超弦理論入門

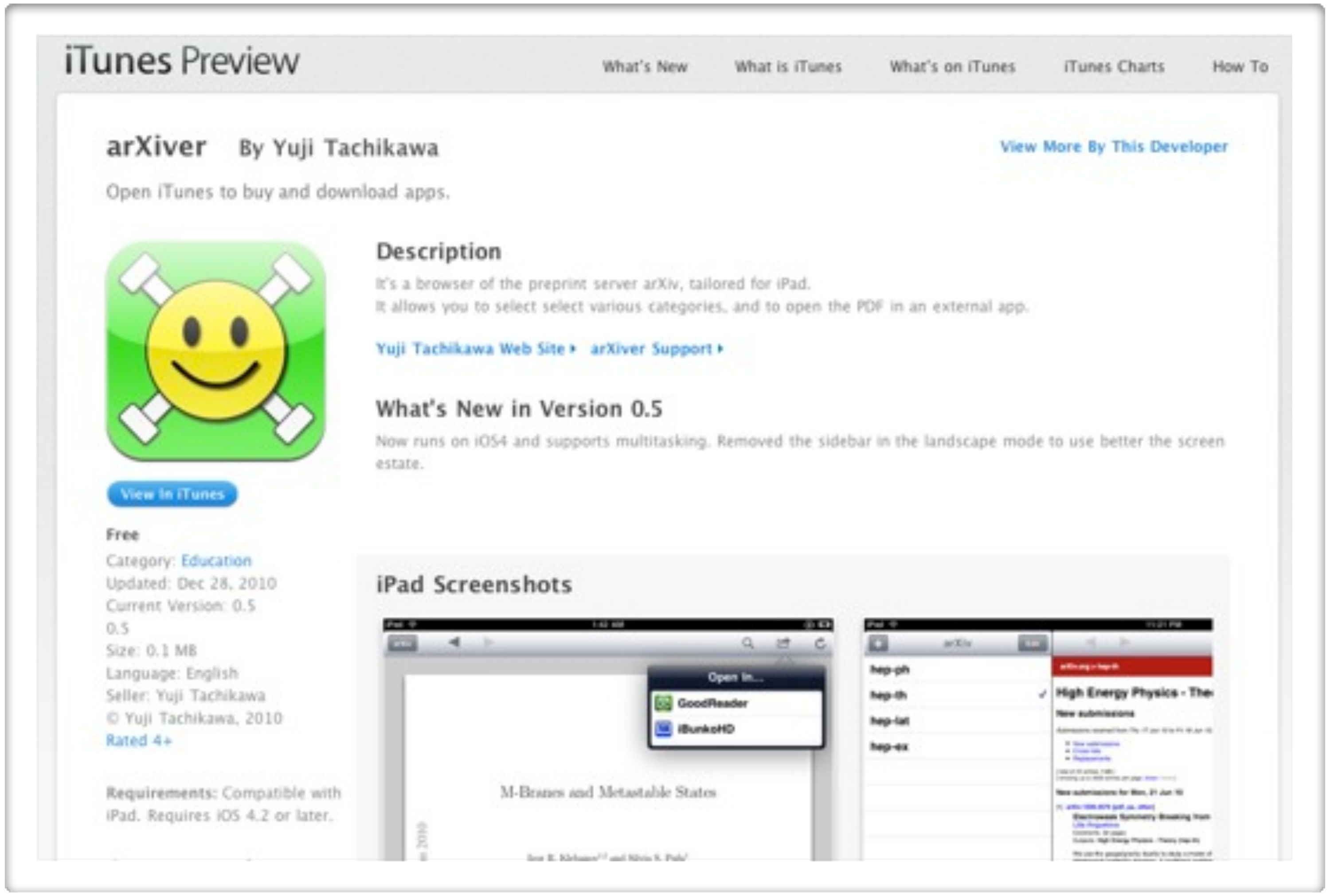
立川裕二 (IPMU & IAS)

at the 17th ICEPP Winter Symposium

Institute for the Physics and Mathematics of the Universe

数物連携宇宙研究機構

数年前できました。宜しくお願いします。



arXiver By Yuji Tachikawa

View More By This Developer

Open iTunes to buy and download apps.

