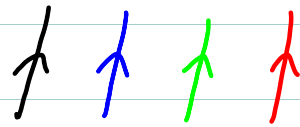
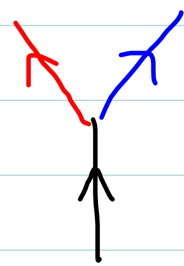
QFT	STRING

QFT



choose types of particles in a theory.

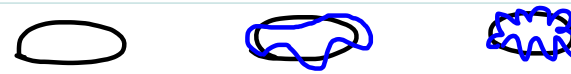
choose how they interact.

$\lambda \sim$ 

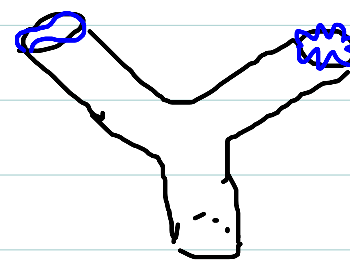
STRING

choose a type of string.

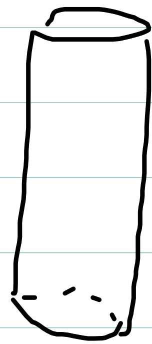
study how they vibrate.



interaction is then determined.



There are two known consistent types of strings.



Type II string

Heterotic string

Both require $1+9$ dimensional spacetime.

have supersymmetry.

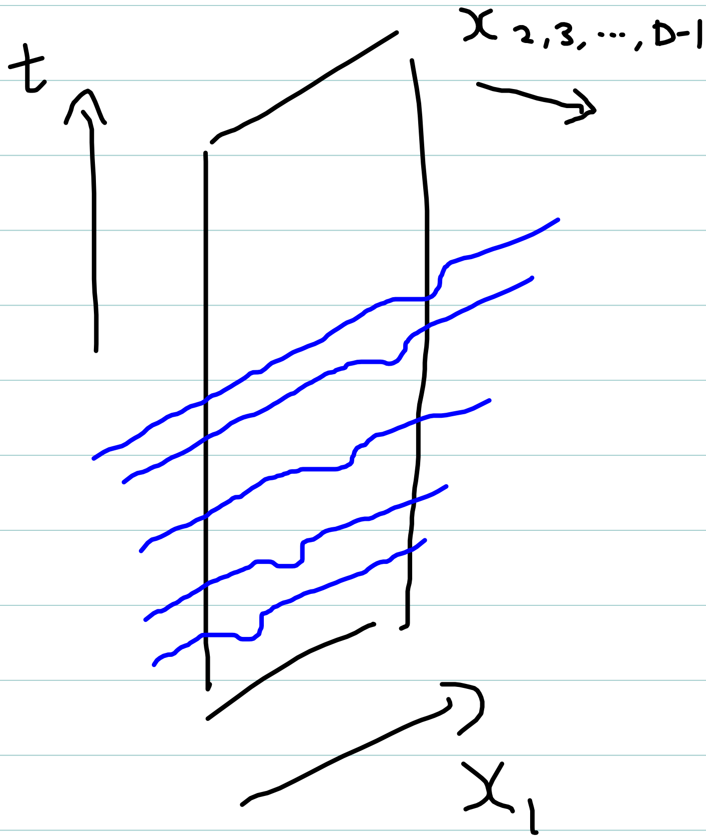
Contrains quantized gravity.

All these follow from the consistency of

Lorentz inv. & Positivity of probability.

I'd like to give you some flavor of the derivation.

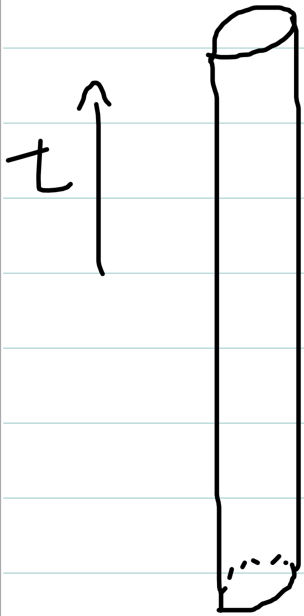
Consider a string moving in D-dim'l spacetime.

