ICEPP シンポジウム 平成24年2月

# 宇宙の構造形成

宇宙の多様な構造はどのようにできたのか
宇宙の進化の歴史を概観

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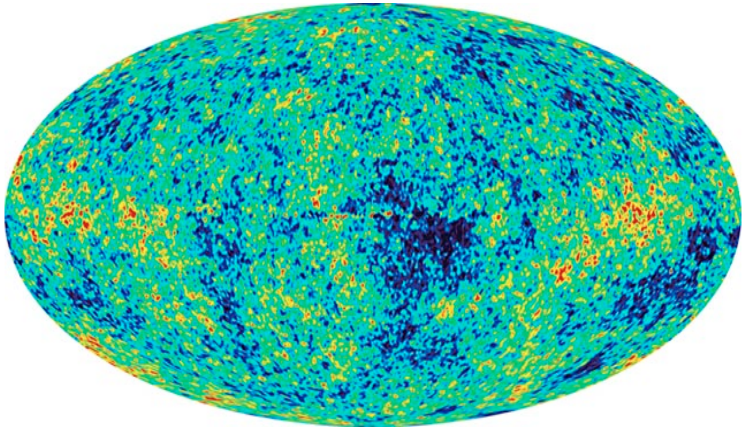
# 講義概要

- 現代宇宙論: 宇宙137億年の歴史
- ダークマターの役割
- ダークエネルギーの性質
- ダークエイジ:最初の3億年

# よく聞く話はそう簡単ではない

宇宙初期の物質分布



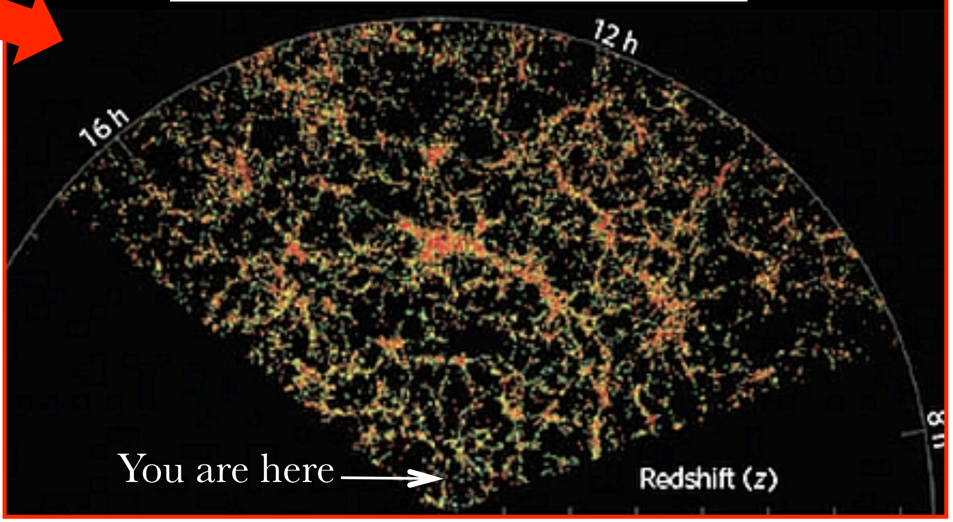
homogeneous universe

標準理論モデル：宇宙初期に生成されたわずかな密度揺らぎをもとにして、重力により大規模構造は作られた。

$$\frac{\Delta T}{T}\Big|_{\mathrm{COBE}} \sim 10^{-5}$$

$$\Delta \rho_{\text{銀河}} = ??$$

現在の宇宙の銀河分布




clumpy universe

# 密度揺らぎの成長

宇宙マイクロ波で見ることができるのは、37万歳の頃

スケールファクターで言えばおよそ 1/1090 ~ 0.01 程度。

(後で学ぶように) 線形理論の予言は「膨張宇宙の中で

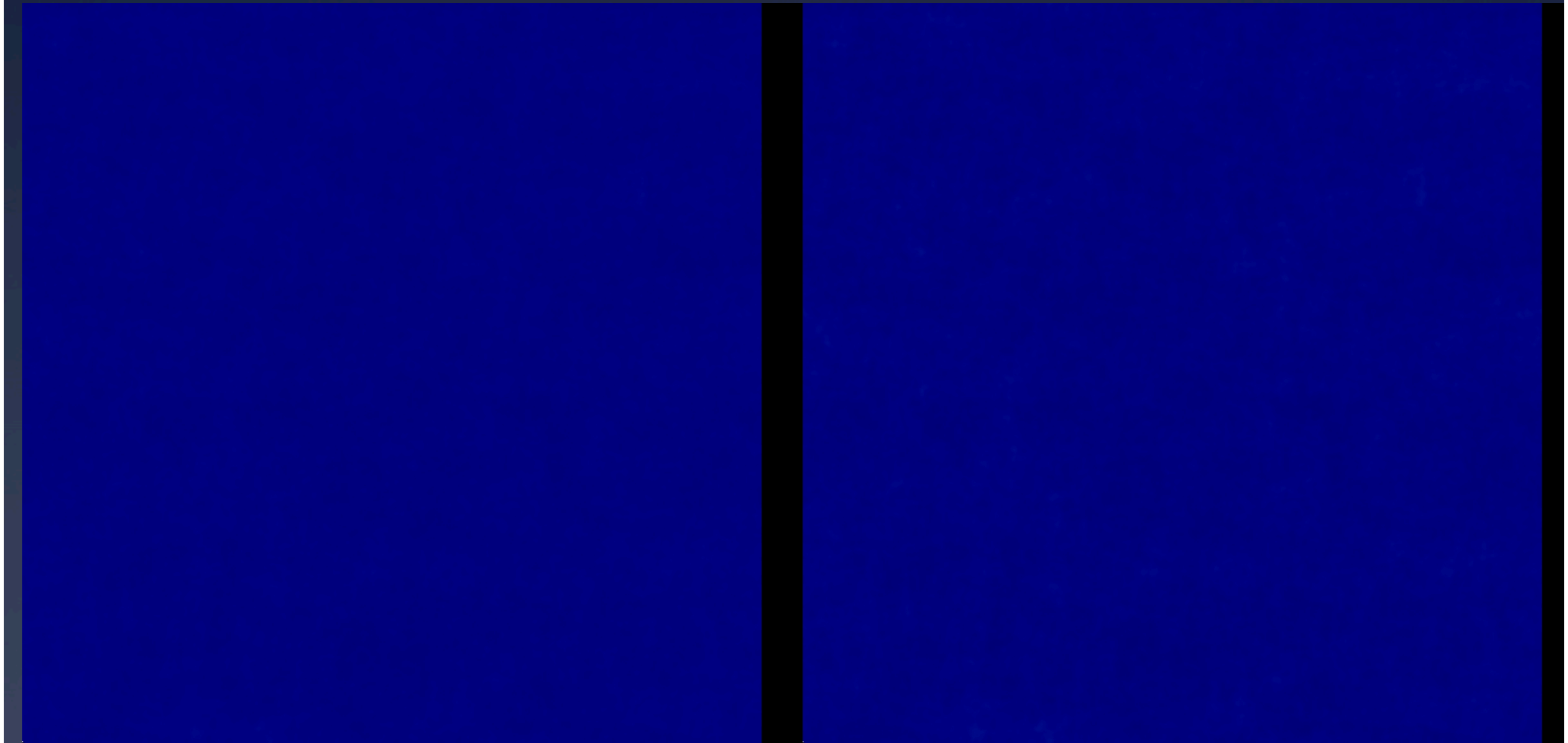
密度揺らぎはスケールファクターに比例して成長する」

はじめに $10^{-5}$ ほどあった密度揺らぎは、宇宙が現在ま

でに 1000倍膨張する間に 0.01 になれるだけ。

宇宙項を足しても状況は悪くなるのみ…

# ダークマターの無い宇宙



# ダークマターがあればオッケーか…

赤方偏移7

30億光年立方に１個ほど

中心には太陽の20億倍のブラックホール

この頃宇宙年齢は7億7千万年

# 第一章
# 標準宇宙モデルとダークマター

# 内容

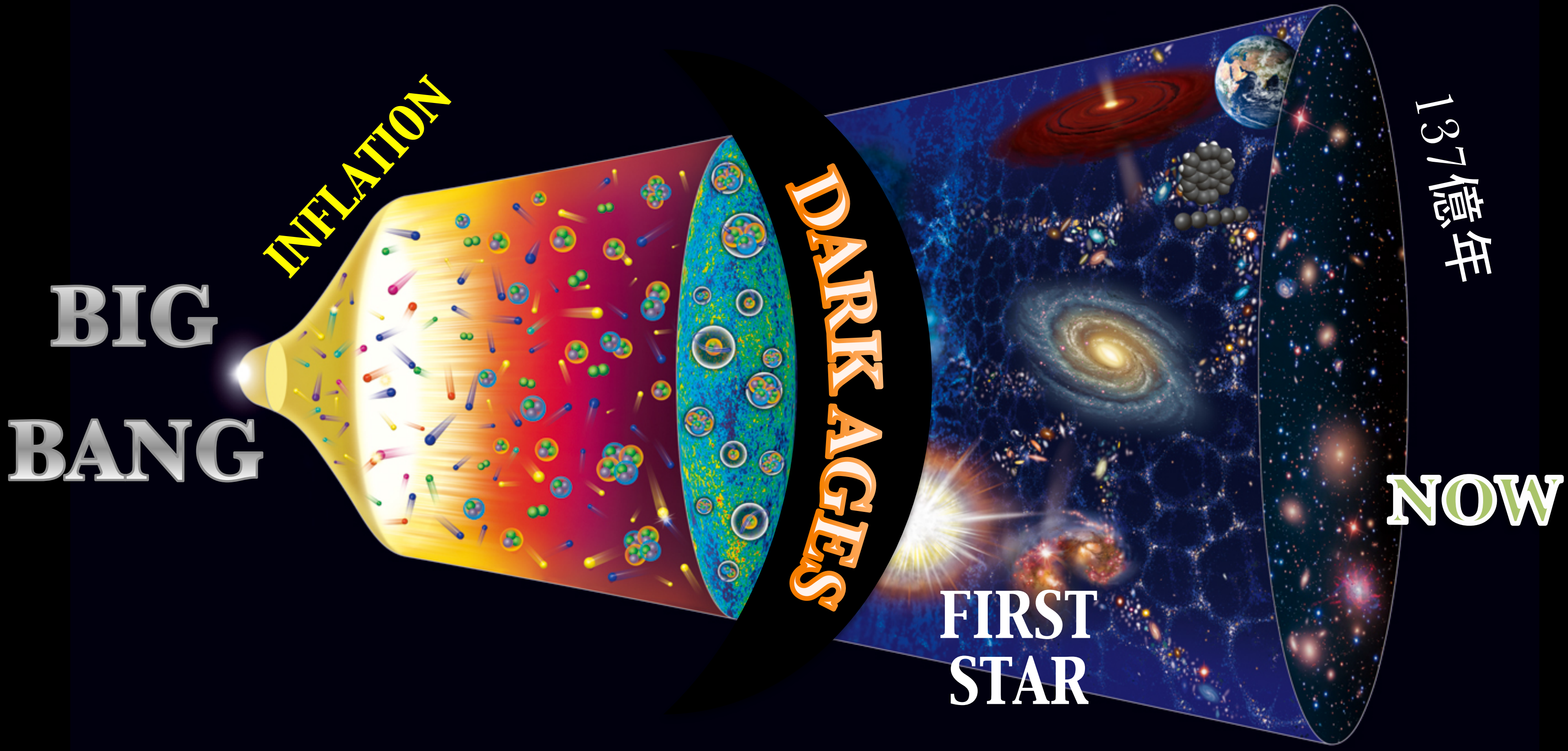
1. 現代宇宙論の基礎

2. フリードマンモデル

3. 密度揺らぎの線形成長

4. ダークマターの性質

# 宇宙の歴史



# To Study Structure Formation…

- The evolution of the universe as a whole:
  - Cosmic expansion history
  - Energy content
- The initial condition
  - Inflation and quantum fluctuations
  - Primordial density perturbations
- The perturbation evolution
  - Linear and nonlinear models