Description

It's a browser of the preprint server arXiv, tailored for iPad.
It allows you to select select various categories, and to open the PDF in an external app.

Yuji Tachikawa Web Site ▸ arXiver Support ▸

What's New in Version 0.5

Now runs on iOS4 and supports multitasking. Removed the sidebar in the landscape mode to use better the screen estate.

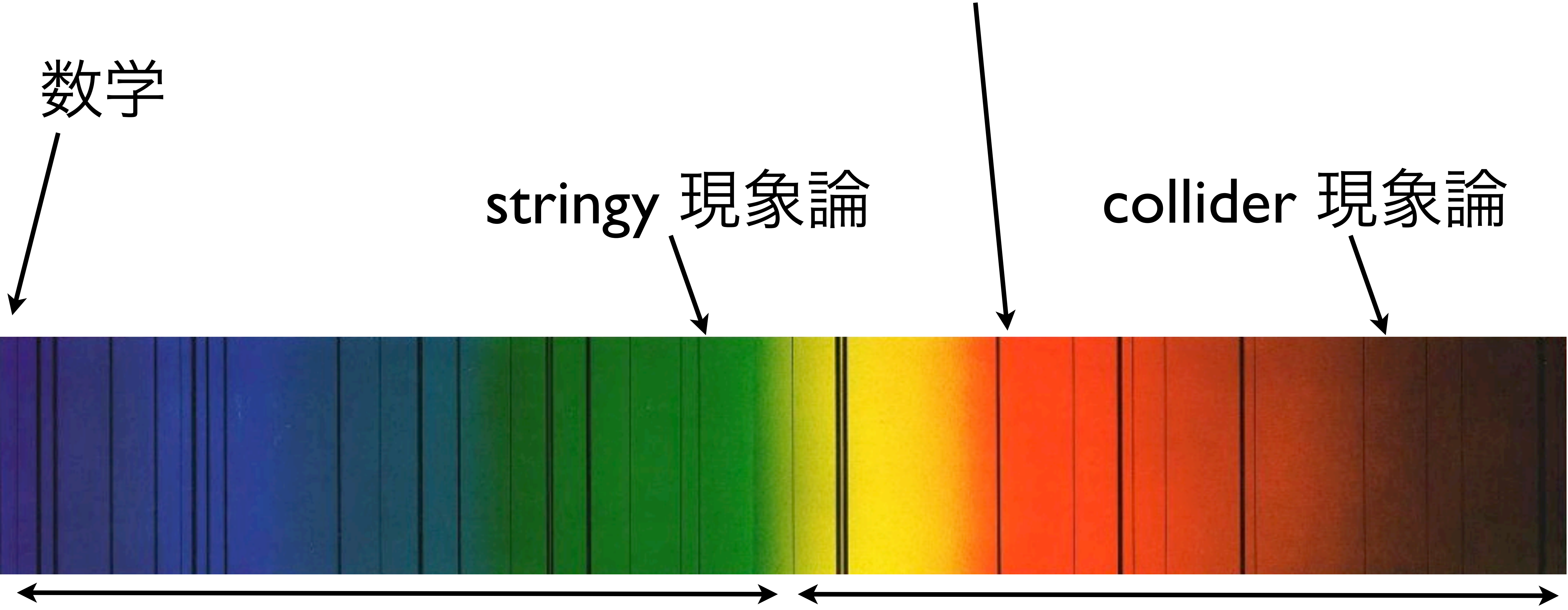
Free
Category: Education
Updated: Dec 28, 2010
Current Version: 0.5
0.5
Size: 0.1 MB
Language: English
Seller: Yuji Tachikawa
© Yuji Tachikawa, 2010
Rated 4+

Requirements: Compatible with iPad. Requires iOS 4.2 or later.

