10d supergravity : $\boxed{\text{3 types.}}$

$\boxed{\text{Type IIA}}$

boson: $g_{(\mu\nu)}$, $B_{[\mu\nu]}$, ϕ , C_μ , $C_{[\mu\nu\rho]}$

128: 35, 28, 1, 8, 56

fermion: $\psi_{\mu\alpha}$, $\psi_{\mu\dot{\alpha}}$, ψ_α , $\psi_{\dot{\alpha}}$

128: 56, 56, 8, 8

$\boxed{\text{Type IIB}}$

boson: $g_{(\mu\nu)}$, $B_{[\mu\nu]}$, ϕ , C , $C_{[\mu\nu]}$, $C_{[\mu\nu\rho\sigma]}$

128: 35, 28, 1, 1, 28, 35

fermion: $\psi_{\mu\alpha}$, $\psi'_{\mu\alpha}$, ψ_α , ψ'_α

128: 56, 56, 8, 8

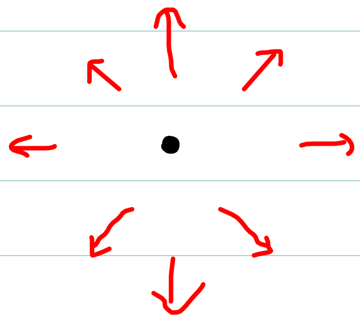
Again, you can't add gauge fields for type IIA, IIB supergravity.

But they have D-branes.

Let's revisit the relation between charged particles and Maxwell field.

A charged particle at the origin creates

$$\Delta V(\vec{x}) = e\,\delta^{(3)}(x)$$

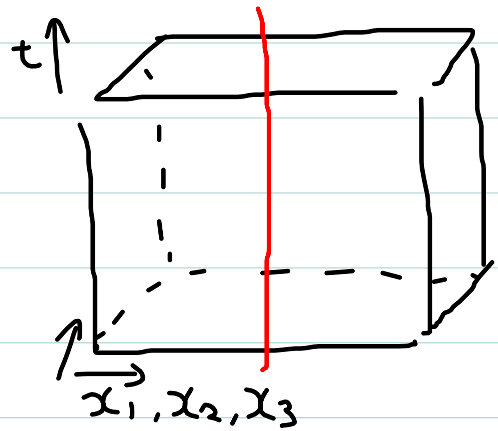


$$\longrightarrow V(\vec{x}) = \frac{e}{r}$$

$$\longrightarrow \vec{E} = \frac{e}{r^2}\,\hat{\vec{x}}$$

$$\longrightarrow \int_{S^2} \vec{E}\cdot d\vec{n} = e.$$

How does it arise from relativistic mechanics?



$$S = \int d^4x \, \vec{E}^2 + e\int dt \, V(x_1 = x_2 = x_3 = 0)$$

where $\vec{E} = \vec{\nabla} V$

$$\vec{\nabla} \cdot \vec{\nabla} V = e\,\delta^{(3)}(x)$$

$$\Downarrow$$

$$S = \int d^4x \, F_{\mu\nu} F_{\mu\nu} + \boxed{e\int d\tau \cdot \frac{dx^\mu}{d\tau} \cdot A_\mu}$$

$$\begin{cases} F_{0i} = E_i \\ F_{ij} = B_k \end{cases}$$

$$A_\mu = (V, A_1, A_2, A_3)$$

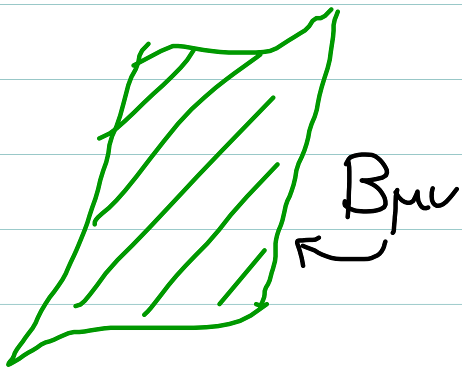
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

A particle worldline

can naturally couple to A_μ.

and produces $F_{\mu\nu} = \partial_\mu A_\nu - \cdots$.

A string worldsheet



can naturally couple to $B_{[\mu\nu]}$

$$\int d\tau\, d\sigma \underbrace{\frac{dx^{[\mu}}{d\tau}\frac{dx^{\nu]}}{d\sigma}}_{\text{Jacobian}} B_{\mu\nu}$$

and produces $G_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \cdots$.

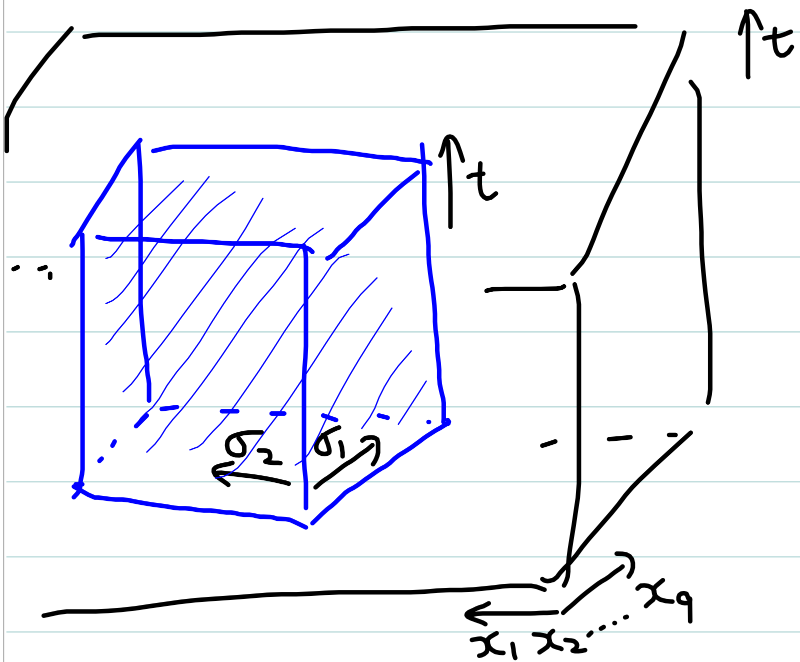
And indeed, 10d supergravities have $B_{\mu\nu}$:

$$
\begin{array}{lllll}
\mathrm{IIA}: & g_{(\mu\nu)}, & B_{[\mu\nu]}, & \phi\ ; & C_\mu\ ,\ C_{[\mu\nu\rho]} \\
\mathrm{IIB} & g_{(\mu\nu)}, & B_{[\mu\nu]}, & \phi\ ; & C\ ,\ C_{[\mu\nu]}\ ,\ C_{[\mu\nu\rho\sigma]} \\
\mathrm{Het} & g_{(\mu\nu)}, & B_{[\mu\nu]}, & \phi & \\
SO(32),\ E_8\times E_8 & & & & A_\mu^a
\end{array}
$$

But then,
what are they??

This question troubled string theorists for about ten years. (1984 ~ 1995)

They couple to D-branes. For example:



$$\int d\sigma_1 d\sigma_2 dt \, \frac{dx^{[\mu} dx^{\nu} dx^{\rho]}}{d\sigma_1 d\sigma_2 dt} C_{\mu\nu\rho}$$

is a natural coupling of a membrane to $C_{\mu\nu\rho}$.

2 spatial dimensions

+ 1 time direction.

