Constraints on R-parity violating interactions in MSSM from leptonic decays of pseudoscalar mesons

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Aida, Asakawa, Cho, HM, PRD82, 115008(2010)
Cho, HM, PLB703, 318(2011)
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Leptonic decays of Ds-meson

- decay rate of a pseudoscalar meson
  \[ \Gamma(P \rightarrow l_i \nu_j) = \frac{1}{8\pi} G_F^2 |V_{ud}|^2 f_P^2 m_{l_i}^2 m_P \left(1 - \frac{m_{l_i}^2}{m_P^2}\right)^2 \]

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CLEO, BaBar, Belle

\[ f_{D_s} = 259.0 \pm 6.9 \text{ [MeV]} \]

PRD80, 112004 (2009),

HPQCD + UKQCD

\[ f_{D_s} = 241 \pm 3 \text{ [MeV]} \]

PRL100, 062002 (2008)

2.4-sigma deviation

New Physics ... !?
Leptonic decays of Ds-meson

- decay rate of a pseudoscalar meson

\[ \Gamma(P \to l_i \nu_j) = \frac{1}{8\pi} G_F^2 |V_{ud}|^2 f_P^2 m_P^2 \left(1 - \frac{m_{l_i}^2}{m_P^2}\right)^2 \]

CLEO, BaBar, Belle

- \[ f_{D_s}^{\text{exp}} = 257.3 \pm 5.3 \text{ [MeV]} \]
- \[ f_{D_s}^{\text{SM}} = 248.9 \pm 3.9 \text{ [MeV]} \]

FNAL, MILC, HPQCD

- Pos LATTICE2010, 317(2010); arXiv: 1008.4018[hep-lat].

1.3-sigma deviation
Leptonic decays of Ds-meson

• decay rate of a pseudoscalar meson

\[
\Gamma(P \rightarrow l_i \nu_j) = \frac{1}{8\pi} G_F^2 |V_{ub}|^2 f_P^2 m_P^2 \left(1 - \frac{m_{l_i}^2}{m_P^2}\right)^2
\]

CLEO, BaBar, Belle

\[ f_{Ds}^{\text{exp}} = 257.3 \pm 5.3 \text{ [MeV]} \]

PRD82, 091103; PRD79, 052001; PRL100, 241801, ...

FNAL, MILC, HPQCD

\[ f_{Ds}^{SM} = 248.9 \pm 3.9 \text{ [MeV]} \]


1.3-sigma deviation

The “f_{Ds}-puzzle” has disappeared.

However, this result constrains NP contributions.

→ “interference effect of multi channels in NP” is important.
Leptonic decays of B-meson

**BaBar,Belle**

\[ \text{Br}(B^+ \to \tau \nu)_{\text{exp}} = (1.64 \pm 0.34) \times 10^{-4} \]


**UTfit**

\[ \text{Br}(B^+ \to \tau \nu)_{\text{SM}} = (0.805 \pm 0.071) \times 10^{-4} \]

ICHEP2010

2.4-sigma deviation
our purpose

• possibilities of New physics to these processes:
  • Two-Higgs doublet models (THDM) Ahn et al (’10), Akeroyed et al (09), Akeroyd et al (’07)...
  • Leptoquark models Dobrescu et al (’08), Benbrik et al (09), ...
  • R-parity violating SUSY (w/ “single coupling dominant hypothesis”) Beak et al (’99), Dreiner et al (’02), ... Bhattacharrya et al (’10)

• investigate possibilities of by R-parity violating SUSY-SM beyond the single coupling hypothesis
R-parity

\[ R_P \equiv (-1)^3(B-L)+2S \]

- B: baryon number, L: lepton number, S: spin quantum number
- SM particle : R-even (+1), SUSY particle : R-odd (-1)
  - as a consequence, even number of SUSY particles should appear in interaction vertices
    - proton is stable
    - lightest SUSY particle (LSP) is stable --> could be a candidate of cold dark matter
- R-parity violating interaction (RPV)
  - as a consequence, there are lepton- & baryon-number violating operators
  - phenomenological interests
    - neutrino mass w/o ultra-heavy Majorana neutrinos \textit{Hall, Suzuki (‘84)}
    - LSP is unstable, but gravitino could be dark matter \textit{Buchmuller etal (‘07)}
set up (1)

R-parity violating superpotential (trilinear int.)