iPad Screenshots

- プログラミングが趣味なので、
同好の方はよろしくおねがいします。
- Mac / iPhone / iPad で Objective-C を
使います。

Spectrum of High Energy Physics Theory



SUSY / extra
dimension 現象論

数学

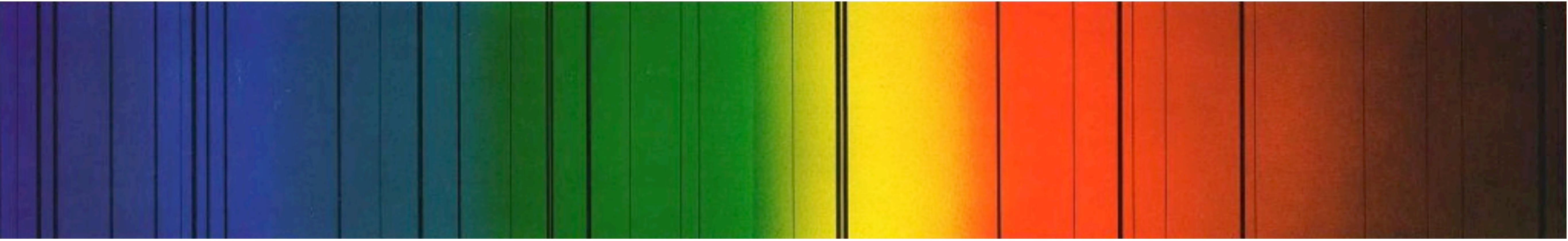
stringy 現象論

collider 現象論

arXiv:hep-th

arXiv:hep-ph

Spectrum of High Energy Physics Theory



数学

僕はこの辺

stringy 現象論

SUSY / extra dimension 現象論

collider 現象論

arXiv:hep-th

arXiv:hep-ph

- というわけで、超弦理論だとするとどういうのが実験で見えるかもしれない、という話は出来ません…
- ごめんなさい。

- そのかわり、なぜ理論家が超弦理論が自然な枠組みだと思っているかを伝えられたらと思います。

- 場の量子論:

  標準模型の書かれている言葉

- 点粒子を相対論的に量子化したもの

- 案外難しかった。

  電磁気: 戦後すぐ、
  弱い力/強い力: ～1970年代
  重力: 出来ていない

- (超)弦理論

  重力を量子化できる。

- 弦を相対論的に量子化したもの

- かなり難しかった。

  はじまり:　1974 ~

  第一次革命:　1984 ~

  第二次革命:　1995 ~

黎明期

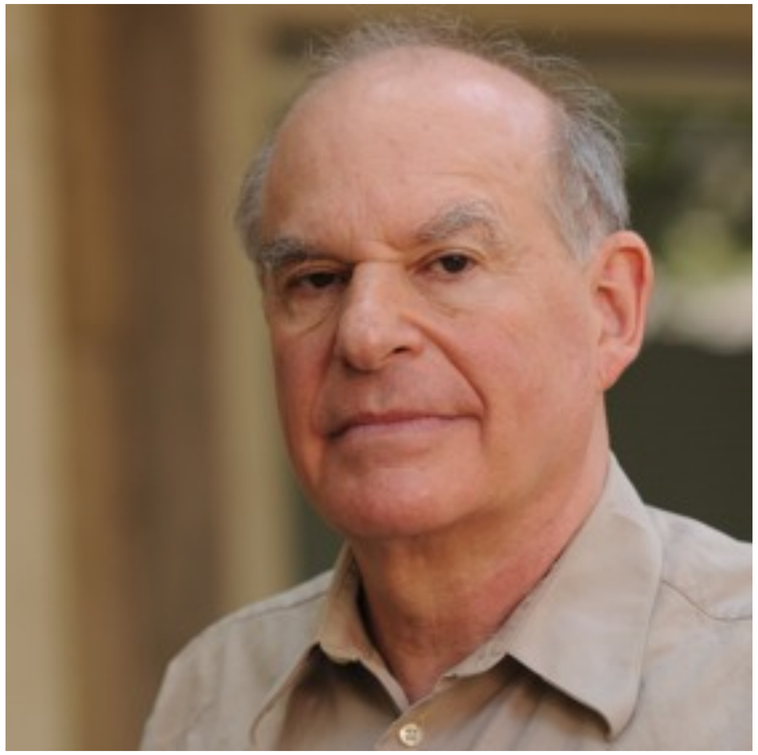
Veneziano

Nambu

Polyakov

1984~

Green



Schwarz

Witten