Produces

$$G_{\mu\nu\rho\sigma} = \partial_\mu C_{\nu\rho\sigma} + \cdots .$$

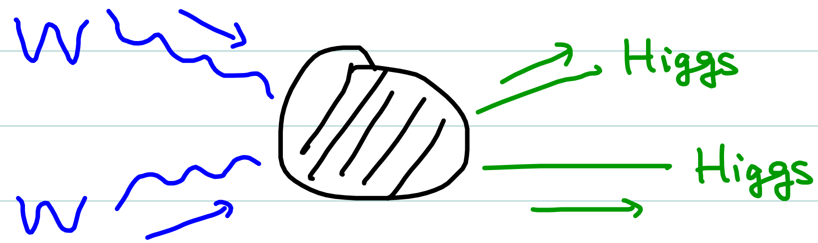
$C \qquad C_\mu \qquad C_{\mu\nu} \qquad C_{\mu\nu\rho} \qquad C_{\mu\nu\rho\sigma} \qquad \cdots$

particle. string. membrane. we didn't have a word.

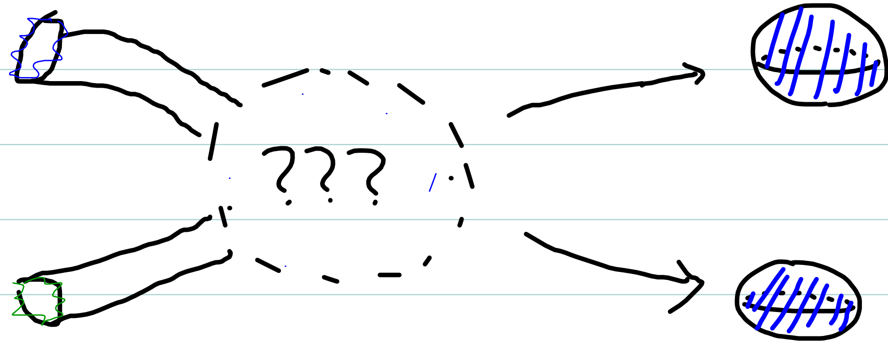
(-1)-brane 0-brane 1-brane 2-brane 3-brane

So, all these things exist in type II string theory.

It's like the SM... W-bosons themselves are not unitary. There need to be the Higgs bosons.



Similary, if you scatter strings really hard,



these <u>branes</u> are pair-produced.

In this way, string theory contains

membranes, 3-branes, 4-branes ...

automatically.

$\boxed{\text{Type IIA}}$

$g_{(\mu\nu)}, B_{[\mu\nu]}, \phi \qquad C_\mu \qquad C_{[\mu\nu\rho]}$

string   0-brane   2-brane

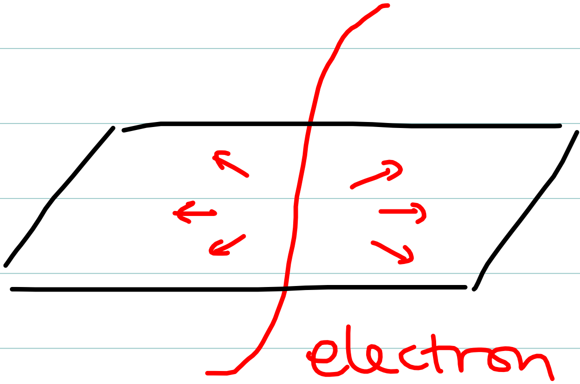
there are also

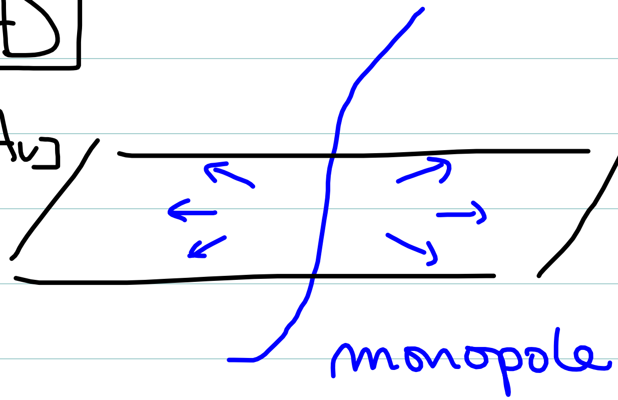
5-brane   6-brane   4-brane

These are like monopoles.

$\boxed{\text{In 4D}}$

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

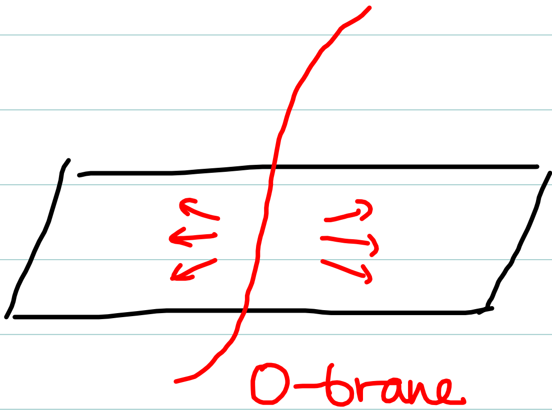


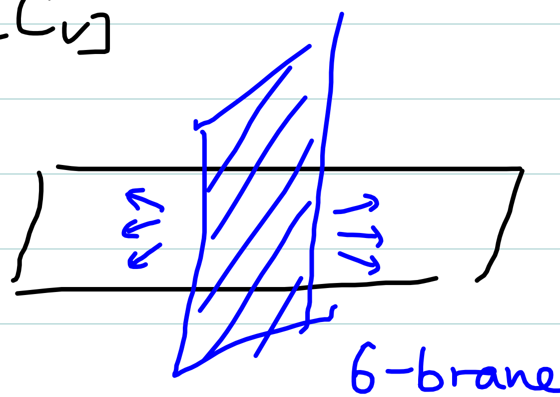


has $E_i = F_{0i}$ has $B_i = F_{kl}\,\varepsilon^{kl}{}_i$

$\boxed{\text{In 10D}}$

$$G_{\mu\nu} = \partial_{[\mu} C_{\nu]}$$





has $E_i = G_{0i}$ has $B_{abcdefg} = G_{pq}\,\varepsilon^{pq}{}_{abcdefg}$

$\boxed{\text{Type IIA}}$

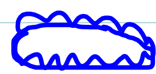
	$g_{\mu\nu}$,	$B_{[\mu\nu]}$,	ϕ,	C_μ,	$C_{\mu\nu\rho}$
"electric"		string		D0-brane	D2-brane
"magnetic"		NS 5-brane		D6-brane	D4-brane

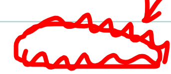
$\boxed{\text{Type IIB}}$

	$g_{\mu\nu}$,	$B_{[\mu\nu]}$,	ϕ,	C,	$C_{\mu\nu}$,	$C_{\mu\nu\rho\sigma}$
"electric"		string			D1-brane	D3-brane
"magnetic"		NS 5-brane			D5-brane	D3-brane