# Elements of Modern Cosmology

# Cosmological Principle

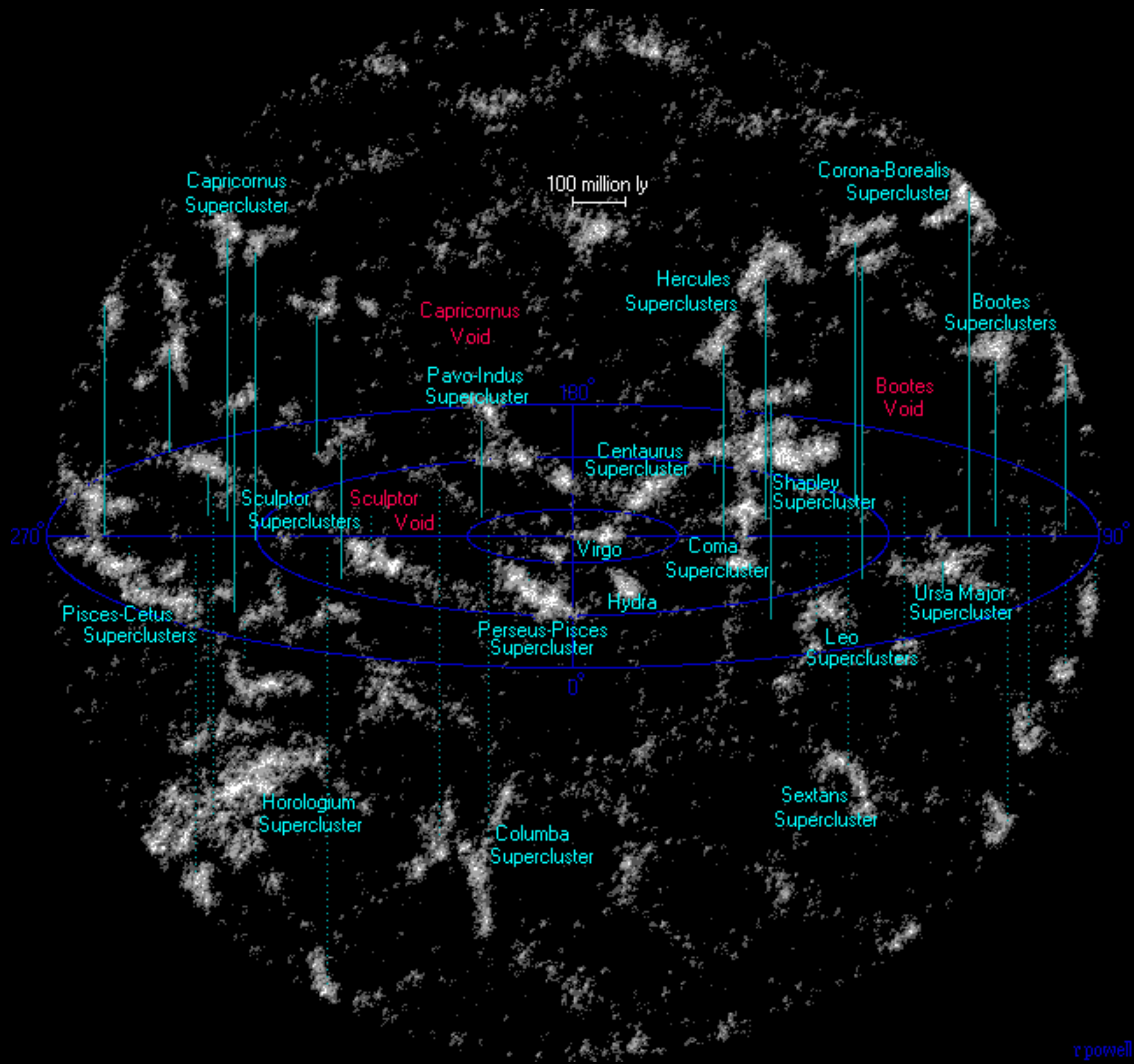
- The universe is homogeneous and isotropic
- This means that the universe does not possess any privileged positions and directions.

  - We are *not* in a special location.

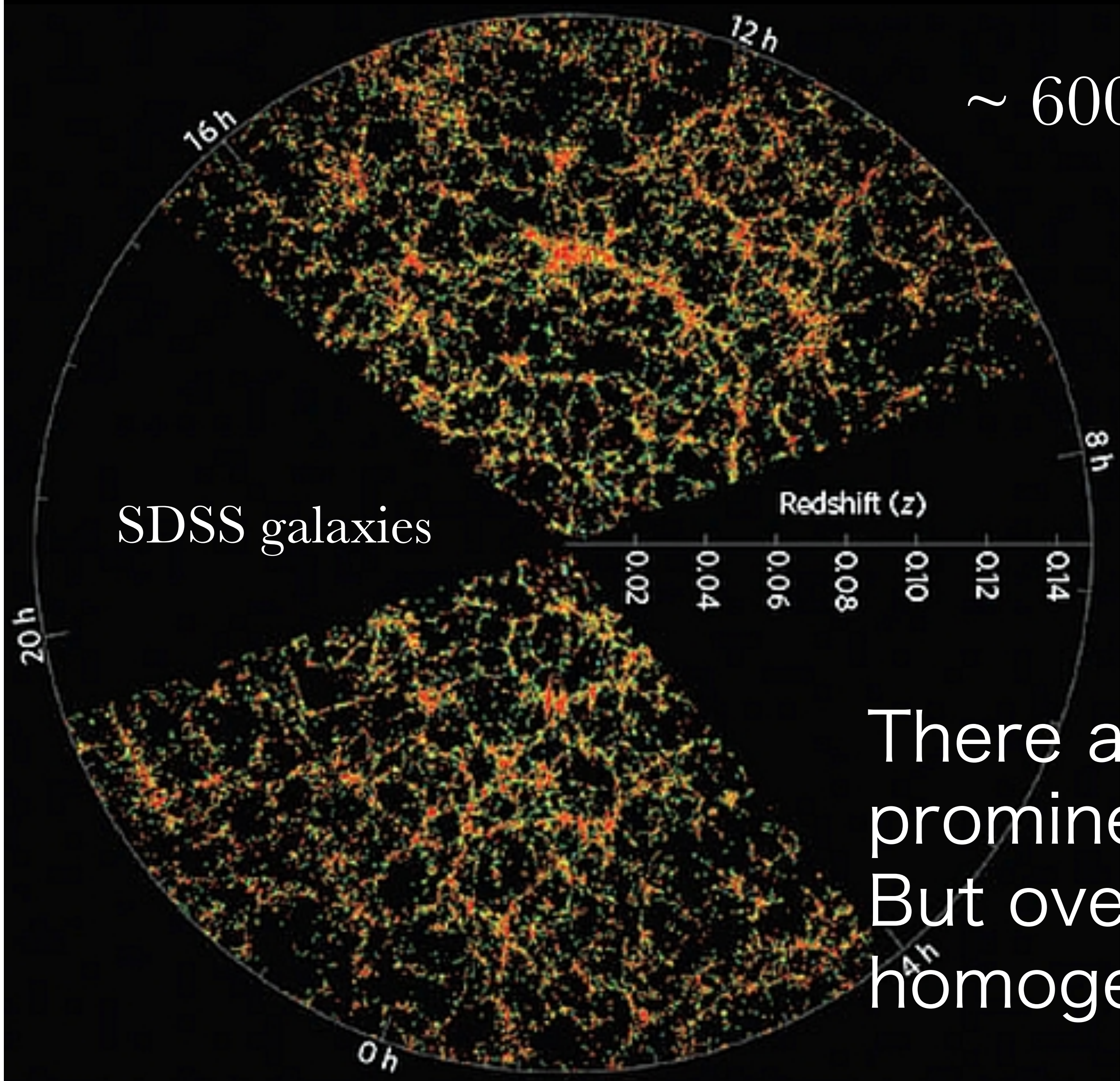
Observationally this is true, or at least a very good approximation. There are galaxies, clusters of galaxies etc., but on very large-length scales, nearly homogeneous and isotropic.

But then another problem arises: in fact, our universe appears homogeneous on scales much larger than the causal *horizon.*

# The Universe within 100 Mpc



~ 600Mpc distance




SDSS galaxies

There are some prominent structures. But overall, it looks homogeneous.

# The Relativistic Universe

Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4}T_{\mu\nu}$$

describe the interaction of gravity with space-time being curved by matter and energy.

This is a set of ten nonlinear equations for functions of four variables. Here, $R_{\mu\nu}$, $R$, $g_{\mu\nu}$ are the Ricci curvature tensor, the Ricci scalar, and the metric.