\[ W_{RParityViolating} = \frac{1}{2} \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \frac{1}{2} \lambda''_{ijk} U_i D_j D_k \quad (i, j, k = 1, 2, 3) \]

\( \lambda, \lambda', \lambda'' : \) RPV couplings

we set the baryon number violating coupling \( \lambda''_{ijk} = 0 \)

\[ \begin{array}{cccc}
SU(3)_C & SU(2)_L & U(1)_Y \\
Q & 3 & 2 & 1/6 \\
U & 3^* & 1 & -2/3 \\
D & 3^* & 1 & 1/3 \\
L & 1 & 2 & -1/2 \\
E & 1 & 1 & 1 \\
\end{array} \]

previous studies on RPV couplings: ...Barger et al.'89, Bhattacharyya ('97), Allanach et al.'99, Dreiner ('07)...

Feynman diagrams of decay \( P(u_a \bar{d}_b) \rightarrow \ell_i \nu_j \)

\[ u_a \xrightarrow{(\bar{d}_R)_k} (\ell_R)_i \xrightarrow{(\bar{L})_k} \nu_j \]
set up (2)

• The new physics contribution is parametrized by

\[ r_P^2 \equiv \frac{|G_F V_{ub}^* + A_{ii}|^2}{G_F^2 |V_{ub}^*|^2} + \sum_{j(\neq i)} \frac{|A_{ij}|^2}{G_F^2 |V_{ub}^*|^2} \]

- second term in r.h.s. → leptons in final state are flavor off-diagonal

• Decay rate:

\[ \Gamma(P \rightarrow l_i \nu_j) = \frac{1}{8\pi} r_P^2 G_F^2 |V_{ub}^*|^2 f_P^2 m_{l_i}^2 m_P \left( 1 - \frac{m_{l_i}^2}{m_P^2} \right)^2 \]
set up(3) : RPV processes

\[
r_P^2 \equiv \frac{|G_F V_{ub}^* + A_{ii}^P|^2}{G_F^2 |V_{ub}|^2} + \sum_{j(\neq i)} \frac{|A_{ij}^P|^2}{G_F^2 |V_{ub}|^2}
\]

\[
W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k
\]

\[
(A_t^P)_{ij} = \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\chi_{iak}^\prime \chi_{jbk}^\prime}{m_{\tilde{d}_{Rk}}^2}
\]

\[
(A_s^P)_{ij} = -\frac{1}{2\sqrt{2}m_i} \frac{m_{\tilde{u}_a}^2}{m_{\tilde{u}_a} + m_{\tilde{d}_b}} \sum_{k=1}^{3} \frac{\lambda_{kji}^* \lambda_{kab}^\prime}{m_{\tilde{l}_{Lk}}^2}
\]
set up(4) : MSSM process

\[ r_P^2 \equiv \frac{|G_F V_{u_a d_b}^* + A_{ii}^P|^2}{G_F^2 |V_{u_a d_b}^*|^2} + \sum_{j(\neq i)} \frac{|A_{ij}^P|^2}{G_F^2 |V_{u_a d_b}^*|^2} \]

\[ A_H^P = -G_F V_{u_a d_b} \frac{m_d}{m_{u_a} + m_d} \frac{m_{P}^2}{m_{H^-}^2} \left( \tan^2 \beta - \frac{m_{u_a}}{m_d} \right) \]
set up(3) : RPV processes

$$r_P^2 \equiv \frac{|G_F V^*_{u_a d_b} + A_{ii}^P|^2}{G_F^2 |V^*_{u_a d_b}|^2} + \sum_{j(\neq i)} \frac{|A_{ij}^P|^2}{G_F^2 |V^*_{u_a d_b}|^2}$$

$$W_R = \frac{1}{2} \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k$$

$$\left(\tilde{d}_R\right)_k$$

$$\left(\tilde{\ell}_L\right)_k$$

$$(A_{t}^P)_{ij} = \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'_{jbk}}{m_{d_{R_k}}}$$

$$(A_{s}^P)_{ij} = -\frac{1}{2\sqrt{2}m_i} \frac{m_P^2}{m_{u_a} + m_{d_b}} \sum_{k=1}^{3} \frac{\lambda^*_{kj i} \lambda'_{kab}}{m^2_{L_{k}}}$$
Numerical Study (I)