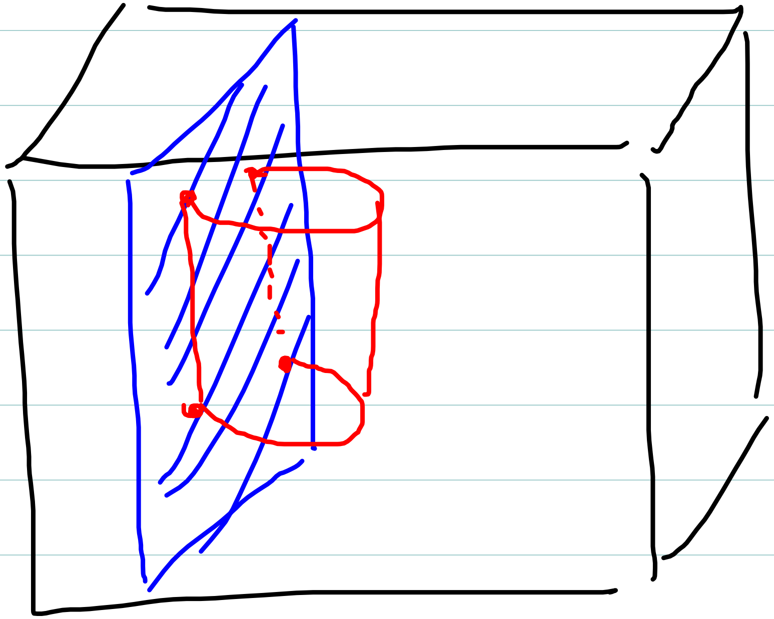
has two types of strings!
with different tensions.

much, much heavier!

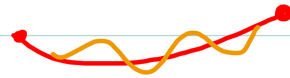




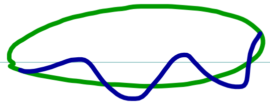
Consider a big D-brane:



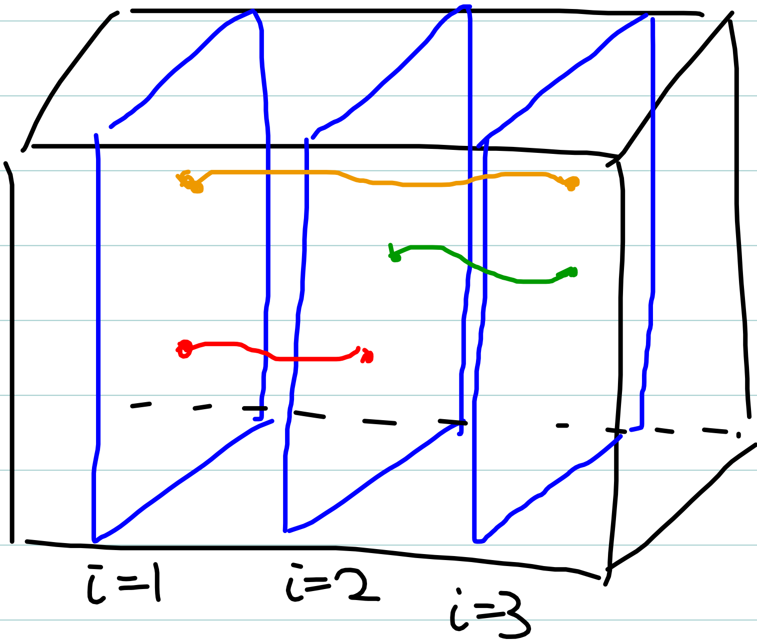
a string can have ends on it. → Open strings.



so far I only talked about closed strings.



Consider N big D-branes.



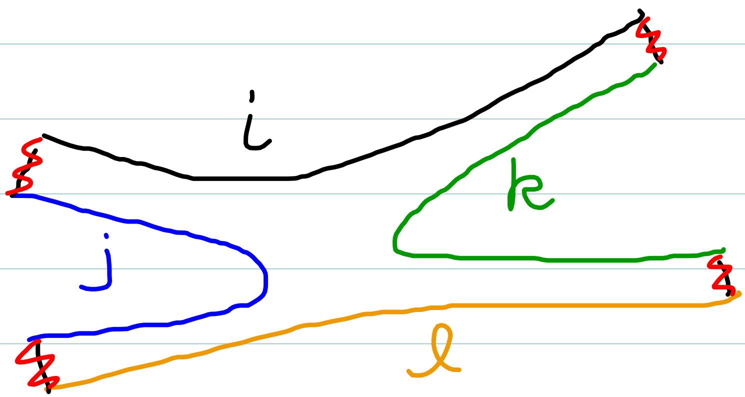
open strings can go from

i-th D-brane to
j-th D-brane.

$N \times N$ types of open strings.

Their scattering can be calculated.

They behave like
$U(N)$ gauge bosons,
which also have $N \times N$ dof.



So far, we saw that:

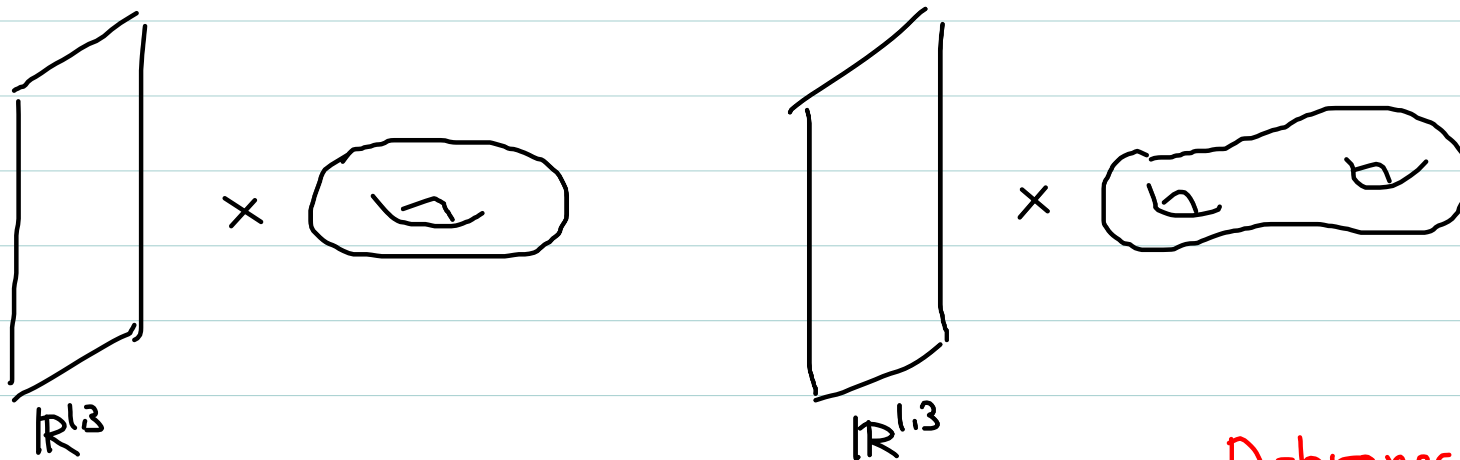
quantizing strings

- needs 10d + SUSY.
- automatically contains quantized gravity.
  - $\boxed{\text{Het } E_8 \times E_8}$ and $\boxed{SO(32)}$ has gauge fields in 10d.
  - $\boxed{\text{Type II}}$ has gauge fields on D-branes.