# The Robertson-Walker Metric

The space-time of a homogeneous, isotropic universe can be described by a simple metric:

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

(note !) which has only one function,

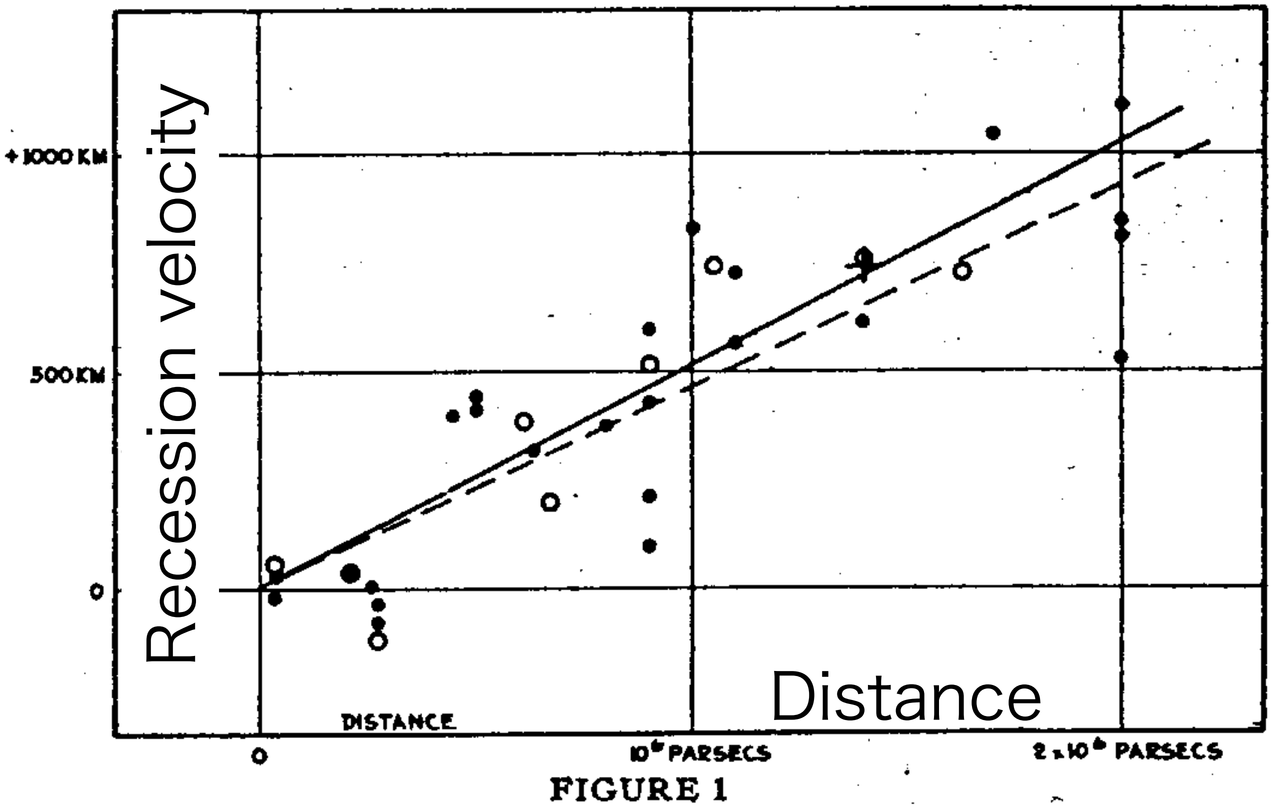
the expansion parameter, and one constant,

the curvature.

Essentially, only the evolution of *a(t)* is important.

# The Expanding Universe : Discovery

$V = H_0\, d$




Nearly all the galaxies are receding from us. Furthermore, the recessaion velocities are greater for more distant galaxies.

ちょっと横道

# 宇宙膨張は本当は誰が発見したか

Who discovered the expanding universe?

Harry Nussbaumer and Lydia Bieri

Does it really matter who discovered the expanding universe? Great discoveries are anyway never done single-handedly. This is a valid attitude. However, those interested in the evolution of our scientific culture are eager to know the intricate patterns that lead to new insights. As the expanding universe is one of the most important discoveries ever made, it is not astonishing that the question of how it happened is still widely discussed.

The debate on this topic has flared up again due to an article in *Nature View*, where Eugenie Reich highlighted two contributions by Sidney van den Bergh (2011) and David Block (2011). Their effect was to reanimate the discussion whether Hubble or Lemaître discovered the expanding universe, or whether it was simply a nearly predictable outcome of the normal scientific activity of those days. We have investigated this question in our book *Discovering the Expanding Universe* [7], where we reconstructed the discovery from original documents.

Arxiv:1107.2281

THE CURIOUS CASE OF LEMAÎTRE'S EQUATION NO. 24 Arxiv:11061195

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In August of 1961 Abbé Georges Lemaître told me (with a twinkle in his eyes) that, being a priest, he felt a slight bias in favor of the idea that the Universe had been created. It must therefore have been a particular pleasure for him (Lemaître 1927) to have been the first to find both observational and theoretical evidence for the expansion of the Universe. His observational discovery was based on the published distances and radial velocities of 42 galaxies. Lemaître's theretical result was based on the finding that the Universe is unstable, so that pertubations tend to grow. These results, which were published in French and in a relatively obscure journal, anticipated the work of Edwin Hubble (1929) by two years. It might therefore have been appropriate to assign the credit for the discovery of the expansion of the Universe to Lemaître, rather than to Hubble (Peebles 1984). The early evolution of our understanding of the expansion (and the scale-size) of the Universe has recently been discussed in detail by Kragh & Smith (2003) and by Nussbaumer & Bieri (2009).

$$\frac{v}{rc} = \frac{625\ x\ 10^5}{10^6 x\ 3{,}08\ x\ 10^{18}\ x\ 3\ x\ 10^{10}},$$

in which v is the radial velocity, r the distance and c the velocity of light is omitted. Of the three numbers given above the speed of light in cm/s, and the length of the parsec are well-known. Only the cosmic expansion term 625 x $10^5$ [corresponding in modern parlance to a Hubble constant of 625 km/s] might possibly be considered to be controversial. The fact that dropping this term from Lemaître's Eqn. 24 was intentional is supported by the fact that a short paragraph in the paper,

# Lost in translation

This clearly ends speculation about who translated the paper and who deleted the paragraphs — Georges Lemaître did both himself.

Lemaître's letter also provides an insight into the scientific psychology of (some of) the scientists of the 1920s. Lemaître was not at all obsessed with establishing priority for his original discovery. Given that Hubble's results had been published in 1929, Lemaître saw no point in repeating his own more tentative earlier findings in 1931. Rather, he preferred to move forward and to publish his new paper, 'The expanding Universe', which he did later that year[11]. Lemaître's request to join the Royal Astronomical Society, at Smart's invitation, was eventually granted; he was elected as an associate on 12 May 1939. ■

# The Expanding Universe : Discovery

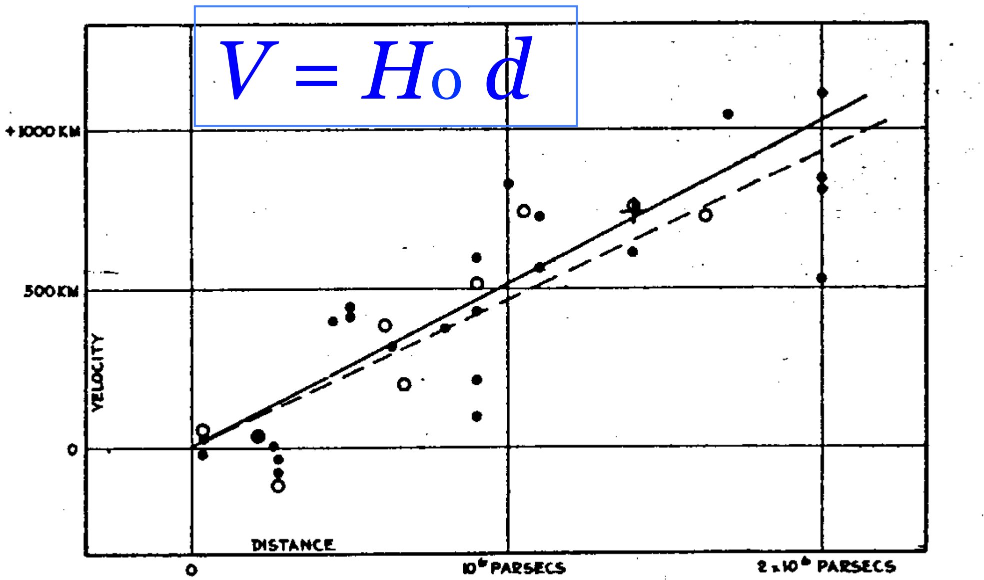


FIGURE 1

Nearly all the galaxies are receding from us.
Furthermore, the recessaion velocities are
greater for more distant galaxies.

# The Expanding Universe : Theory

For the Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Einstein equation reduces to two equations:

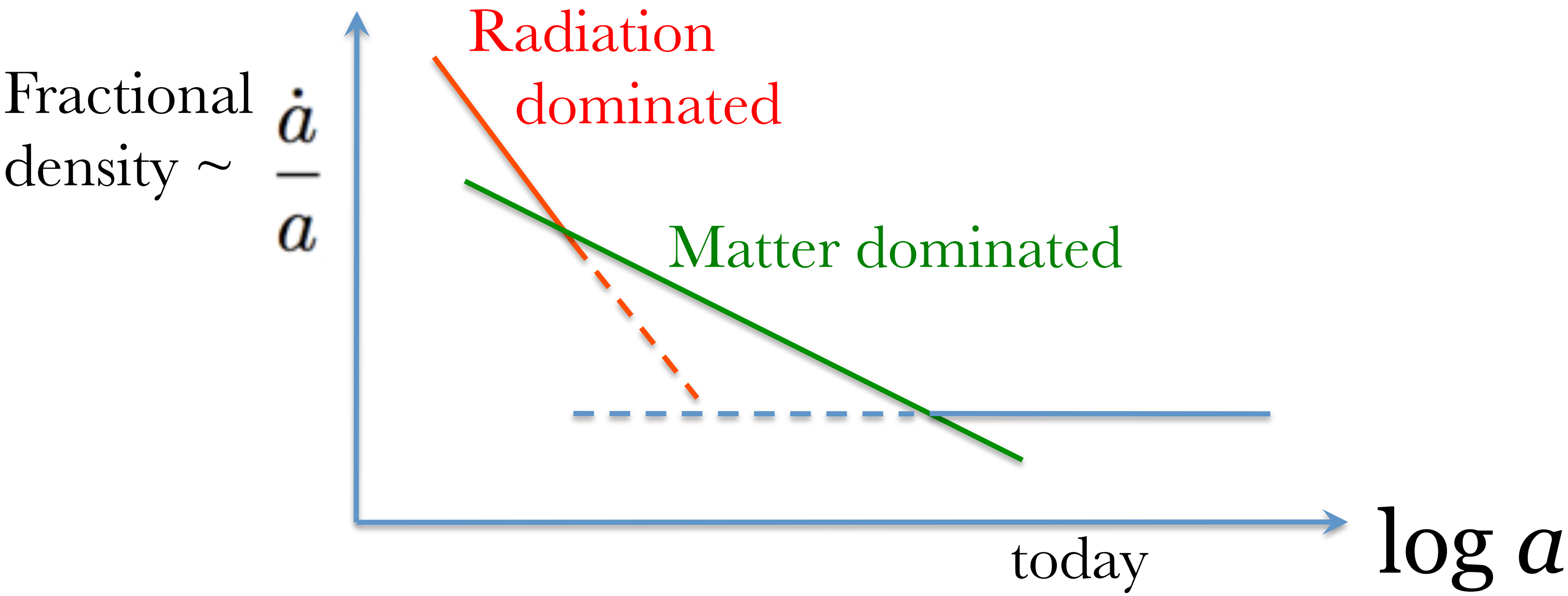
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{c^2 K}{a^2} + \frac{c^2 \Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p) + \frac{c^2 \Lambda}{3}$$