inputs

\[ |V_{cs}| = 1.023 \pm 0.036, \quad |V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}, \]
\[ m_{D_s} = 1968.47 \pm 0.33 \text{ MeV}, \quad m_{B^+} = 5279.17 \pm 0.29 \text{ MeV}, \]

adopt the central values as references

assumptions

• \textbf{final state: } \tau \nu \tau
• \textbf{we set } A_{ij}^P = 0 \text{ for } i \neq j \quad \text{(constraints from LFV)}
• \textbf{t-channel: } sbottom exchange
• \textbf{s-channel: } smuon exchange \quad (\lambda_{k33}, \quad k \neq 3)

constraints on new physics

\[ r_{D_s} = 1.03 \pm 0.04 \]
\[ r_{B^+} = 1.43 \pm 0.21 \]
Numerical Study (2)

[assumptions]

- $\lambda^2 \equiv |\lambda_{323}^\prime|^2 = \lambda_{222}^\prime \lambda_{233}^{*}\lambda_{233}$
- squark/slepton masses: $\tilde{m} = 100$ GeV

\[
D_s \to \tau \nu
\]

\[
r_P = \frac{|G_F V_{u_a d_b}^* + A^{P}_{i i}|}{G_F |V_{u_a d_b}^*|}
\]

- $\sum_{k=1}^{3} \frac{\lambda_{iak}^* \lambda_{jbk}^*}{m_{d_{Rk}}^2}$
- $\sum_{k=1}^{3} \frac{\lambda_{kji}^* \lambda_{kab}^*}{m_{l_{Rk}}^2}$

Cancellation between the s- and t-channel amplitudes!
\[ D_s \rightarrow \tau \nu \]

- **squark/slepton masses:** \( \tilde{m} = 500 \text{ GeV} \)
- \( \lambda' \lambda' \) (t-ch.) of \( D_s \rightarrow \) always positive
- signs of \( \lambda' \lambda' \) (t-ch.) and \( \lambda \lambda' \) (s-ch.)
  - **same:**
  - due to **destructive** interference between s-ch and t-ch,
  - RPV couplings can be large
  
- **opposite:**
  - due to **constructive** interference between s-ch and t-ch,
  - RPV couplings are constrained

\[ 0 < |\lambda'_{323}|^2 < 1.2 \quad (t\text{-ch}) \]
\[ -0.4 < \lambda^*_{233} \lambda'_{222} < 0 \quad (s\text{-ch}) \]
\( B^+ \rightarrow \tau \nu \)

- combination of opposite sign of s, t-channel couplings is constrained
\[ B^+ \rightarrow \tau \nu \]

- combination of opposite sign of s, t-channel couplings is constrained

\[
\begin{align*}
(A^P_t)_{ij} &= \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'^*_{jbk}}{m^2_{d_{Rk}}} \\
(A^P_s)_{ij} &= -\frac{1}{2\sqrt{2}m_i} \frac{m^2_P}{m_{u_a} + m_{d_b}} \sum_{k=1}^{3} \frac{\lambda'^*_{kji} \lambda'^*_{kab}}{m^2_{l_{Lk}}} \\
r_{D_s} &= |G_F V^*_{cs} + A_t + A_s/G_F V^*_{cs}| 
\end{align*}
\]
\( B^+ \to \tau \nu \)

- Combination of opposite sign of s, t-channel couplings is constrained

\[
(A_t^P)_{ij} = \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'_{jbk}}{m_{d_{Rk}}^2}
\]

\[
(A_s^P)_{ij} = -\frac{1}{2\sqrt{2}m_i} \frac{m_P^2}{m_{u_a} + m_{d_b}} \sum_{k=1}^{3} \frac{\lambda'_{kji} \lambda'_{kab}}{m_{l_{Lk}}^2}
\]

\[
r_D = |G_F V_{cs}^* + A_t + A_s / G_F V_{cs}^*|
\]
$B^+ \rightarrow \tau \nu$

- combination of opposite sign of $s, t$-channel couplings is constrained
- $t$-channel coupling is constrained by $B^+ \rightarrow \pi^+ \nu \bar{\nu}$

\[
\lambda'_{313} \lambda'_{333} < 2.5 \times 10^{-2}
\]

for BR\(<1 \times 10^{-4} \ \text{@BaBar, (PRL94, 101801 (2005))}

\[
(A_t^P)_{ij} = \frac{1}{4\sqrt{2}} \sum_{k=1}^{3} \frac{\lambda'_{iak} \lambda'_{jbk}}{m^2_{d_{Rk}}}
\]

\[
(A_s^P)_{ij} = -\frac{1}{2\sqrt{2}m_t} \frac{m^2_P}{m_{u_a} + m_{d_b}} \sum_{k=1}^{3} \frac{\lambda'_{kji} \lambda'_{kab}}{m^2_{l_{Lk}}}
\]

\[
r_{D_s} = |G_F V_{cs}^* + A_t + A_s/G_F V_{cs}^*|
\]
\[ B^+ \rightarrow \tau \nu \]