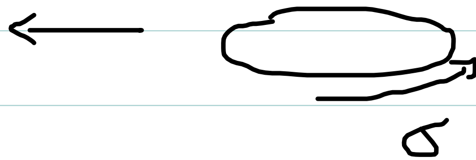


The config. of a string is specified by giving

$x_2(t,x_1)$, $x_3(t,x_1)$, $\cdots$, $x_{D-2}(t,x_1)$, $x_{D-1}(t,x_1)$

We have $\boxed{\text{D-2 scalar field}}$ on the 2d space (t, x_1).



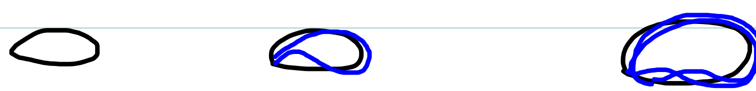


$$X_i(\sigma) = \sum a_{i,k}\, e^{2\pi i k\sigma} + a_{i,k}^\dagger e^{-2\pi k i\sigma}$$

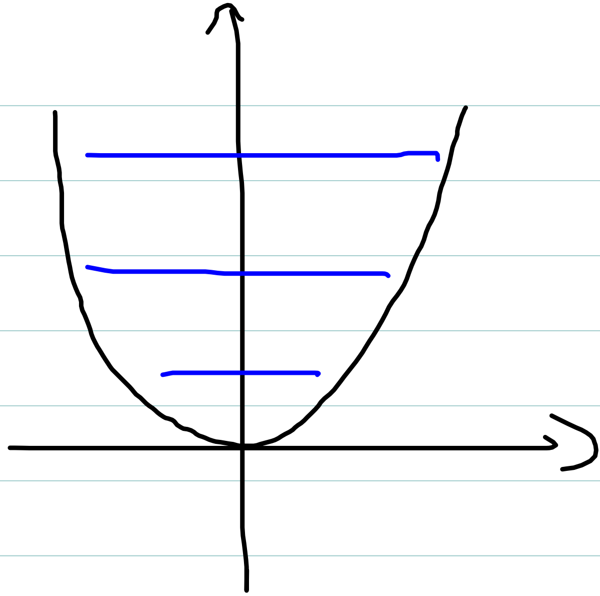
$$\begin{cases} i = 2, 3, \cdots, D-2 \\ k = 1, 2, 3, \cdots \end{cases}$$

Each $a_{i,k}$, $a_{i,k}^\dagger$ is a harmonic oscillator.

$|0\rangle$, $a_{1,1}^\dagger|0\rangle$, $(a_{1,1}^\dagger)^2|0\rangle$, $\cdots$



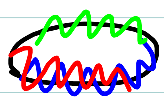
The same mode excited twice!



$\leftarrow \frac{5}{2}\hbar\omega$

$\leftarrow \frac{3}{2}\hbar\omega$

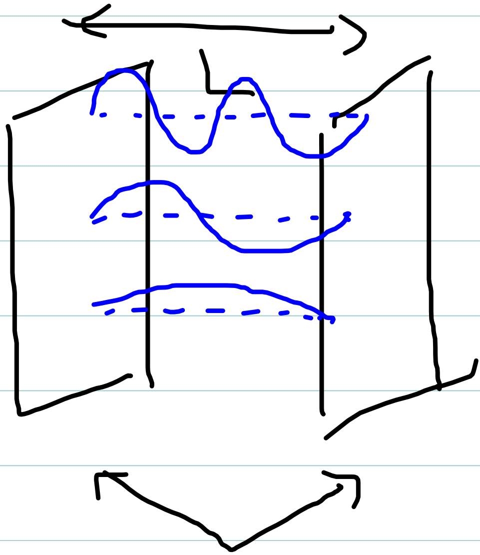
$\leftarrow \frac{1}{2}\hbar\omega$: zero point energy.



$$a^\dagger_k a^\dagger_{k'} \cdots a^\dagger \cdots a^\dagger_{k''''} |0\rangle$$

has $m^2 = \frac{1}{l_s^2}\left[(k+k'+\cdots+k'''') + \text{zero pt energy.}\right]$

ASIDE the Casimir effect



two conducting plate

The sum of the zero pt energy / area

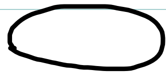
$$\sim \frac{1}{L^3}\left(1^3+2^3+3^3+\cdots\right)$$

$$= -\frac{\pi^2}{720}\frac{1}{L^3} \quad \text{after renormalization}$$

This is MEASURED.