# The Friedmann Equation

For a flat universe with matter, radiation and cosmological constant,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\frac{\Omega_{\rm m}}{a^3} + \frac{\Omega_{\rm r}}{a^4} + \Omega_\Lambda\right]$$



# 講義1 クイズ

物質と宇宙定数（暗黒エネルギー）に満たされた宇宙に対するフリードマン方程式

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\frac{\Omega_\mathrm{m}}{a^3} + \Omega_\Lambda\right]$$

を解き、スケールファクター $a$ を時間の関数として図に表そう。また、現在のハッブル定数の値 $H_0$ = 70.2 km/sec/Mpc を用いて、

$\Omega_\mathrm{m} = 1$, $\Omega_\Lambda = 0$ の宇宙の年齢を求めよう。

# Pillars of Modern Cosmology

The Large-Scale Structure
of the Universe

The Cosmic Microwave
Background Radiation

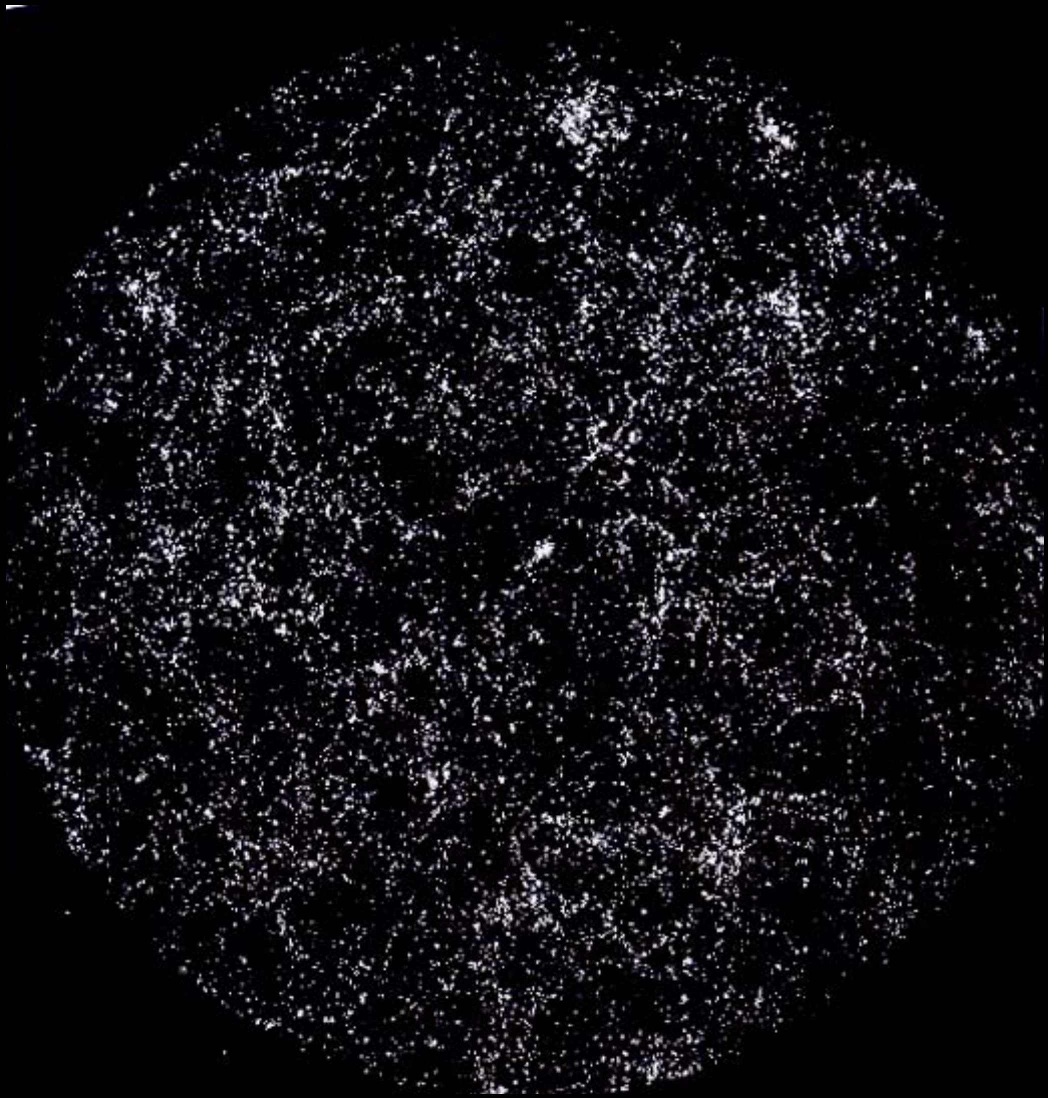
Big Bang
Nucleosynthesis

The Cosmic Expansion

*These are all relevant to structure formation.*

# Modern Cosmology I:
# The Large-Scale Structure

# The Distribution of Galaxies



Lick Galaxy Catalogue (1970's)

showed the projected

distribution of galaxies

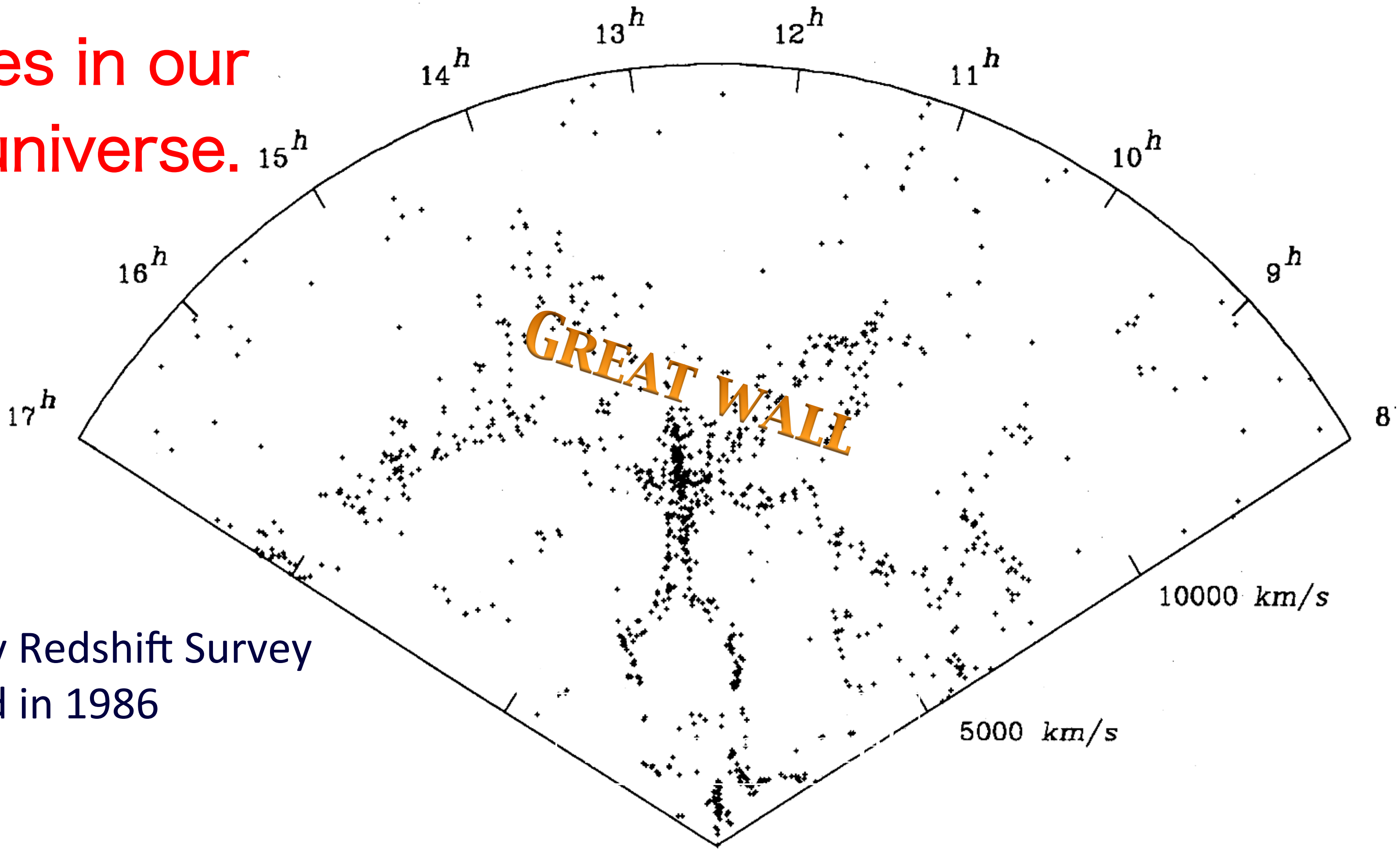
in the sky.

Galaxies are NOT

distributed randomly.