- combination of opposite sign of s, t-channel couplings is constrained
- t-channel coupling is constrained by \( B^+ \rightarrow \pi^+ \nu \bar{\nu} \)

\[
\lambda_{313}' \lambda_{333}'^* < 2.5 \times 10^{-2} \quad \text{PLB681,44 (2009)}
\]

for BR<1 \times 10^{-4} \ @BaBar,(PRL94, 101801 (2005))

\[
\rightarrow \text{for positive t-ch coupling,}
\]

\[
1.9 \times 10^{-2} < \lambda_{213}' \lambda_{233}^* < 2.6 \times 10^{-2}
\]

\[-0.5 \times 10^{-2} < \lambda_{213}' \lambda_{233}^* < 0.1 \times 10^{-2}\]

\[ r_{D_s} = |G_F V_{cs}^* + A_t + A_s/G_F V_{cs}^*| \]
\[ r_P^2 \equiv \left| G_F V_{u_a d_b}^* + A_{ii}^p \right|^2 \frac{G_F^2 |V_{u_a d_b}^*|^2}{2} + \sum_{j(\neq i)} \frac{A_{ij}^P|^2}{G_F^2 |V_{u_a d_b}^*|^2} \]

**assumptions**

- **final state:** \( \tau \nu_j \ (j = e, \mu) \)
- **we set** \( (r_p^2)_{ii} = 1 \)
- **squark/slepton masses:** \( \tilde{m} = 500 \ \text{GeV} \)
RPV-LFV process: $B^+ \rightarrow \tau \nu$

Taking account of contributions from $\lambda_{k13} \lambda_{k13}' (B^0 \rightarrow \tau \mu)$ and $\lambda_{k23} \lambda_{k13}' (B^0 \rightarrow \tau \mu)$, $r_{B^+} \sim 1.3$. 

$\lambda_{k23} \lambda_{k13}' (B^0 \rightarrow \tau \mu) < 4.3 \times 10^{-3}$

for BR < $28 \times 10^{-6}$ @BaBar(PRD77, 091104(2008))

$\lambda_{k13} \lambda_{k13}' (B^0 \rightarrow \tau e) < 4.9 \times 10^{-3}$

for BR < $22 \times 10^{-6}$ @BaBar(PRD77, 091104(2008))

$r_{B^+} \sim 1.1$
RPV-LFV process: $B^+ \rightarrow \tau \nu$

\[ \lambda_{i1k} \lambda_{j3k} \left( i, j \neq 3 \right) \] is shown in the diagram.

The process is described by the equation:

\[ \lambda_{i1k} \lambda_{j3k} \times 10^{-2} \]

for BR < $1 \times 10^{-4}$ @BaBar (PRL94, 101801 (2005))

B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 94, 101801 (2005)

$B^+ \rightarrow \pi^+ \nu \bar{\nu}$

\[ r_{B^+} \sim 1.1 \]
RPV-LFV process: $D_s \rightarrow \tau \nu$

sdown t-channel exchange

$\lambda'_{j2k}$

$\lambda'_{32k}$

$\lambda'_{12k}$

$\tau \rightarrow \mu \gamma$

$\nu_j$

$r_{D_s}^{22} \sim 1.0 + \mathcal{O}(10^{-4})$

Bhattacharyya et al, NPB831, 344 (’10)

$\tau \rightarrow l_j + \gamma$

for $Br < 3.3 \times 10^{-8}$ @ BABAR (PRL104 , 021802(2010))

$\lambda'_{32k} \lambda'_{22k}$

$\tau \rightarrow e \gamma$

$\lambda'_{32k} \lambda'_{22k}$

$\tau \rightarrow \mu \gamma < 0.29$

for $Br < 4.4 \times 10^{-8}$ @ BABAR (PRL104 , 021802(2010))
RPV-LFV process: $D_s \rightarrow \tau \nu$