1995~

Polchinski

Maldacena

- (超)弦理論

  重力を量子化できる。

- 弦を相対論的に量子化したもの

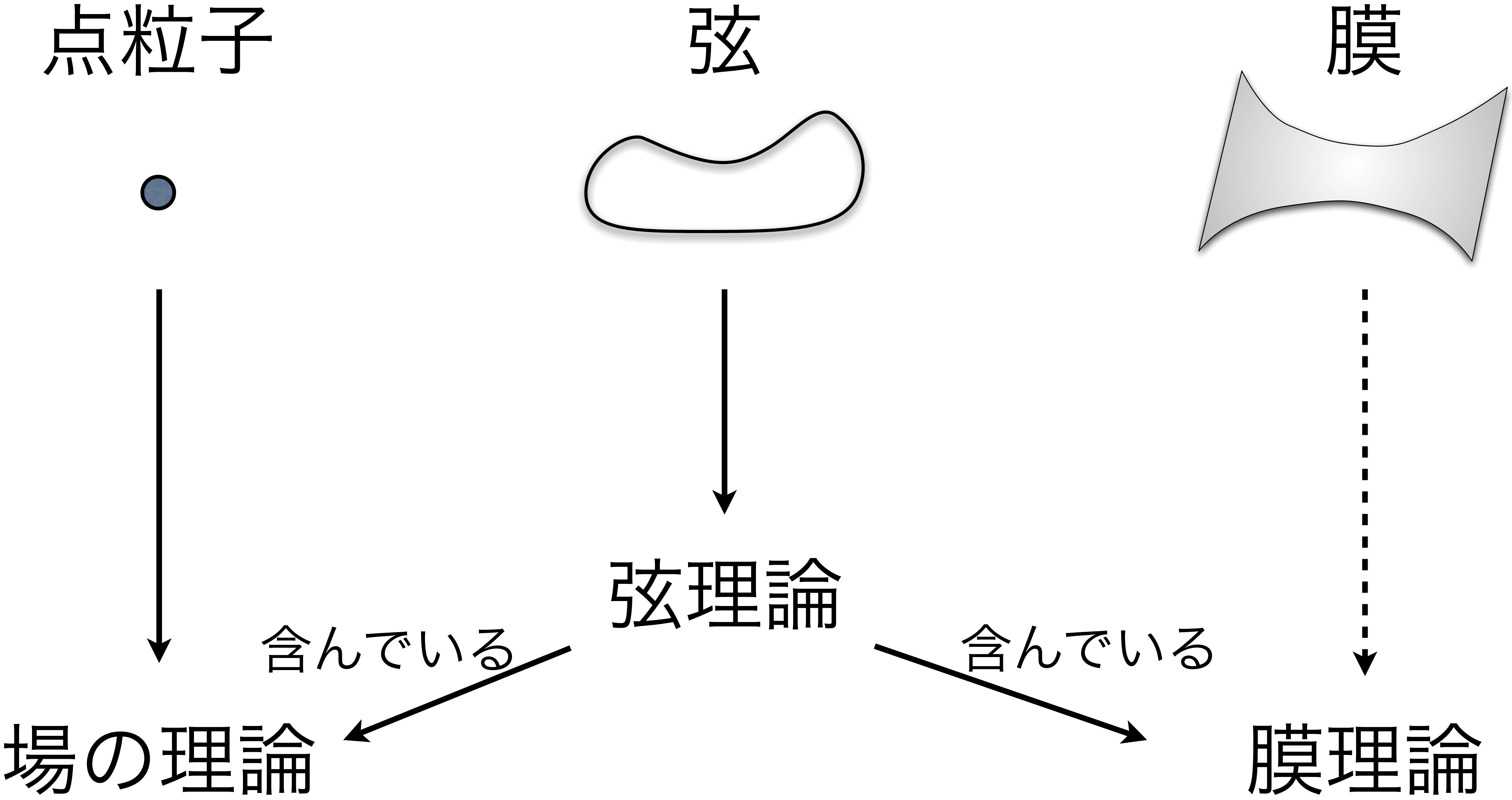
- かなり難しかった。

  はじまり:　1974 ~

  第一次革命:　1984 ~

  第二次革命:　1995 ~

- 2000年ごろには枠組みはほぼ完成。



点粒子

弦

膜

弦理論

含んでいる

場の理論

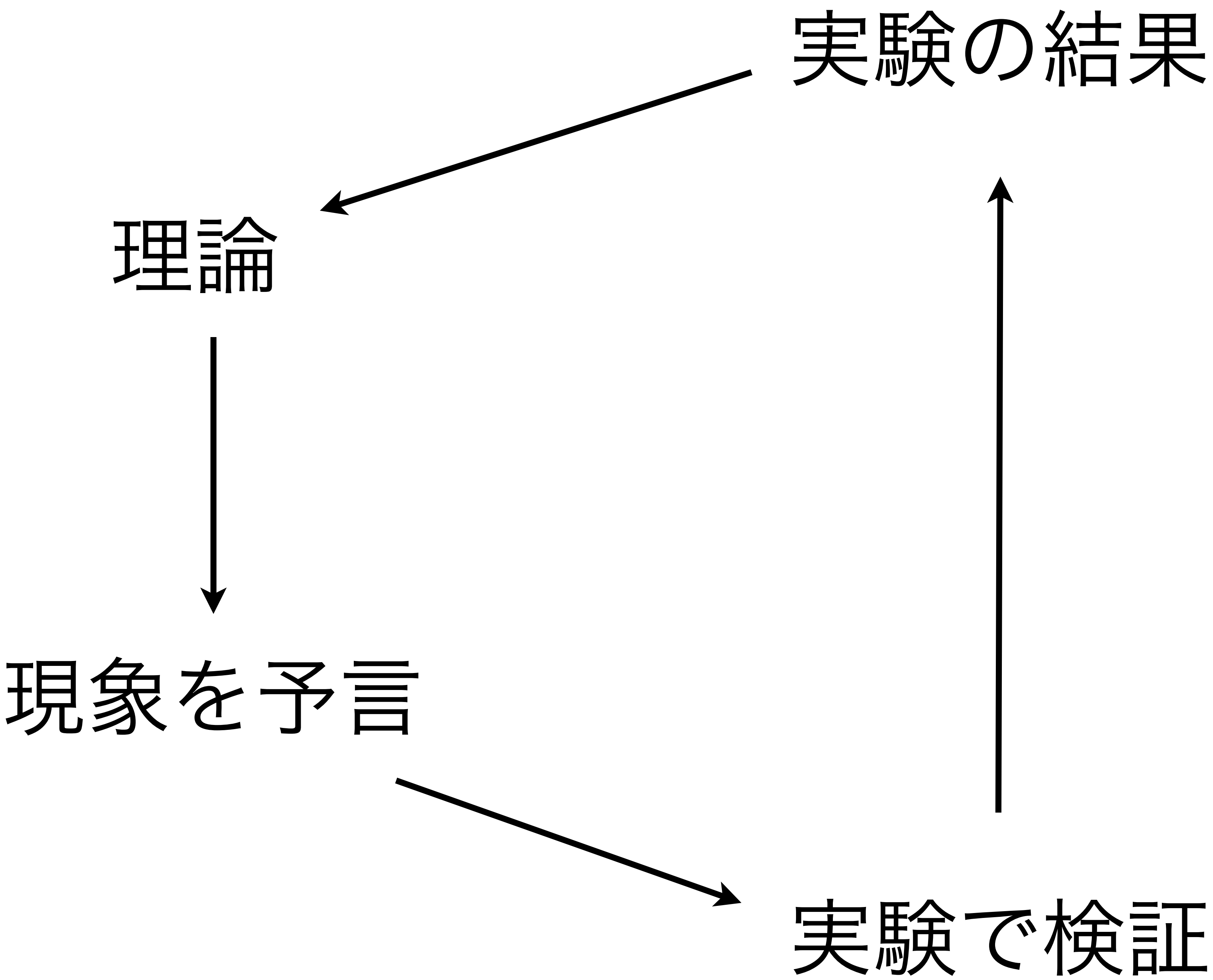
含んでいる

膜理論

- 超弦理論は自動的に量子重力を含む !!