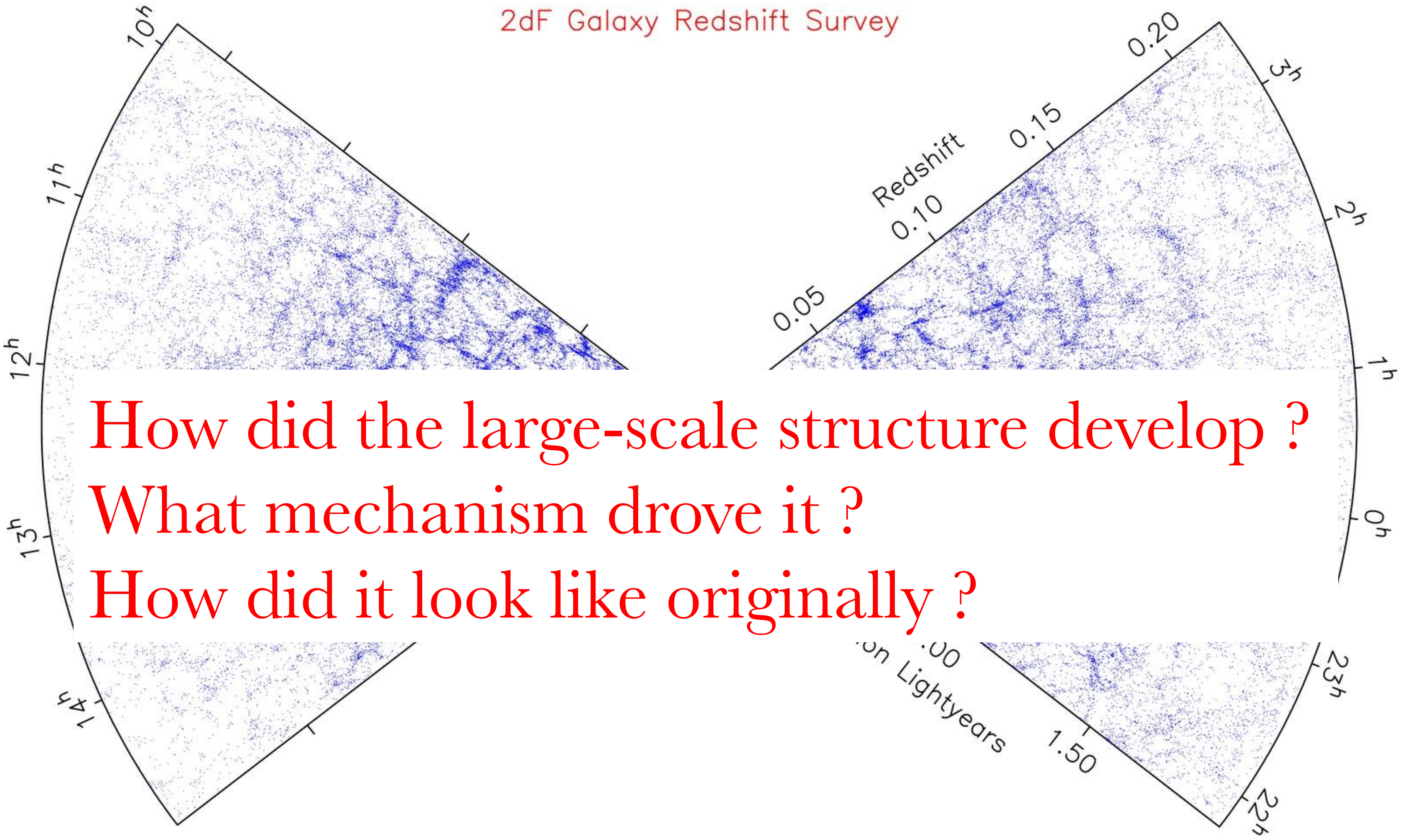
# Galaxy Redshift Survey: 3D

The distribution of galaxies in our local universe.




CfA Galaxy Redshift Survey
completed in 1986

# A Larger Map from 2dF Survey




How did the large-scale structure develop ?

What mechanism drove it ?

How did it look like originally ?

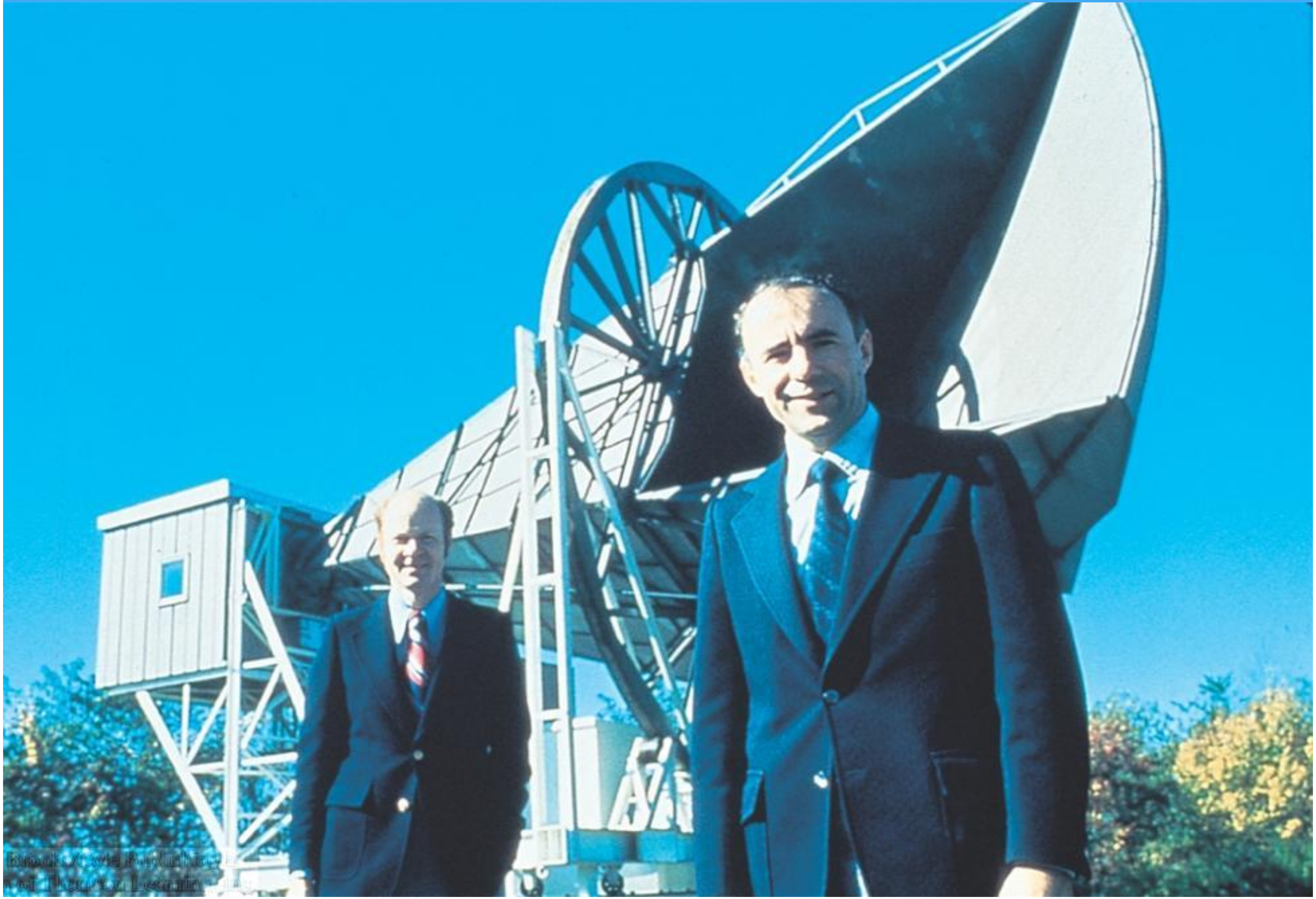
# Modern Cosmology II:
# The Cosmic Microwave Background

From the intensity ratio of the CN lines, a temperature of 2.3 K follows, which has of course only a very restricted meaning.
- G. Herzberg, 1950

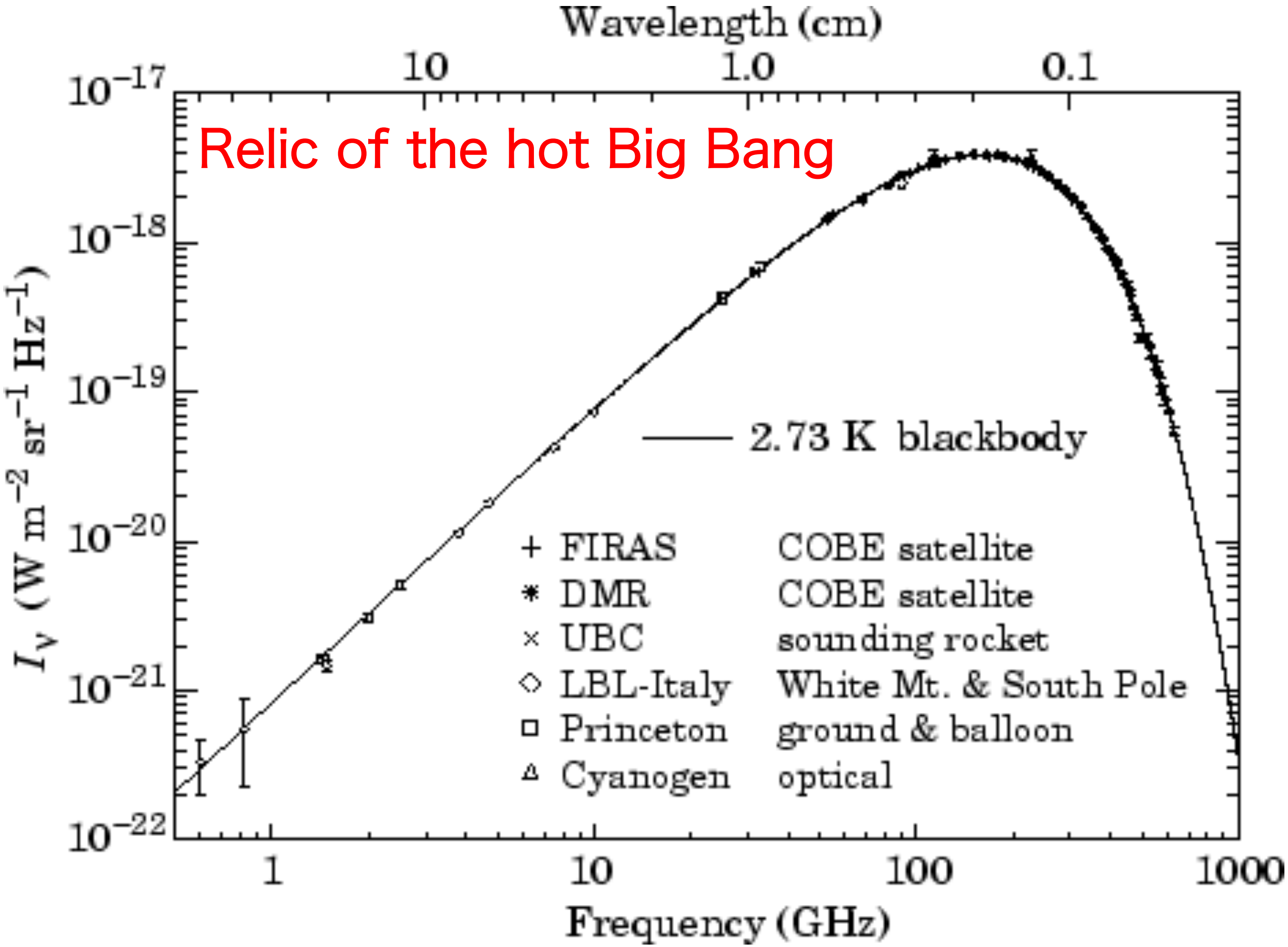
This error estimate 2±1K is based on the temperature ‘not otherwise accounted for’ in previous experiments.
- E. A. Ohm, 1961