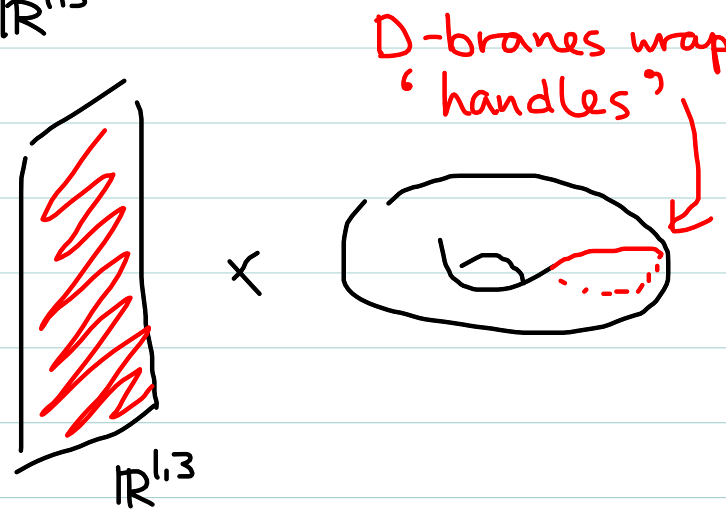
By making 6 out of 10 dim. very small, we have consistent quantum theory of

gravity + gauge fields + fermions.

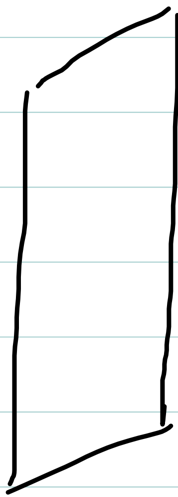
Depending on the shape of the extra 6 dimensions, we get different physics in 4d.



You can also put D-branes :

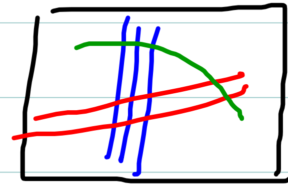


Two major approaches



$\mathbb{R}^{3,1}$ $\times$

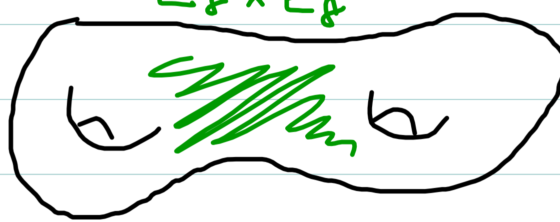
$\boxed{\text{Intersecting brane models}}$



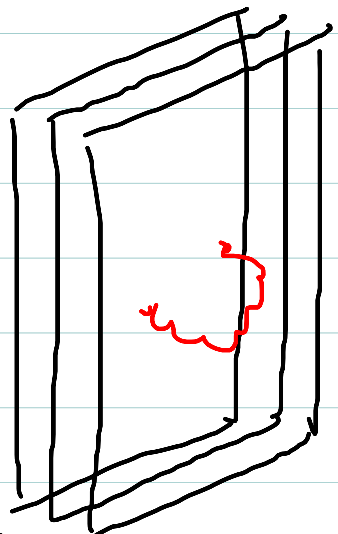
$SU(3) \times SU(2) \times U(1)$

$\boxed{\text{Heterotic compactifications}}$

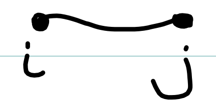
$E_8 \times E_8 \rightarrow$ GUT models



Consider N D p-branes (i.e. 1+p dimensions in 1+9 spacetime)