- 残念ながら、低エネルギー (TeV)
  では場の量子論とほとんど変わらない…

- (でも数学には沢山応用がある)

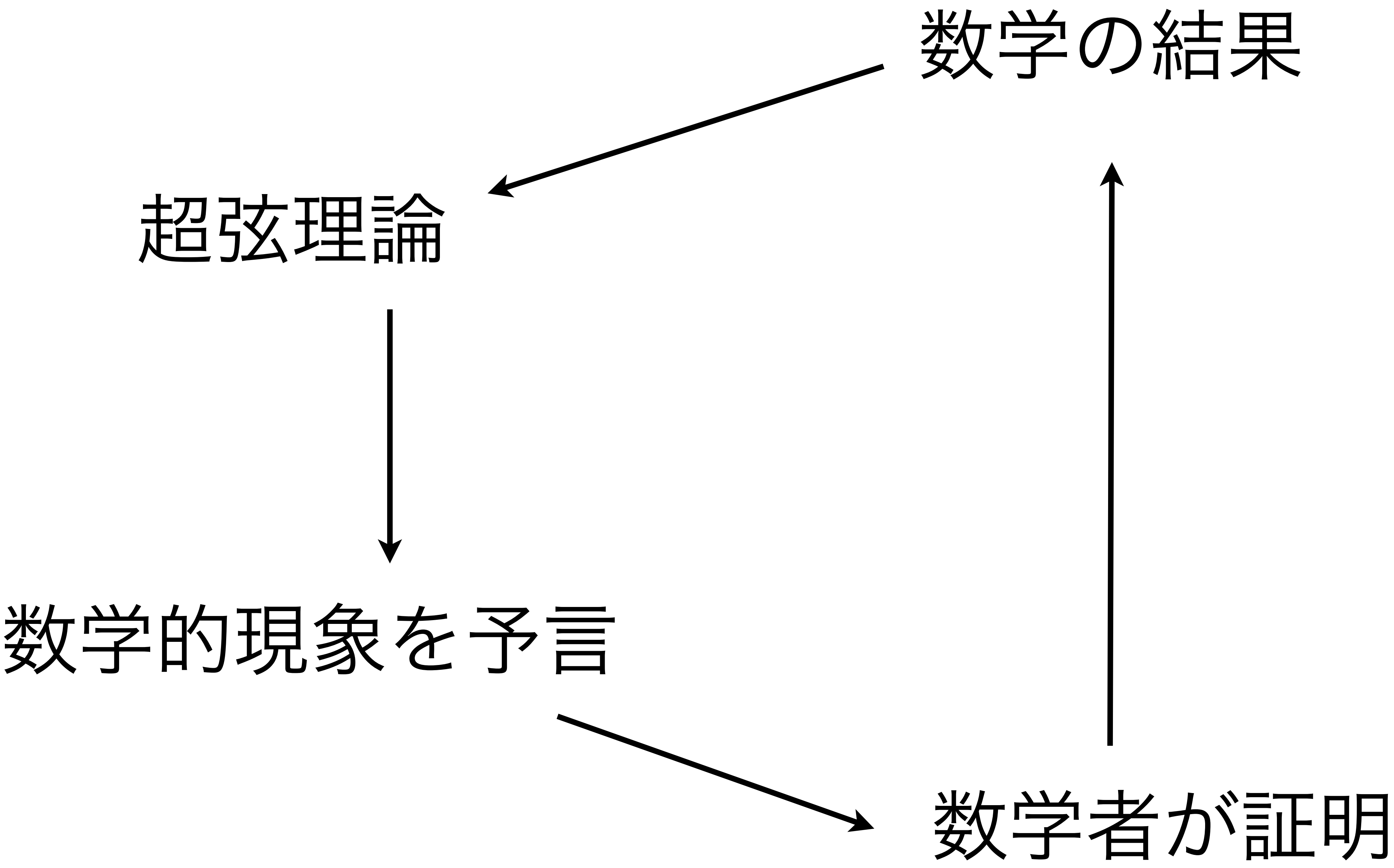


実験の結果

理論

現象を予言

実験で検証