Measurements of the temperature have yielded a value about 3.5 K higher than expected. A possible explanation is given by Dicke et al.
- A. Penzias and R. Wilson, 1965

# Penzias and Willson, with the horn telescope that discovered CMB



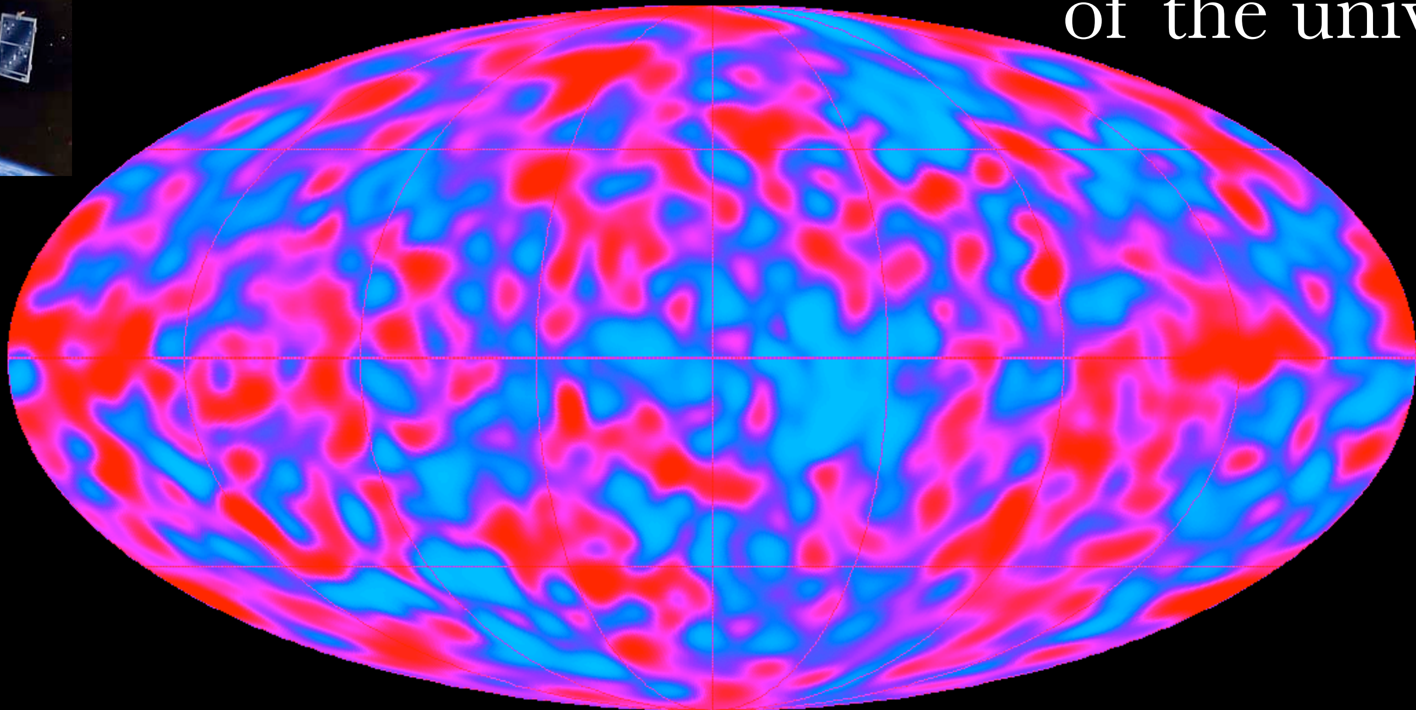
# Cosmic blackbody radiation

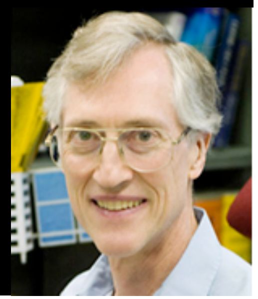


# CMB discovered late

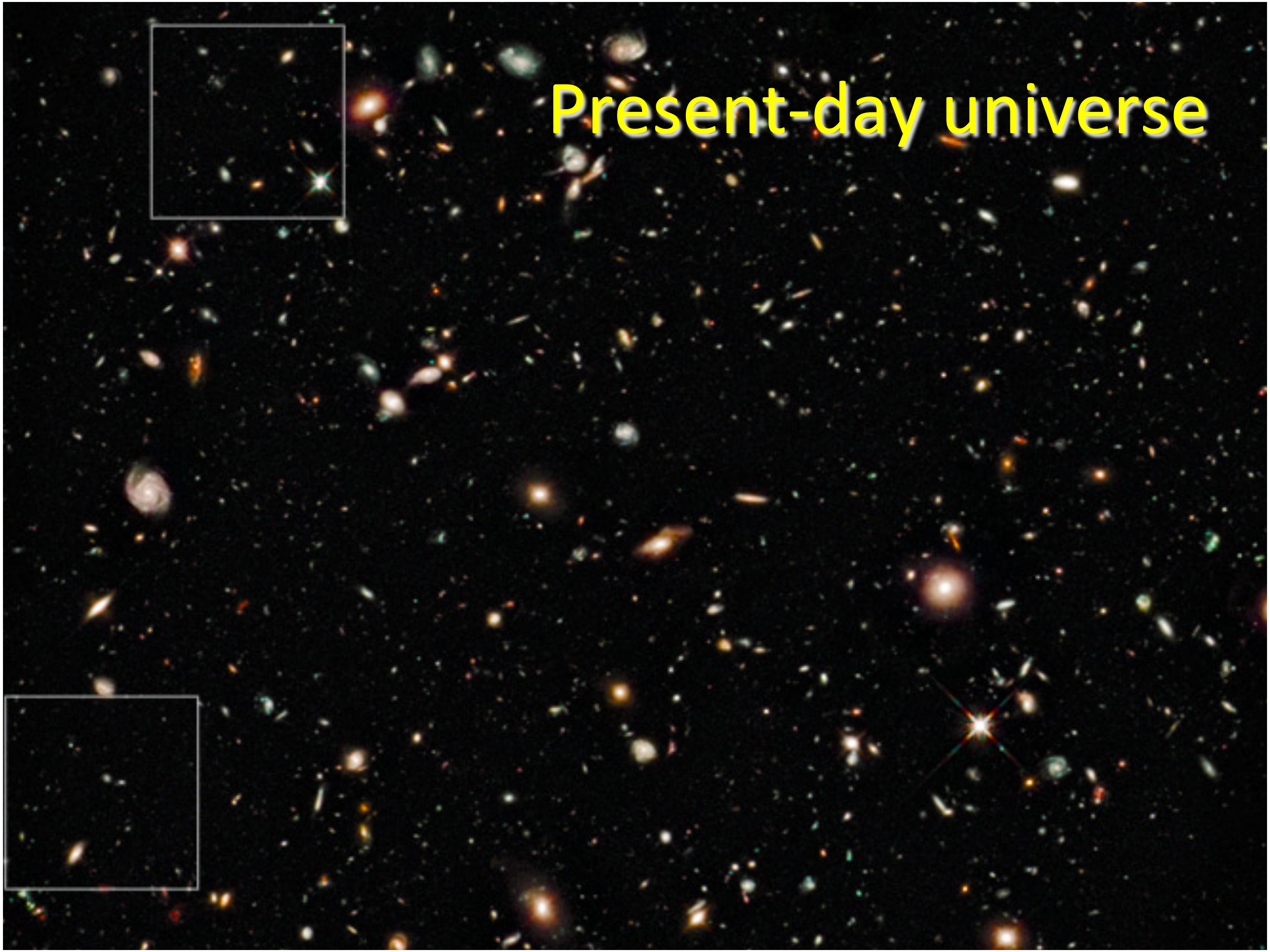
# CMB anisotropies

Tiny density fluctuations left over from the Big Bang : Origin of the structure of the universe

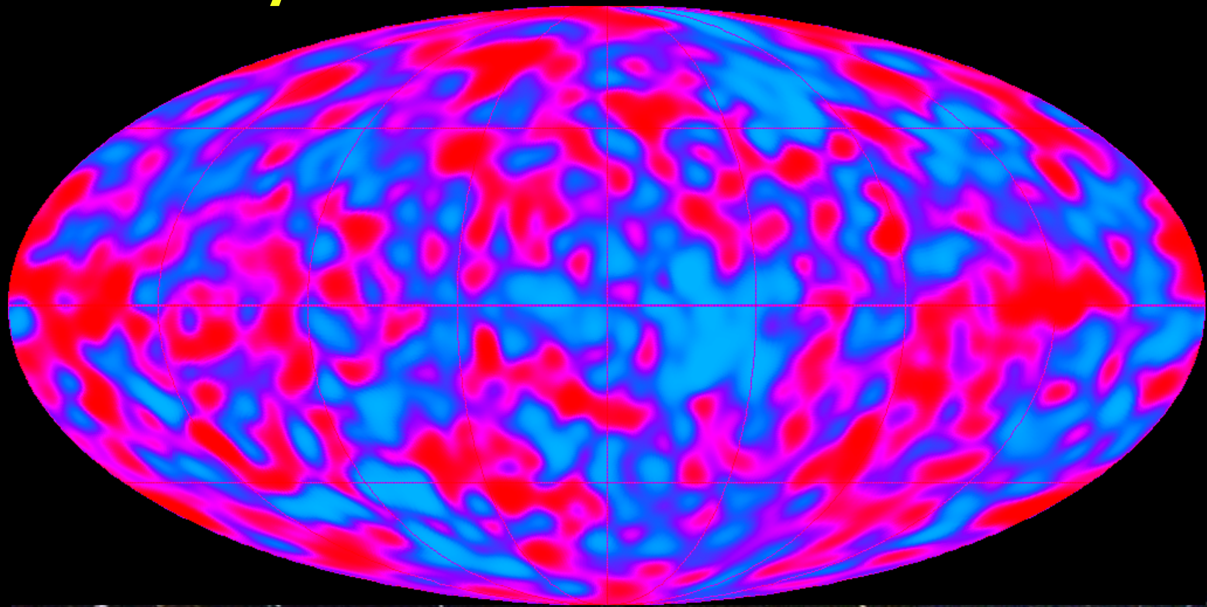




2006 Nobel Prize for Physics G. Smoot、J. Mather


Present-day universe

# Very smooth initial state



# Ending as a clumpy universe

# Growth of linear perturbations in an expanding universe

The continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

The Euler equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{\nabla P}{\rho} + \nabla \phi = 0$$

The Poisson equation

$$\nabla^2 \phi = 4 \pi G \rho$$

# The growing mode solution

The combined equations (continuity, Euler, and Poisson) lead to the second-order ordinary differential equations:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\,\bar{\rho}\,\delta = 0$$

which has two solutions. (Derivation left for exercise.)
The one we are interested in is the so-called "growing mode" which evolves as

$$\delta = D(t)\,\delta_{\text{initial}}$$

For a flat, matter-dominated universe, $D = a$
where $a$ itself scales as $\sim t^{2/3}$
Density fluctuations grow with time in the early epoch.