- $\lambda_{k13} \lambda'_{k22} (\tau \rightarrow e\eta)$
  
  \begin{equation*}
  \text{for } Br < 4.4 \times 10^{-8} \quad \text{@ BELLE (ICHEP2010)}
  \end{equation*}

- $\lambda_{k23} \lambda'_{k22} (\tau \rightarrow \mu\eta) < 7.8 \times 10^{-3}$
  
  \begin{equation*}
  \text{for } Br < 2.3 \times 10^{-8} \quad \text{@ BELLE (ICHEP2010)}
  \end{equation*}

- $r_{D_s} \sim 1.0 + \mathcal{O}(10^{-6})$

Li et al, PRD73,073005
summary

- leptonic decays of $D_s$ and $B^+$ mesons are studied in SUSY-SM with RPV interactions (beyond “single coupling dominance hypothesis”)

- both constructive and destructive interference between s- and t-channel amplitudes are possible

- because of cancellation between s- and t-channels, size of RPV couplings cannot be restricted so that production/decay of SUSY particles through RPV interactions may be enhanced without conflicting the data of leptonic decays

- contributions from the flavor off-diagonal interactions are discussed under constraints from LFV experiments
back up
charged higgs

\[ A_H^P = -G_F V_{u_a d_b}^{*} \frac{m_{d_b}}{m_{u_a} + m_{d_b}} \frac{m_{P}^2}{m_{H^-}^2} \left( \tan^2 \beta - \frac{m_{u_a}}{m_{d_b}} \right) \]

\[ D_s \rightarrow \tau \nu \]

\[ B^+ \rightarrow \tau \nu \]

\[ \tau_{D_s} \]

\[ \tau_{B^+} \]

\[ m_{H^-} (GeV) \]

\[ m_{H^-} (GeV) \]
**charged higgs :** \( B^+ \rightarrow \tau \nu \)

- **blue : RPV only**
- **green : RPV + charged Higgs**

**input**
- \( m_{H^-} = 300 \text{GeV} \)
- \( \tan \beta = 50 \)

- **taking account of**
- (RPV + charged Higgs) contributions,

\[
0.5 \times 10^{-2} < \chi'_{213} \lambda^*_{233} < 2.0 \times 10^{-2} \\
-1.2 \times 10^{-2} < \chi'_{213} \lambda^*_{233} < -0.5 \times 10^{-2}
\]

for positive t-ch. coupling

\[
A_H^p = -G_F V_{ud}^* \frac{m_d}{m_u + m_d} \frac{m_p^2}{m_{H^-}} \left( \tan^2 \beta - \frac{m_u}{m_d} \right) \\
\rho_P = |G_F V_{ui}^* d_j + A_t + A_s + A_H/G_F V_{ui}^* d_j| \]

**1- sigma allowed regions**
RPV-LFV process: $B^+ \rightarrow \tau \nu$

BR($B^0 \rightarrow l_i l_j$) $[\times 10^{-5}]$

$\lambda_{k3} \lambda_{k13}'$ $[\times 10^{-3}]$ s-ch.

$B^0 \rightarrow \tau e$ ($j = 1$)

$B^0 \rightarrow \tau \mu$ ($j = 2$)

$slepton \ s-channel \ exchange$

$\lambda_{k23} \lambda_{k13}' \ (B^0 \rightarrow \tau \mu) < 4.3 \times 10^{-3}$

for BR $< 28 \times 10^{-6}$ \textit{@BaBar}(PRD77, 091104)

$\lambda_{k13} \lambda_{k13}' \ (B^0 \rightarrow \tau e) < 4.9 \times 10^{-3}$

for BR$<22 \times 10^{-6}$ \textit{@BaBar}(PRD77,091104)
result

\[ D_s \rightarrow \tau \nu_{\tau} \quad s-t \text{の干渉} \]

\[ r_{D_s} = 1.07 \pm 0.04 \] の領域に等高線をひく
and tonic decays of to the flavor off-diagonal part, flavor violation

3.2. Constraints on the RPV couplings from the final state with lepton decay processes of SUSY particles due to the RPV interactions at large simultaneously, it may lead to observation of productions or channel diagrams. However, since several RPV couplings could be of the relevant RPV couplings, the size of the couplings cannot the

\[ \Delta \tau_{D_s} \]