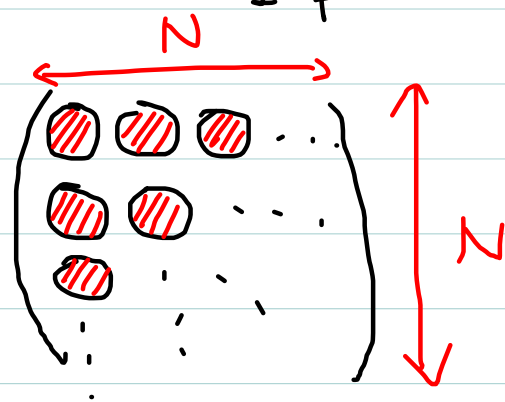
$i = 1, 2, \cdots, N$

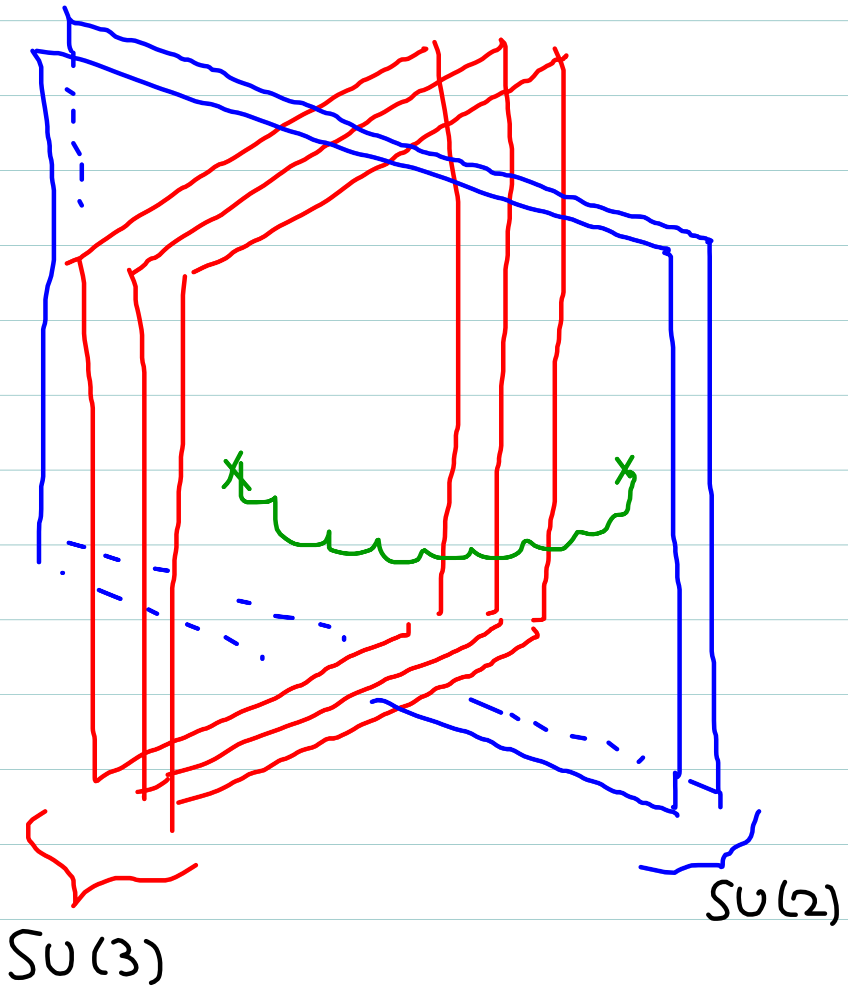


a string stretched between i & j

$\Downarrow$

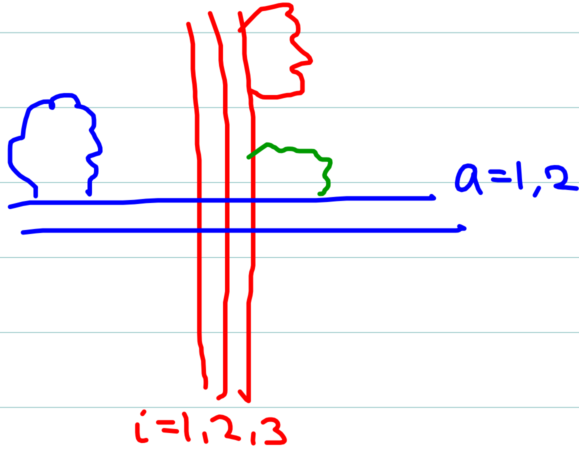
U(N) gauge theory on 1+p dimension

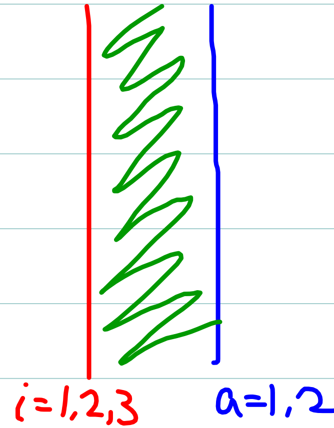
$A_\mu^a =$ 



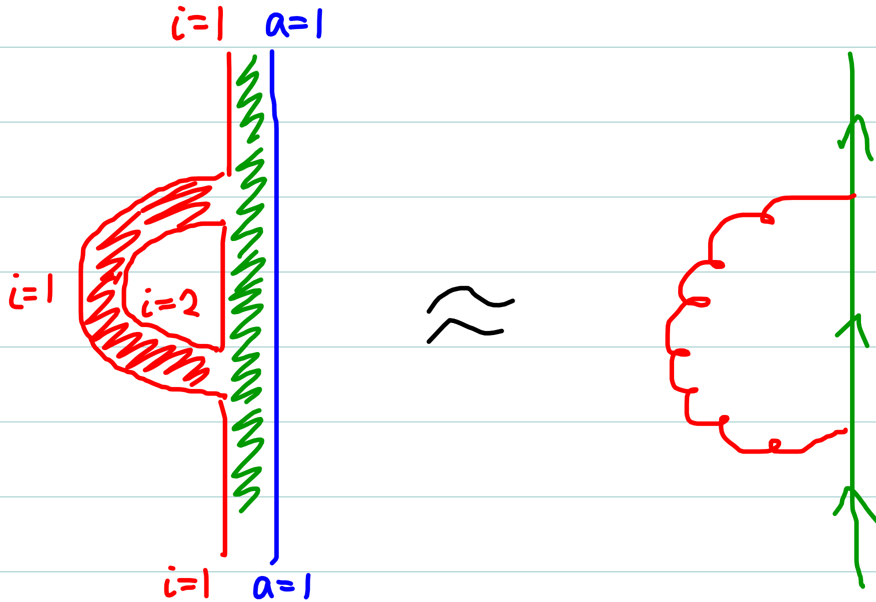
Strings can stretch between two stacks of branes.

They are known to give rise to chiral fermions.

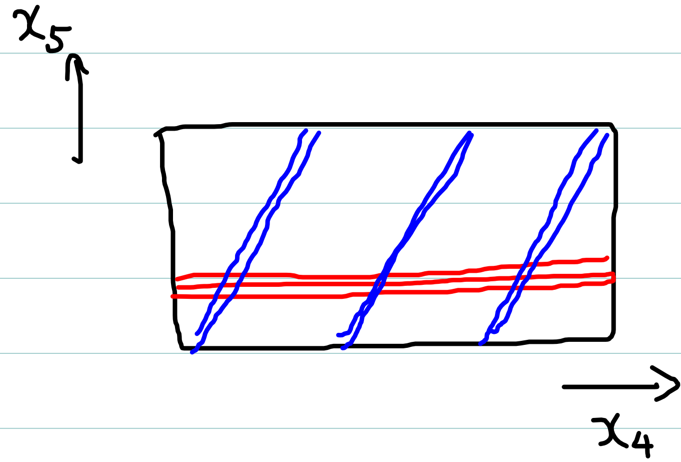




Fermion in
$(\mathcal{B}, \mathcal{D})$

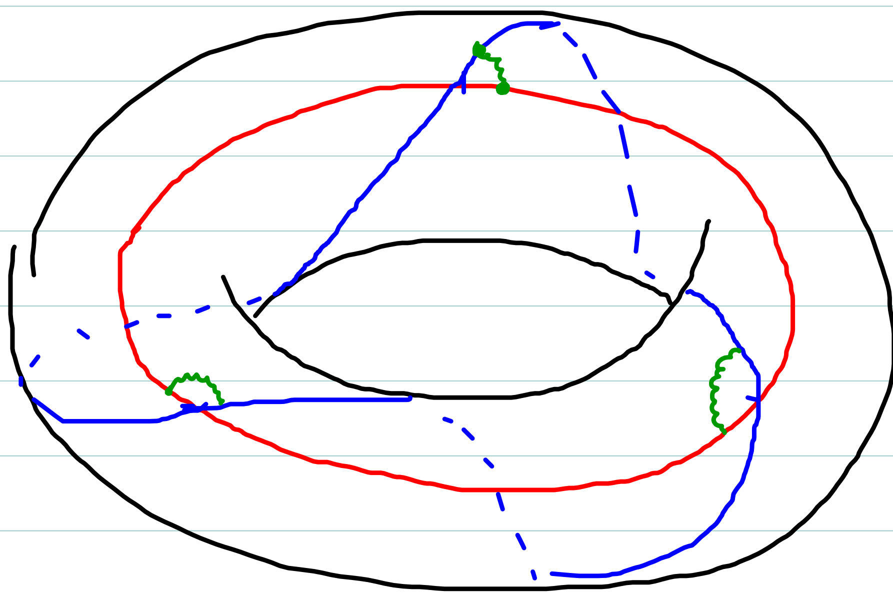


They interact with
gauge fields
as is expected.



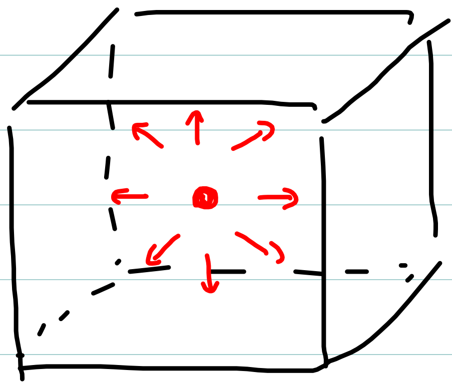
$$x_4 \sim x_4 + 2\pi R_4$$
$$x_5 \sim x_5 + 2\pi R_5$$

$\Rightarrow$ 3 'generations' of $(\mathcal{B}, \mathcal{D})$ of $SU(3) \times SU(2)$.



But there's a difficulty I haven't talked about.

Consider electromagnetism in 4d.



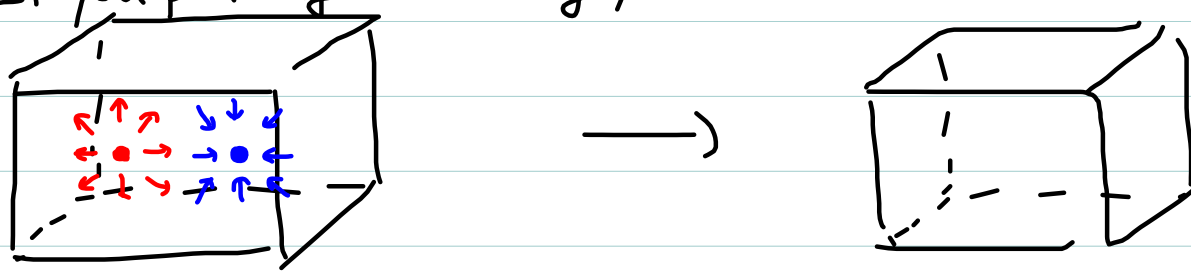
But with periodic boundary conditions:

$$\begin{cases} x_1 \sim x_1 + L \\ x_2 \sim x_2 + L \\ x_3 \sim x_3 + L \end{cases}$$

Then you can't just put one electron.