# The growing mode solution

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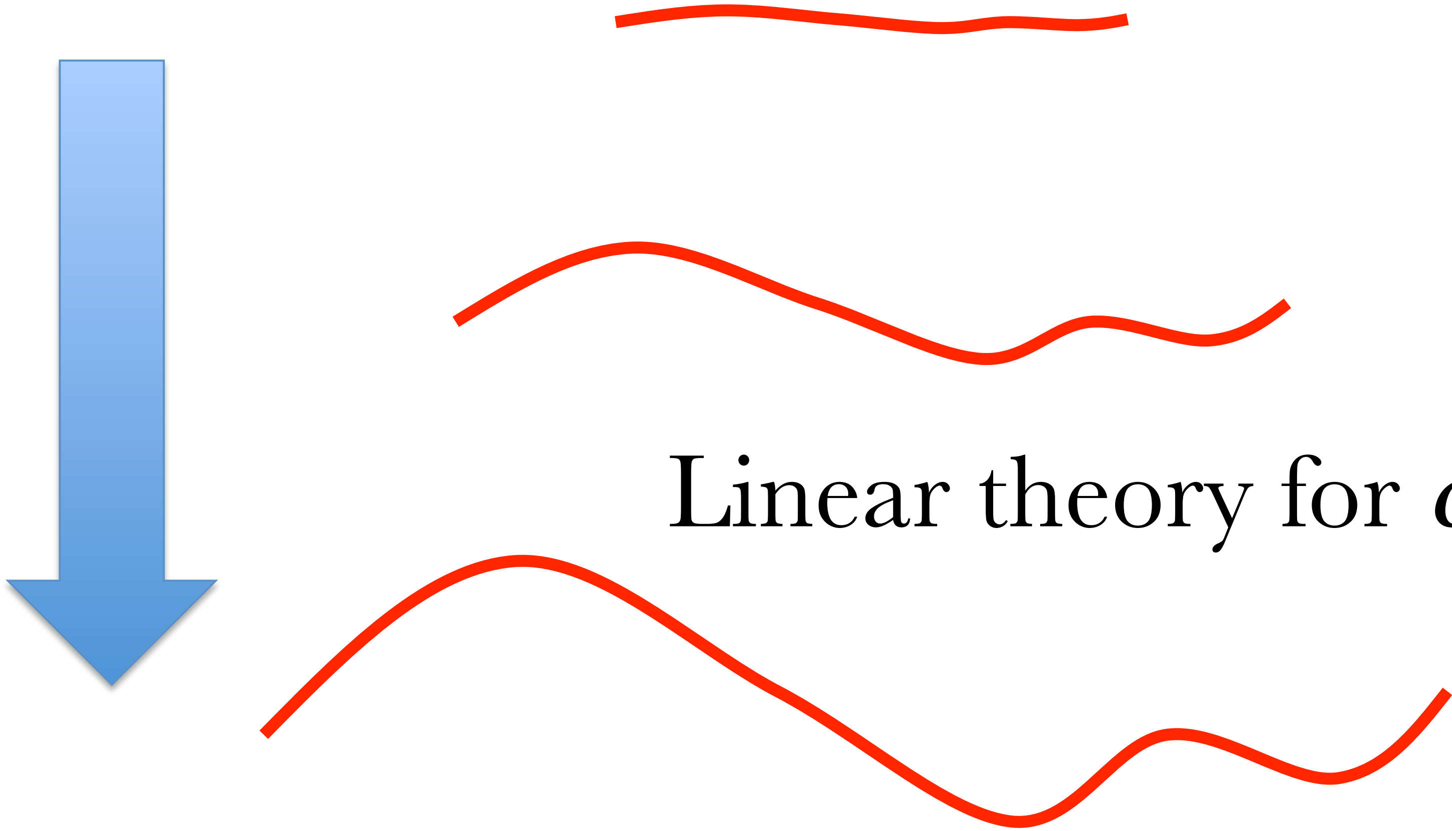



Amplitude ~ $a$ in the matter dominated era.

# The Standard Cosmological Model

# To follow structure formation

One needs to know :

1. The expansion history of the universe

   Present-day expansion rate $H_0$

2. The energy contents

   Matter, radiation, neutrinos, and the fractions

3. The initial conditions

   Statistics of the density fluctuations

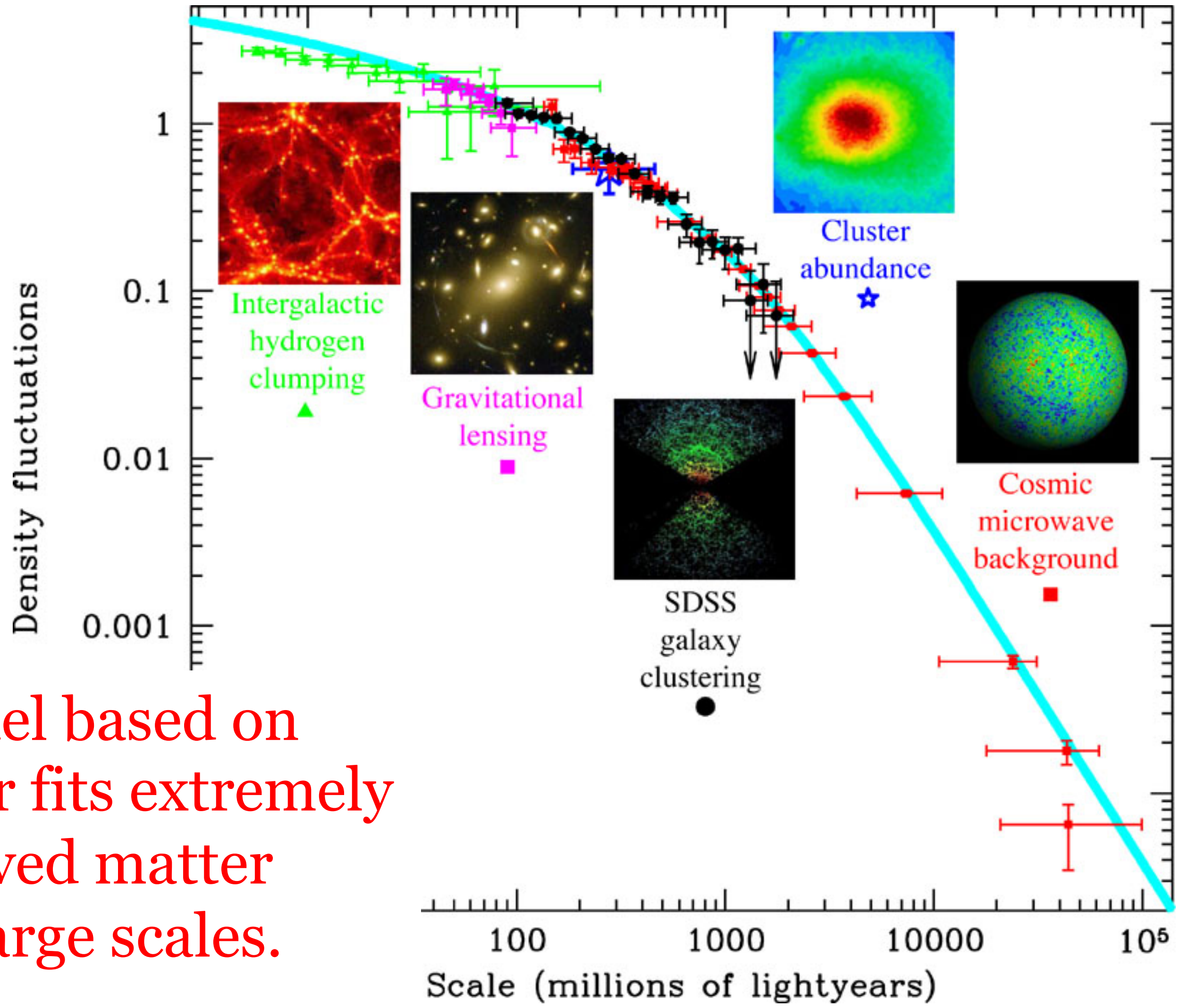
# Inflation + Cold Dark Matter

In its very early phase, the universe went through a very rapid expansion phase called “inflation”, where the density fluctuations were generated from quantum fluctuations of a scalar field.

The fluctuations are of a Gaussian random field (like a noise) and scale invariant. Just like the CMB anisotropies we observe today.

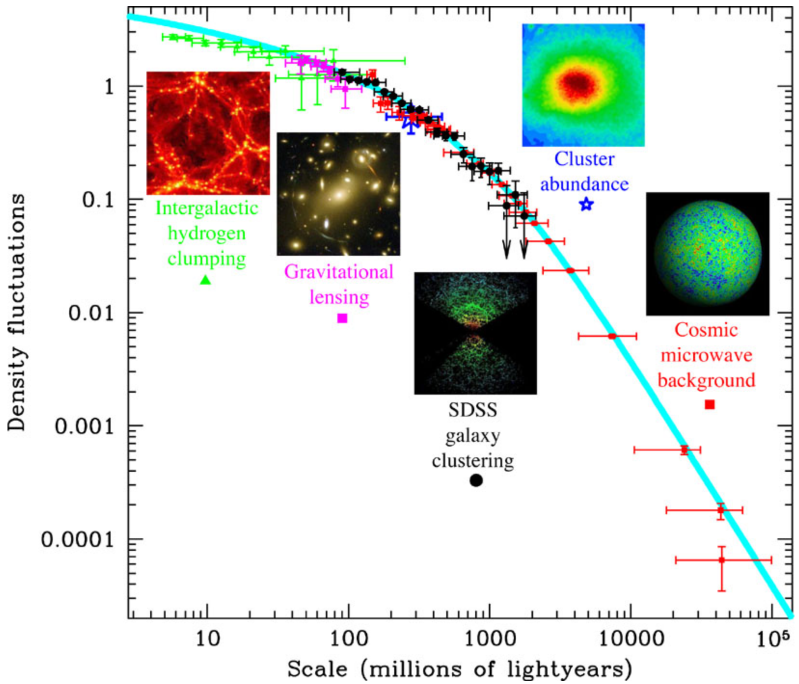
Eventually heavy Cold Dark Matter decoupled from the rest of matter and radiation. CDM dominates the matter content since then.