(Lamoreaux, PRL 78 ('97) 5)

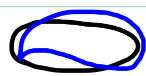
So, there's Casimir energy on the string:

$|0\rangle$  without any excitation

$$\cdots\; m^2 = \frac{1}{l_s^2} \cdot L\left(-\frac{1}{24L}\right) \times (D-2)$$

Casimir energy of one $X_i(t, x_1)$

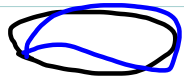
$i = 2, 3, 4, \cdots D-1.$

$a_{i,1}^\dagger|0\rangle$  with one excitation of the lowest mode

$$\cdots\; m^2 = \frac{1}{l_s^2}\left(-\frac{D-2}{24} + 1\right).$$

Note that there are states for $i = 2, 3, \cdots, D-1$

It's a <u>vector boson</u>.

 gives $D-2$ polarizations of a vector boson
with $\text{mass}^2 = \frac{1}{l_s^2}\left(-\frac{D-2}{24}+1\right)$.

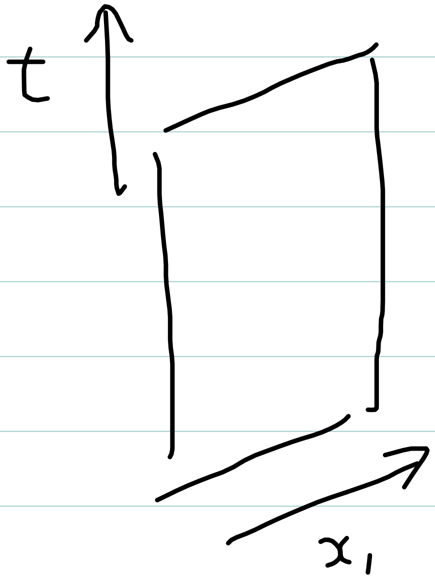
This makes sense only when $m^2=0 \longrightarrow D=26$

Recall: in 4d, massless vector: 2 polarizations
massive vector: 3 polarizations.

In general in D dim, massless $\longrightarrow$ $D-2$ pol.
massive $\longrightarrow$ $D-1$ pol.

But, even when $D=26$,
The lowest mode
 : $m^2 = -\frac{1}{l_s^2}$: extremely tachyonic!

<u>The cure</u> add supersymmetry.



$$\underbrace{X_2(t,x_1) \cdots X_{D-2}(t,x_1)}_{\text{spacetime fluctuation}}$$

$$\underbrace{\psi_2(t,x_1)}_{\text{fermionic fluctuation.}}$$

Casimir energy:

$$(D-2)\sum(1+2+\cdots+) - \left(\begin{matrix}\#\text{ fermion}\\ \text{components}\end{matrix}\right)\sum(1+2+\cdots)$$

This can be 0, removing the tachyon.

$$\rightsquigarrow \quad D=10$$

Summary

Quantizing particles can be done in any D,

Quantizing strings can only be done in

$D = 10$.

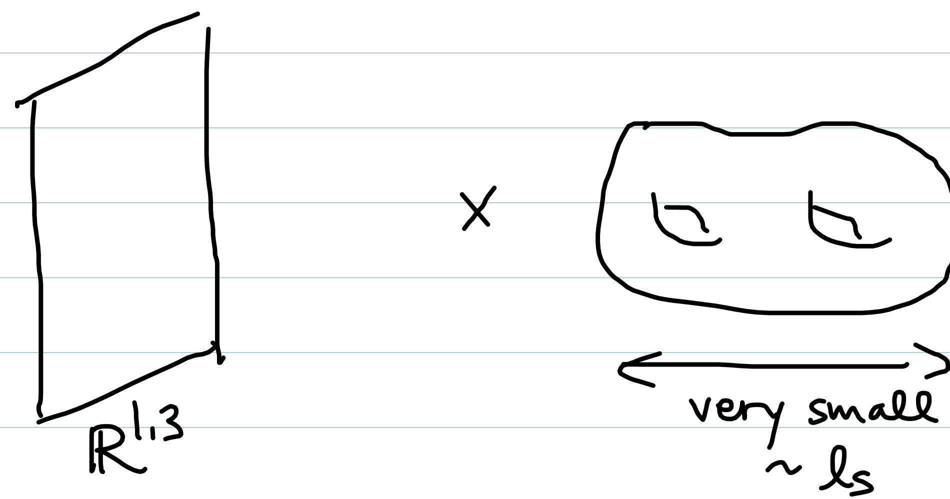
You say, wait, our spacetime is $1+3$ d.