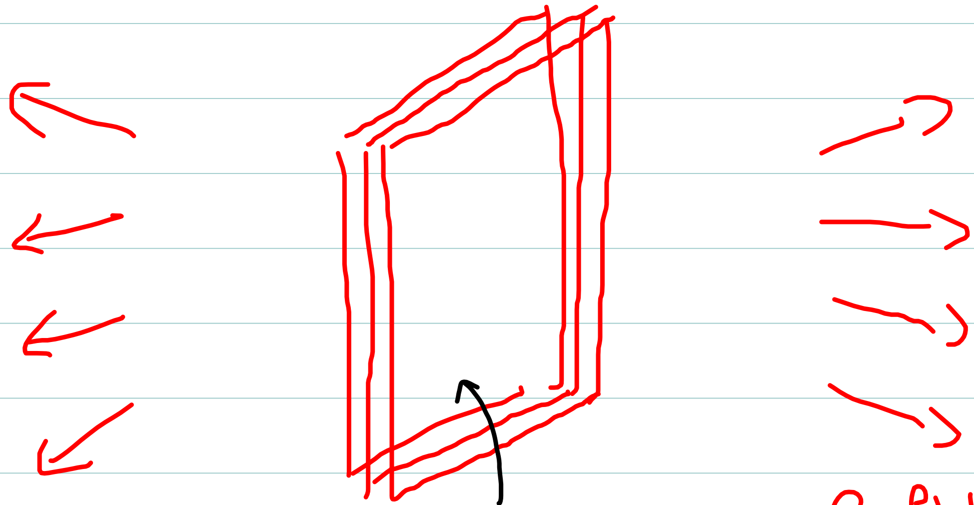
Electric field lines have nowhere to go! It's periodic!

If you put negative charge,



annihilate!

As I told you previously, D-brane is charged under C-fields.

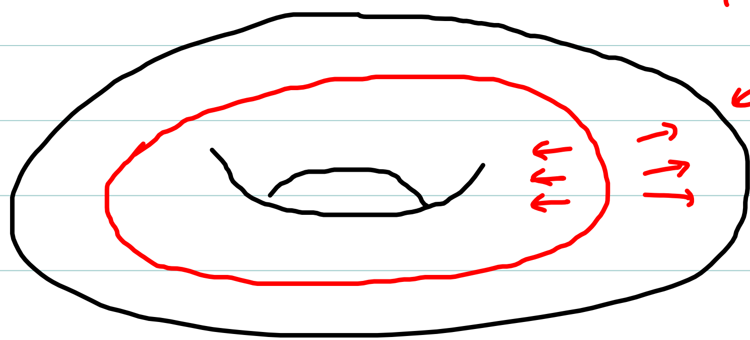


SU(N) gauge field on them.

C-field flux!

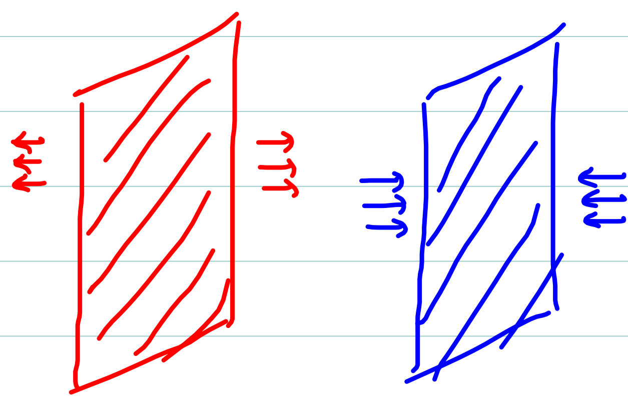
They are distinct!

So, just as was the case for electrons, you can't just put it on a compact space.



Field lines of C-field have nowhere to go!

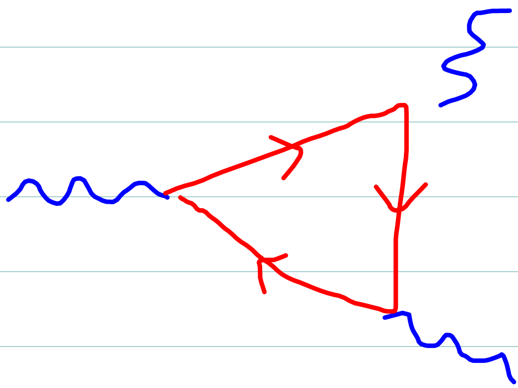
We need to cancel the total charge.



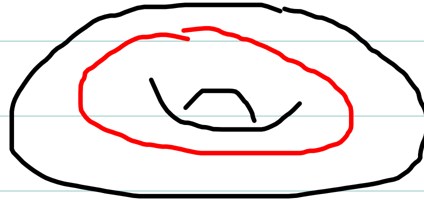
But then, they tend to annihilate each other.

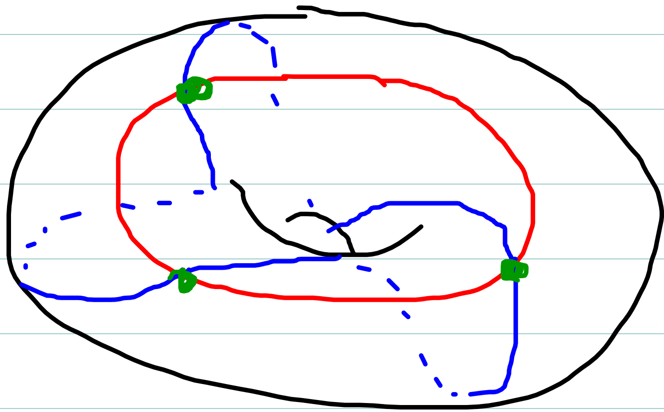
These problems can be taken care of.
But not easy.

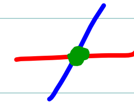
This cancellation of the D-brane charge is the stringy counterpart to the <u>anomaly</u> <u>cancellation</u>



$$\boxed{\sum q_L^3 - \sum q_R^3 = 0}$$

Indeed, if  were consistent,

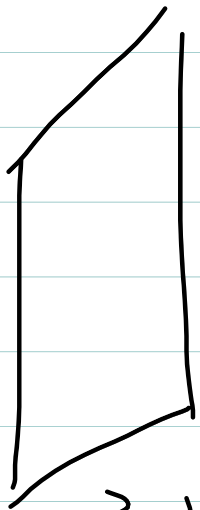


Each  gives a chiral fermion with $q_L = 1$.

$$\Rightarrow \underbrace{\sum q_L^3}_{\text{non-zero}} - \underbrace{\sum q_R^3}_{\text{zero}} \neq 0$$

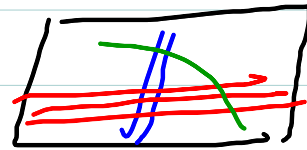
$\Rightarrow$ fermion content would be inconsistent!