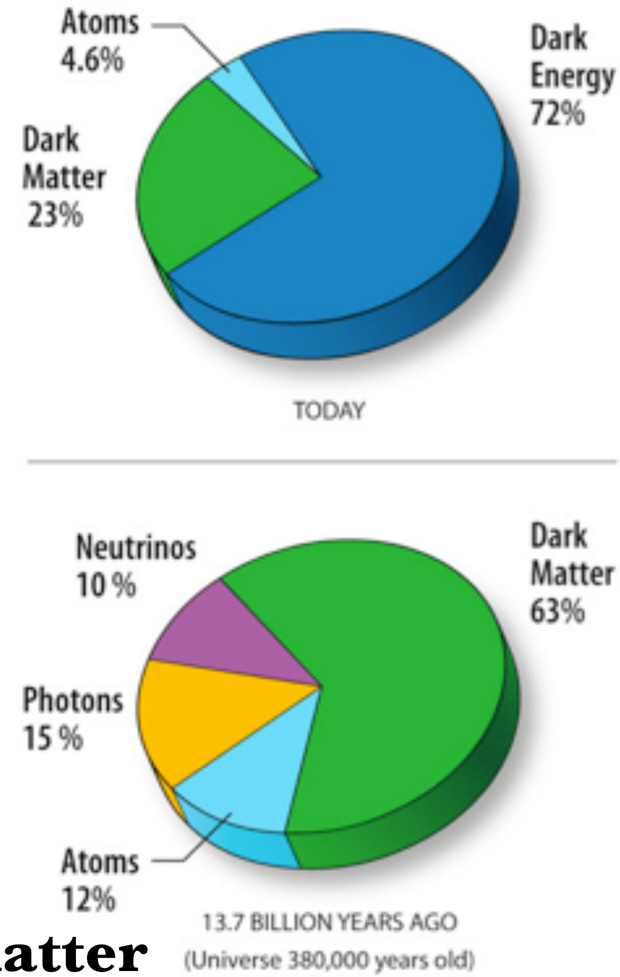
# The standard ΛCDM model




A particular model based on Cold Dark Matter fits extremely well to the observed matter distribution on large scales.

# The standard ΛCDM model



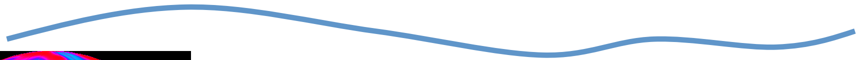



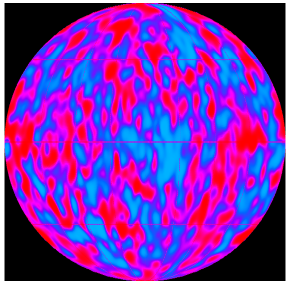

**4 % ordinary matter**
**23 % exotic (unknown) dark matter**
**72 % yet unknown dark energy**

# Gravitational Instability

## Real world

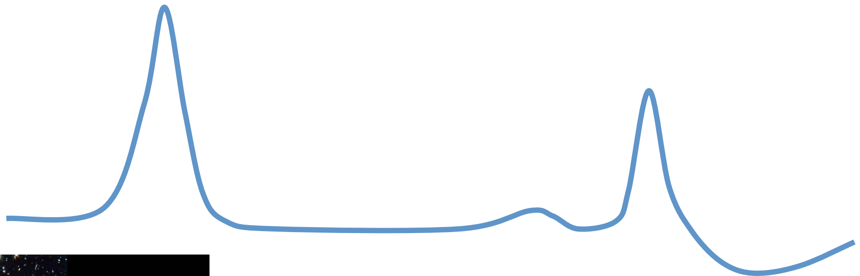
Almost homogeneous distribution

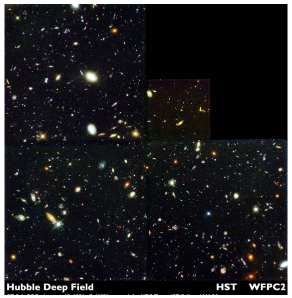




1/100,000

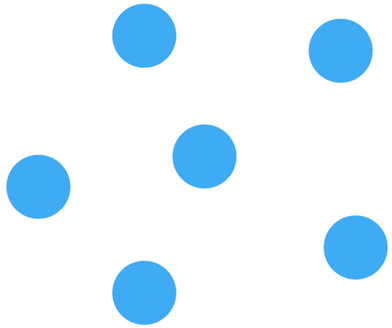
Gravity



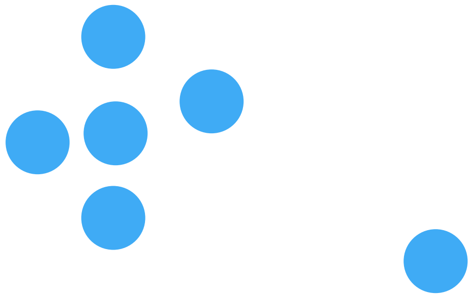



## On a computer

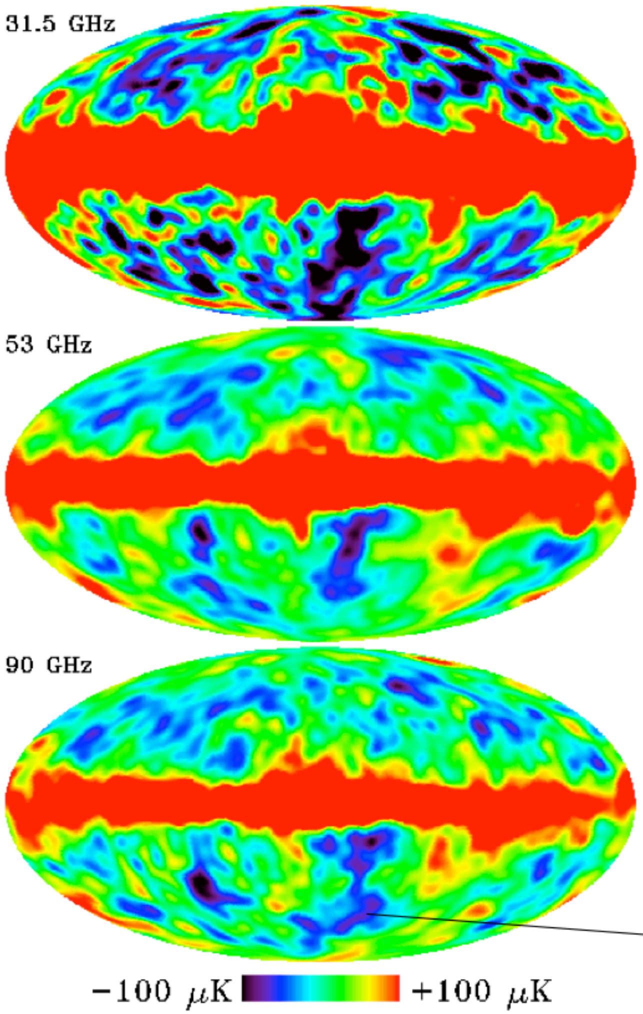
Discretized



Solve
*N*-body
interactions



# The "initial" amplitude




$$\frac{\Delta T}{T}\Big|_{\mathrm{COBE}} \sim 10^{-5}$$

In an over-dense region, there are slightly more photons, but the photons need to climb out of the gravitational potential well.
So, *does an over-dense region appear as hotter or colder ?*

Cold spot

# Fluctuation growth since z=1089

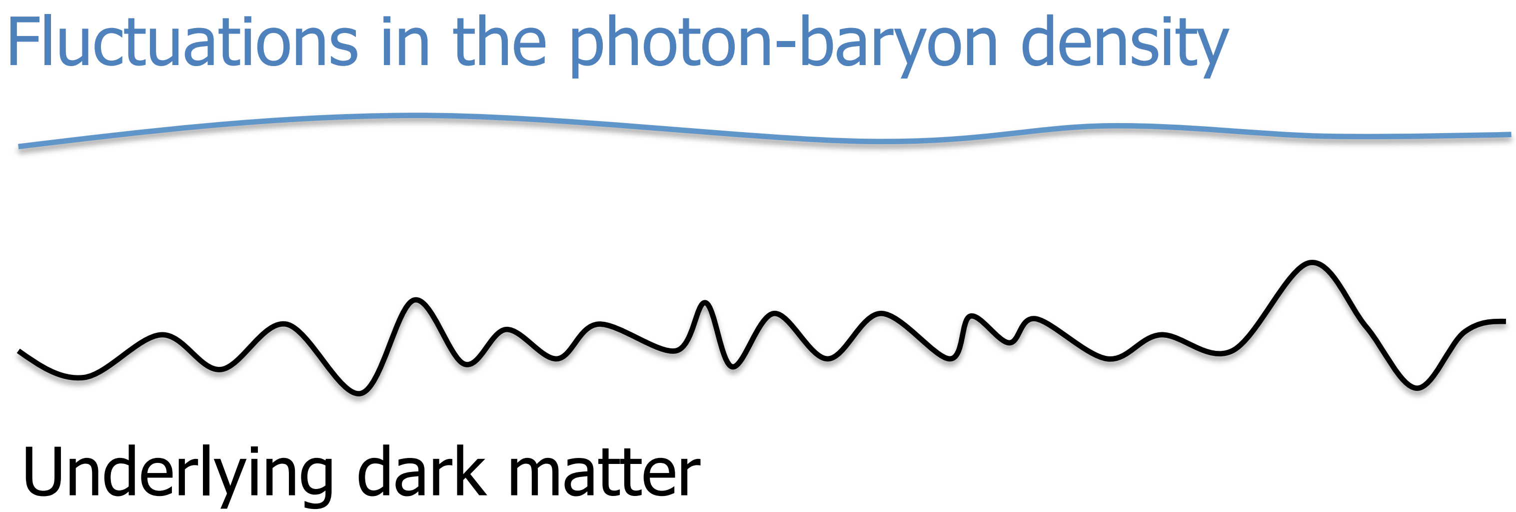
We have learned that the density fluctuations grow as $\sim$ *a* in a matter-dominated universe. The expansion parameter increases by a factor of $\sim$ 1090 since the recombination epoch when the fluctuation amplitudes were ~ $10^{-5.}$

Then the amplitude today would be ~ $10^{-2}$

Adding $\Lambda$ will only suppresses the growth at late time…

# Dark matter helps

## - Fluctuations on small-scales




Small-scale fluctuations with large amplitude can develop to clumps. These fluctuations cannot be seen in CMB because of tight photon-baryon coupling, i.e., distribution different from dark matter.

# Summary of Lecture 1.a

- Pillars of modern cosmology
  Expanding universe, CMB, large-scale structure
- The standard cosmological model based on cold dark matter
- Growth of density perturbations and nonlinear gravitational models