⇝ We need to make 6 out of 10 dimensions <u>very small</u> so that they are not observed.

(NOTE THAT if we lived in 30 dimensions, string theory would have been already experimentally falsified.)

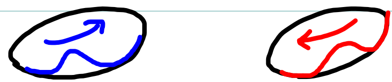
Quantizing strings $\longrightarrow$ Only fully consistent in 10 d and with SUSY.

To say anything about 4d physics, we need to <u>compactify</u> the extra <u>6d</u>.



Then it's important to study the geometry of the 6d internal space.
That's why a lot of math is necessary.

Before going there, let's study 10d physics first.



Strings have right-moving & left-moving excitations.

Yesterday I only talked about one of them.

$a^\dagger_{j,1} a^\dagger_{i,1} |0\rangle$ has two spacetime index, i & j.

$i, j = 2, 3, \cdots, D-1 = 9$

$\rightsquigarrow$ contains graviton !

(cf. $g_{\mu\nu}$ has two indices)

The system also has supersymmetry.

$\rightsquigarrow$ 10d SUPERGRAVITY.

SUPERSYMMETRY

$$\begin{tikzcd} \text{Boson} \arrow[r, bend left, "Q_\alpha"] & \text{Fermion} \arrow[l, bend left, "Q_\beta"] \end{tikzcd}$$

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = \gamma^\mu_{\alpha\dot\beta} \underbrace{P_\mu}_{\text{translation}} \quad \rightarrow \quad Q_\alpha \text{ is like } \sqrt{\text{translation}}$$

squark $\longleftrightarrow$ quark

gauge boson $\longleftrightarrow$ gaugino

graviton $\longleftrightarrow$ gravitino

$\vdots$

This is possible only in 11d or less !

BOSON $\leftrightarrow$ FERMION $\rightsquigarrow$ # of boson degrees of freedom
= # of fermion d.o.f.

$g_{\mu\nu}, A_\mu, \cdots$: D, D^2 components, roughly speaking.

ψ_α : How many components? γ^μ needs to act on it.

In 4d, γ^μ is a 4×4 matrix. $\{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu}$.

In D dimensions, let $b_1 = \gamma^0 + \gamma^1$, $b_1^\dagger = \gamma^0 - \gamma^1$

$$b_2 = \gamma^2 + i\gamma^3, \quad b_2^\dagger = \gamma^2 - i\gamma^3$$

$$\vdots$$

$$b_{D/2} = \gamma^{D-1} + i\gamma^D, \quad b_{D/2}^\dagger = \gamma^{D-1} - i\gamma^D$$

They satisfy standard fermion commutator: $\{b_i, b_j^\dagger\} = \delta_{ij}$.

So, a Dirac spinor in D spacetime dimensions is like having D/2 fermions. eg when $D=4$, $D/2=2$.

$$|00\rangle \quad |01\rangle$$
$$|10\rangle \quad |11\rangle \quad \rightsquigarrow 4 \text{ states.}$$

This was why γ^μ is 4×4. $4=2^2$

In general, $\underbrace{\text{a}}_{\text{Dirac}}$ spinor has $2^{D/2}$ components.

You can impose Weyl or Majorana conditions ...

Boson : $\phi, A_\mu, g_{\mu\nu}$: $1, D, D^2$ components

$\updownarrow$

Fermion $\psi_\alpha, \psi_{\alpha\mu}$: $2^{D/2}, D\,2^{D/2}$ components

grows much faster than bosons.

SUSY $\longrightarrow$ # boson d.o.f = # fermion dof

$\longrightarrow$ D can't be very big.

Maximum is $D=11$. There's only one theory there.

11d supergravity

		dof		dof
graviton $\longrightarrow$	$g_{(\mu\nu)}$	44		
			$\psi_{\alpha\mu}$	128
	$C_{[\mu\nu\rho]}$	84	gravitino	
C-field				

That's it. You can't add gauge field, scalar, ...

If we lived in 11d, and had SUSY, the theory is

uniquely defined.

