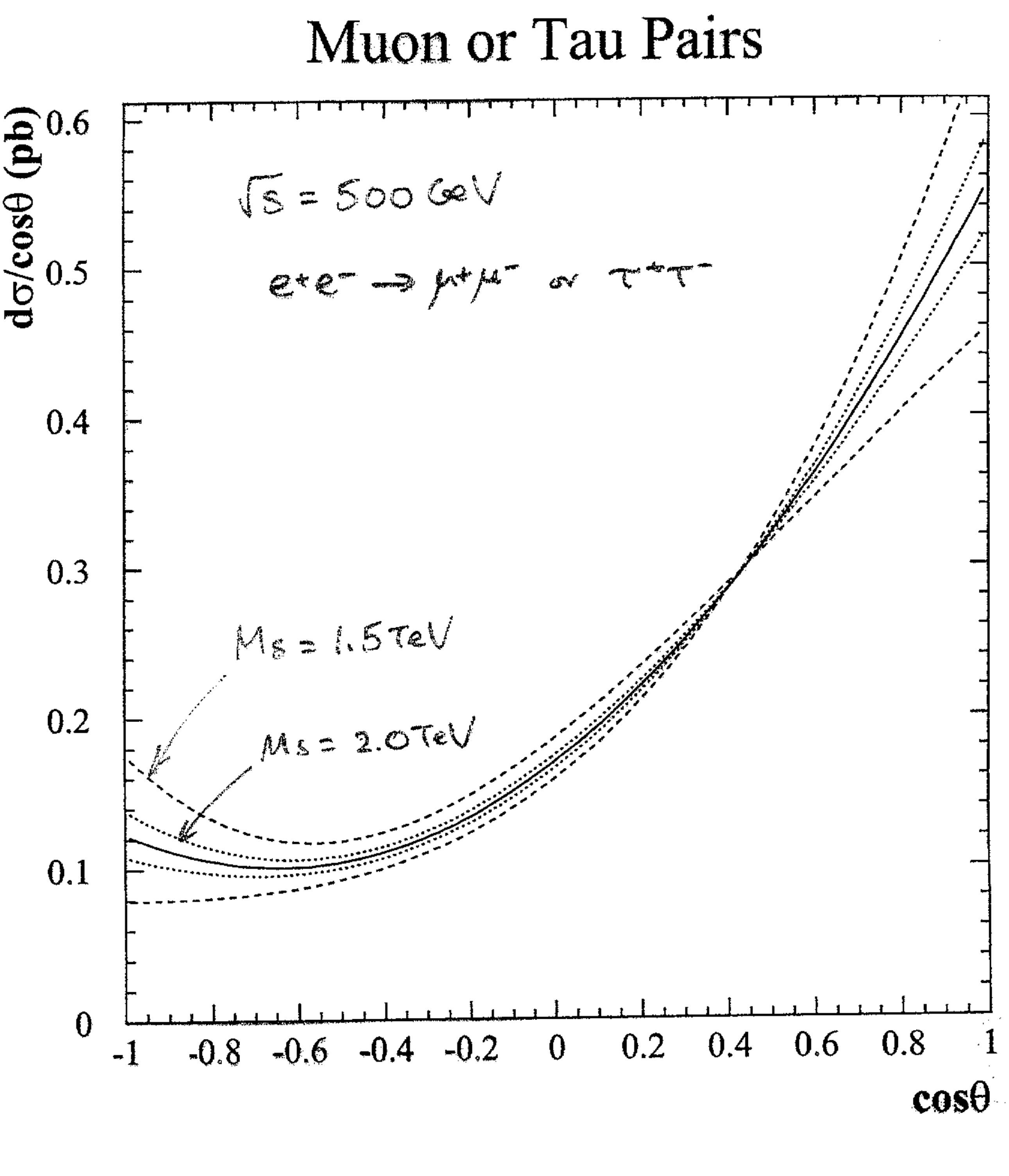
$2P_T^\gamma/\sqrt{s} > 0.05$

Single gamma



$\sqrt{s} = 500$ GeV

$2P_T^\gamma/\sqrt{s} > 0.05$



WITH RAD. CORRECTIONS
WITHOUT BEAMSTRAHLUNG

IF LUMINOSITY IS MEASURED IN 3%.

WE CAN REACH $M_s \sim 2\,\mathrm{TeV}$ @ 95% CL.

FROM STATISTICS

$$\Delta\sigma \simeq B' \frac{s}{M_s^4}$$

$$\sigma \simeq A' \frac{1}{s}$$

$$\frac{\Delta\sigma \cdot L}{\sqrt{\sigma \cdot L}} \qquad L = \int \mathcal{L}\, dt$$

$$= \frac{B' \frac{s}{M_s^4} L}{\sqrt{A' \frac{1}{s} \cdot L}} \sim 5 \qquad \text{(} 5\sigma \text{ SIGNIFICANCE)}$$

$$M_s \propto s^{3/8} L^{1/8}$$

$\sqrt{s} = 189\,\mathrm{GeV}$ $\quad L = 0.1\,\mathrm{fb}^{-1}$ $\quad M_s \gtrsim 700\,\mathrm{GeV}$ (95% CL)

$\Rightarrow$ $\sqrt{s} = 500\,\mathrm{GeV}$ $\quad L = 100\,\mathrm{fb}^{-1}$

$$M_s > 700 \cdot \left(\frac{500}{189}\right)^{3/8} \left(\frac{100}{0.1}\right)^{1/8}\,\mathrm{GeV}$$

$$\simeq 2.4\,\mathrm{TeV}$$

[illegible]

IF THE THEORY IS
STILL ATTRACTIVE
WHEN LC(S) IS TURNED ON,
IT CAN BE SERIOUSLY TESTED.

Theory papers

N. Arkani-Hamed, S. Dimoloulos, G. Dvali

PL B429 ('98) 263 hep-ph/9803315

basic idea:

confine SM fields into 1+3 dim

a simple model for $n=2$

I. Antoniadis + A, D, D

PL B436 ('98) 257 hep-ph/9804398

relation to string theory

proton stability

gauge coupling unification

susy breaking

Very hard to understand

A, D, D

hep-ph/9807344

51 pages too long to read

Many trivial tests of the theory

cosmic ray

rare decay

SN 1987

cosmology

strong CP etc.

Phenomenology Papers (from Nov. '98)
relatively new!

G. F. Giudice, R Rattazzi, J.D. Wells

CERN-TH / 98-354 hep-ph/9811291

Feynman rules

$e^+e^- \to f\bar{f}$ (incl. $e^+e^- \to e^+e^-$), $\gamma\gamma$

$e^+e^- \to \gamma G,\ Z, G$

$pp \to j + \text{missing} + X$ etc.

J. L. Hewett

hep-ph / 9811356

$e^+e^- \to f\bar{f}$

$gg \to \ell^+\ell^-$

$q\bar{q} \to \ell^+\ell^-$

E.A. Mirabelli, M. Perelstein, M.E. Peskin

hep-ph / 9811337

$e^+e^- \to \gamma G$

$p\bar{p} \to j + \text{missing} + X$

$\bar{q}g \to \bar{q}G$

$gg \to gG$

T. Rizzo

hep-ph / 9901209

$e^+e^- \to f\bar{f}$

HERA

$\gamma\gamma \to f\bar{f}$

K. Cheung W-Y Keung

hep-ph/9903294

$$e^+e^- \to Z G$$
$$\to \gamma G$$