$10d \to 4d$



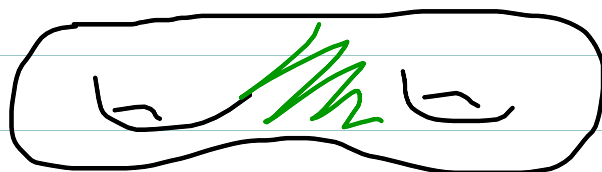
$\mathbb{R}^{3,1}$

$\boxed{\text{Intersecting brane models}}$



$\times$

$\boxed{\text{Heterotic compactifications}}$



gauge deg. of freedom in 10d!

What's the E_8 group anyway?

$O(N)$: transformations preserving

$$|x_1|^2 + |x_2|^2 + \cdots + |x_N^2| . \qquad x \in \mathbb{R}$$

$U(N)$: transformations preserving

$$|z_1|^2 + |z_2|^2 + \cdots + |z_N|^2 . \qquad z \in \mathbb{C}$$

$Sp(N)$: transformations preserving

$$|u_1|^2 + |u_2|^2 + \cdots + |u_N|^2 . \qquad u \in \mathbb{H}$$

quaternions (四元数)

$$u = a + bi + cj + dk$$

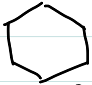
These are called Classical groups.

Mathematicians at the beginning of the 20th century wondered:
what's the list of possible continuous symmetries ??

They found that, in addition to $O(N)$, $U(N)$, $Sp(N)$ there are five others and that was it !

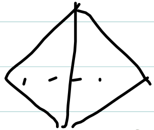
$$E_6 , \quad E_7 , \quad E_8 , \quad F_4 , \quad G_2 .$$

It's rather like the classification of symmetric objects in 3d:

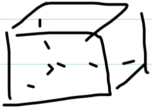


n-gon for each n

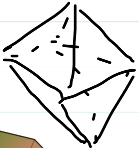
,



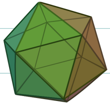
,



,

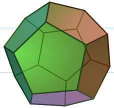


,



icosahedron

,



dodecahedron

$$SU(3)_C \times SU(2)_L \times U(1)_Y \qquad (\mathbf{3},\mathbf{2})_{1/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{-4/3} \oplus (\bar{\mathbf{3}},\mathbf{1})_{2/3}$$

$$\oplus (\mathbf{1},\mathbf{2})_{-1} \oplus (\mathbf{1},\mathbf{1})_{2}$$

$$\downarrow$$

$$SU(5) \qquad \bar{\mathbf{5}} + \mathbf{10}$$

$$\downarrow$$

$$SO(10) \qquad \mathbf{16}$$

spinor rep. $\rightarrow \mathbf{16}$

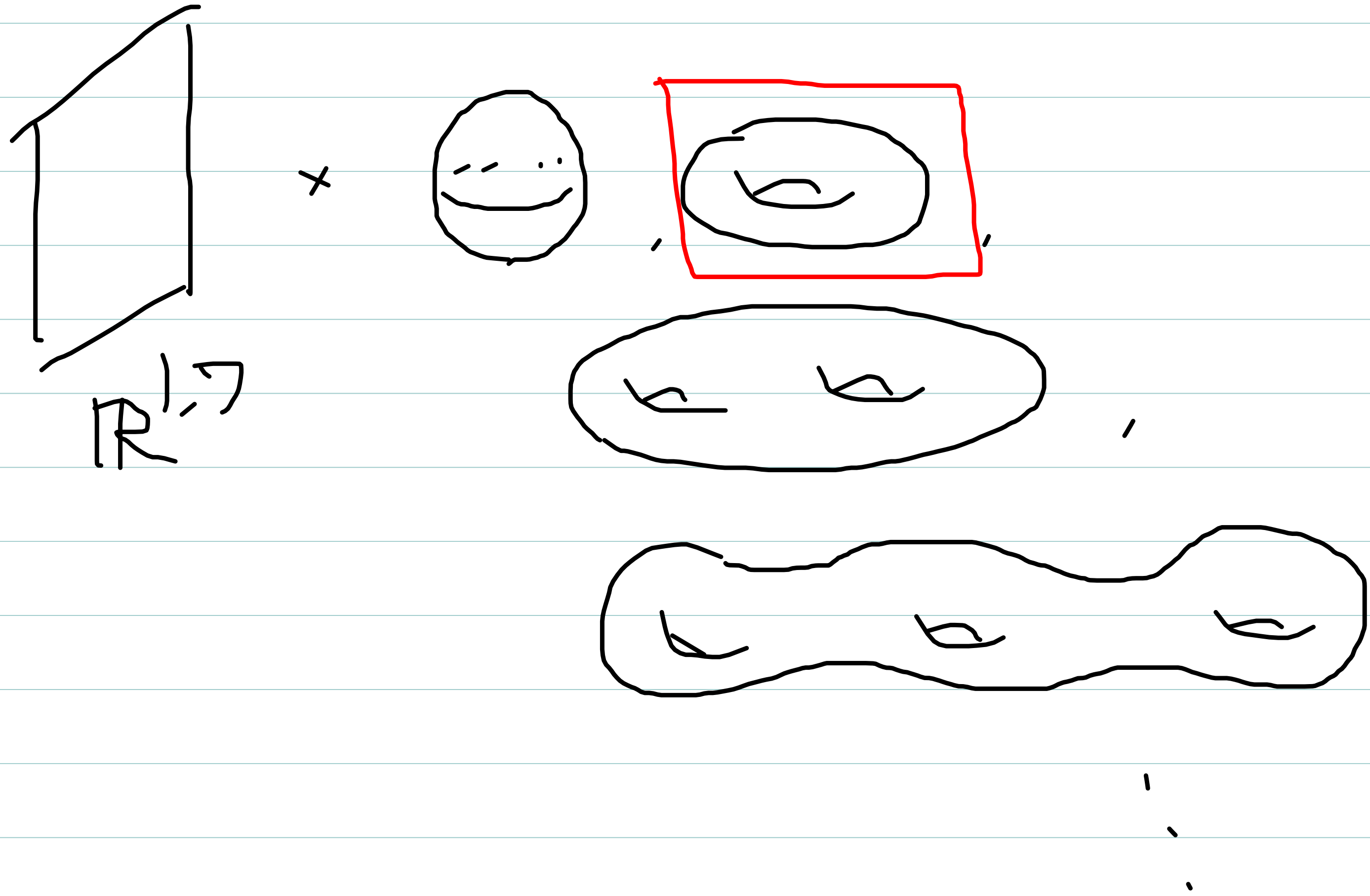
$$\downarrow$$

$$E_6 \qquad \mathbf{27}$$

smallest rep of E_6 $\rightarrow \mathbf{27}$

is the standard (?) path GUT theorists propose...