10d supergravity : $\boxed{\text{3 types.}}$

$\boxed{\text{Type IIA}}$

boson	$g_{(\mu\nu)}$,	$B_{[\mu\nu]}$,	ϕ,	C_μ,	$C_{[\mu\nu\rho]}$	
128	35	28	1	8	56	
fermion	$\psi_{\mu\alpha}$	$\psi_{\mu\dot\alpha}$	ψ_α	$\psi_{\dot\alpha}$		
128	56	56	8	8		

$\boxed{\text{Type IIB}}$

boson	$g_{(\mu\nu)}$,	$B_{[\mu\nu]}$,	ϕ,	C,	$C_{[\mu\nu]}$,	$C_{[\mu\nu\rho\sigma]}$	
128	35	28	1	1	28	35	
fermion	$\psi_{\mu\alpha}$,	$\psi'_{\mu\alpha}$,	ψ_α,	ψ'_α			
128	56	56	8	8			

Again, you can't add gauge fields for type IIA, IIB supergravity.

But they have D-branes.

Type I = Heterotic

boson	$g_{(\mu\nu)}$,	$B_{[\mu\nu]}$,	ϕ
64	35	28	1
fermion	$\psi_{\mu\alpha}$,	ψ_β	
64	56	8	

← superpartners of gravitons

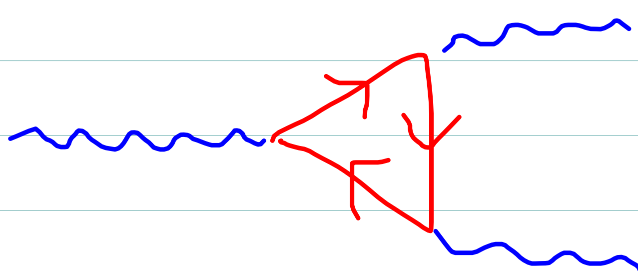
boson	A_μ^a
8	8
fermion	λ_β^a
8	8

← superpartners of gauge fields

Now we can have gauge fields. But only when

$$G = SO(32) \quad \text{or} \quad E_8 \times E_8 .$$

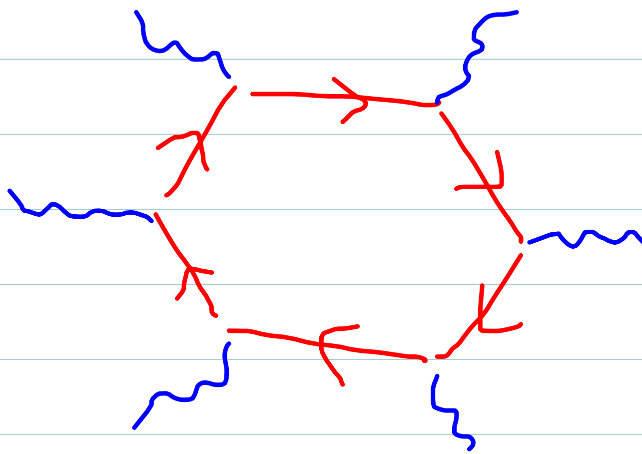
In 4d, with chiral fermions,



caused trouble when

$$\sum q_L^3 - \sum q_R^3 \neq 0.$$

In 10d type I, fermions are always chiral.



causes trouble ...

unless $G = SO(32)$

or $E_8 \times E_8$.

So, in 10d + SUSY, there're only 4 possibilities:

$\boxed{\text{Type IIA}}$, $\boxed{\text{Type IIB}}$

$\boxed{\text{Heterotic } SO(32)}$, $\boxed{\text{Heterotic } E_8 \times E_8}$.

Then, every interaction vertex is fixed.

It's not like in 4d, where we can choose gauge group, fermions, scalars, Yukawa couplings...

These theories are very NON-RENORMALIZABLE in the standard QFT sense. (contains gravity!)

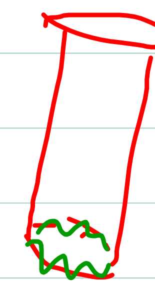
But in fact, there're corresponding string theories

Type IIA SUGRA

Type II String 

Type IIB SUGRA

Het SO(32) SUGRA

Heterotic String 

Het $E_8 \times E_8$ SUGRA

10d supergravities are successfully quantized by strings!

This is one reason why string theorists hope that similar things might work in 4d ...