The flavor off-diagonal contributions to the \( A_{ij} \) parameter as a function of RPV couplings. The vertical line denotes the experimental bound on the RPV couplings from

\[ \tau \rightarrow \mu \gamma \]

\[ \lambda_{\tau \lambda}^{i} \lambda_{\lambda \lambda}^{i} + \lambda_{\tau \lambda}^{i} \lambda_{\lambda \lambda}^{i} \]

\[ \Delta \tau_{D_s} \]

\[ \times \lambda_{32}^{i} \lambda_{22}^{i} \]

\[ 10^{-6} \]

\[ 10^{-5} \]

\[ 10^{-4} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 10^{-1} \]
3.2. Constraints on the RPV couplings from the final state with lepton decay processes of SUSY particles due to the RPV interactions at large simultaneously, it may lead to observation of productions or channel diagrams. However, since several RPV couplings could be the size of the couplings cannot the site signs are strongly constrained. As can be seen in Fig. 3.

The flavor off-diagonal contributions to the A\_ij P\_ii term in Eq. (3) is negligible due to the cancellation between the \( s R \rightarrow t \) and \( t R \rightarrow s \) mesons are useful to constrain the sign \( r, \) when the flavor diagonal conservative, one may expect that the deviation from the SM prediction of the relevant RPV couplings, the size of the couplings cannot the

First we examine the lepton flavor off-diagonal terms in Eq. \( \lambda 32. \) We have so far neglected the contributions of RPV interactions to the flavor off-diagonal part, \( \lambda 313 \)′. We introduced a parameter \( \lambda 0 \) in the following

They found the constraint on the RPV couplings from \( \text{B-factory with the integrated luminosity 5 ab}^{-1} \) as a function of RPV couplings. The vertical line denotes the experimental bound on the RPV couplings from Fig. 2.

In this subsection we

The Feynman diagram of the \( \tau R \rightarrow \gamma s \) → \( \mu \eta \) channel coupling is constrained to be positive (\( \lambda 321 \)′). We replace \( \Delta \tau D_s \) with \( \lambda 22 \) and \( \lambda 23 \) → \( \lambda 22 \). For \( \lambda 32 \) → \( \lambda 32 \), replacing \( \Delta \tau D_s \) with \( \lambda 22 \).

As can be seen in Fig. 3.
Back up slides

\[ \Gamma(Q \to q\bar{q}e\nu_e) = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{Qq}|^2 f \left( \frac{m_q^2}{m_Q^2} \right) \]

\[ f(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln r \]

\[ \text{Br}(b \to c\nu e) = 0.11 \]

\[ \Gamma(b \to d\ell^+_i \ell^-_j) = \frac{1}{12} \frac{m_b^5}{512\pi^3} \left( \frac{\lambda'_{k13} \lambda_{k23}}{m_{\nu_k}^2} \right)^2 \]

\[ \text{Br}(b \to d\ell^+_i \ell^-_j) = \frac{\Gamma(b \to d\ell^+_i \ell^-_j)}{\Gamma(b \to c\nu e)} \text{Br}(b \to c\nu e) \]

\[ \text{Br}(B^+ \to \pi^+ e^\pm \mu^\mp) < 0.17 \times 10^{-6} \]

HFAG
\[
\text{Br}(b \to d l_i^+ l_j^-) = \frac{\Gamma(b \to d l_i^+ l_j^-)}{\Gamma(b \to c e \nu_e)} \text{Br}(b \to c e \nu_e)
\]

\[
= \frac{1}{32 G_F^2 |V_{cb}|^2 f(m_c^2/m_b^2)} \left( \frac{\lambda'_{k13} \lambda_{k23}}{m_{\bar{\nu}_k}^2} \right)^2 
\approx 3.7 \times 10^2 \times \left( \frac{\lambda'_{k13} \lambda_{k23}}{(m_{\bar{\nu}_k}/100 \text{GeV})^2} \right)^2
\]

\[
\rightarrow \lambda'_{k13} \lambda_{k23} < 2.1 \times 10^{-5}
\]

\[
f(m_c^2/m_b^2) \approx 0.484
\]

\[
V_{cb} = (40.6 \pm 1.3) \times 10^{-3}
\]

\[
\text{Br}(B^+ \to \pi^+ e^\pm \mu^\mp) < 0.17 \times 10^{-6}
\]

\text{HFA}G

x