$10d \to 8d$



The Einstein equation in 10d says

$$\underline{R_{MN} = 0}$$

$\boxed{\text{Curvature}}$: R_{MNRS}

$$\downarrow$$

$\boxed{\text{Ricci curvature}}$ $R_{MN} = R_{MRSN}\, g^{RS}$

$$\downarrow$$

$\boxed{\text{scalar curvature}}$ $R = R_{MN}\, g^{MN}$

$$R_{MN} = 0 \longrightarrow R = 0$$

$$R_{MN}=0 \longrightarrow R=0$$

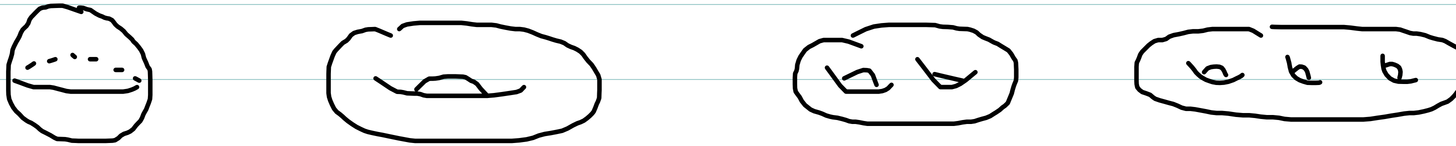
$$M,N=0,1,2,\cdots,9$$

$$\underbrace{\mu,\nu=0,\cdots,d}_{\text{this world}} \qquad \underbrace{a,b=d+1,\cdots,9}_{\text{extra dimension}}$$

$$R=-R_{00}+R_{11}+R_{22}+\cdots+R_{99}$$

$$=\underbrace{-R_{00}+\cdots+R_{dd}}_{R_{\text{this world}}}+\underbrace{R_{d+1,d+1}+\cdots+R_{99}}_{+\;R_{\text{extra dim}}}=0.$$

$$R_{\text{total}} = R_{\text{this world}} + R_{\text{extra dim}} = 0.$$



$R_{\text{extra}} = +1 \qquad 0 \qquad -1 \qquad -2 \quad \cdots$

$R_{\text{this world}} = -1 \qquad 0 \qquad +1 \qquad +2$

$\underbrace{-1}$ → decelerating expansion ← with scale determined by $l_s \sim l_{\text{Planck}}$. → accelerating expansion ← $\underbrace{+1 \quad +2}$

So, we want $R_{\mu\nu}^{\text{extra dim}} = 0$.

$10 = 8 + 2$ 

$6 + 4$  $\times$  , "K3"

only 1 type.
has 24 "holes"?

$4 + 6$  $\times$  $\times$  , "K3" $\times$  , "Calabi-Yau manifolds"

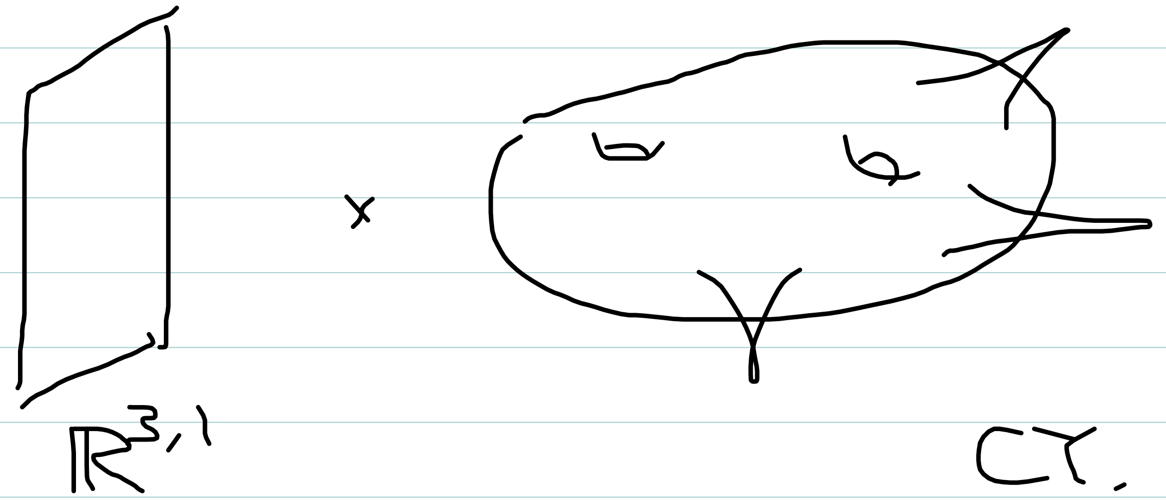
thousands of them are known.

$\mathcal{N}=4$ SUSY

$\mathcal{N}=2$ SUSY

$\mathcal{N}=1$ SUSY

So, take a Calabi-Yau manifold.



and consider heterotic $E_8 \times E_8$ theory on it.

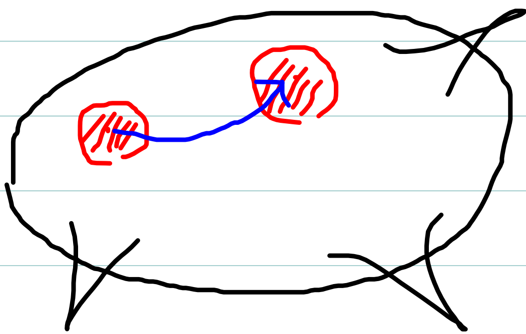
It has metric g_{MN} and A_M^a, $A_M^{a'}$

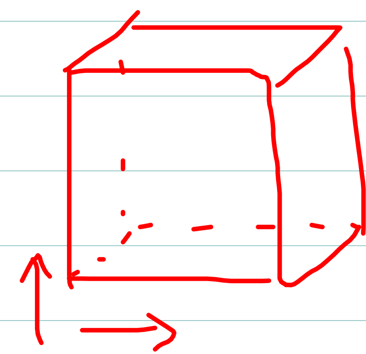
and gauginos λ_α^a $\lambda_\alpha^{a'}$

and ...

The heterotic equations of motion requires $A_\mu^a \neq 0$ in a specific way.

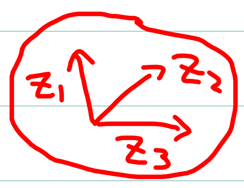
(You can't just make spacetime curved. We also need to make gauge fields curved.)



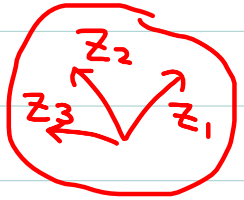


$$z_1 = x_4 + i x_5$$
$$z_2 = x_6 + i x_7$$
$$z_3 = x_8 + i x_9$$

3×3 complex matrix.



$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix} \begin{pmatrix} z_1' \\ z_2' \\ z_3' \end{pmatrix}$$



$E_8 \supset SU(3)$ .... set this gauge field to be equal to the spacetime curvature.

$\downarrow$

breaks the gauge group to E_6.

$$E_8 \quad \left(\begin{array}{c|c} E_6 & 27 \\ \hline 27 & \end{array}\right) \quad \leftarrow SU(3)$$

$\downarrow$

GUT!

chiral fermions in 27 of E_6

generations = # holes in CY.

Let's quantize strings!

We need 10d + SUSY. $E_8 \times E_8$.

Compactify 6d. Needs to be Calabi-Yau.

$\boxed{x_4\ x_5\ x_6\ x_7\ x_8\ x_9}$

Need to turn on gauge fields.

$\boxed{z_1\ z_2\ z_3}$: $SU(3) \subset E_8$

E_8 broken down to E_6, with chiral fermions in 27

\# generations = \# holes in CY.

When \# holes = 3, SUSY E_6 GUT model. (more or less.)