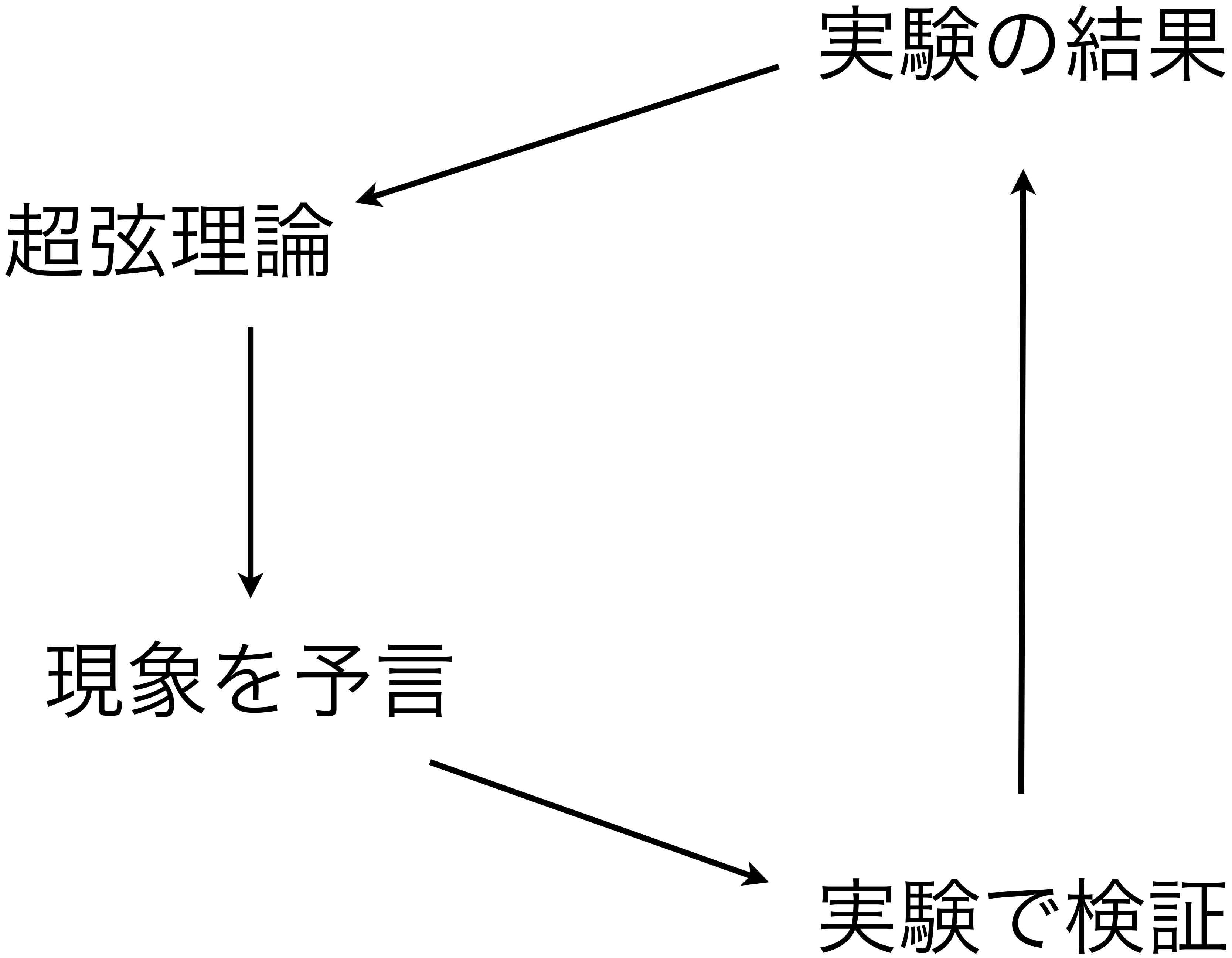


超弦理論

数学的現象を予言

数学者が証明

数学の結果



実験の結果

超弦理論

現象を予言

実験で検証

と行きたいものです…

- 今日

  場の理論のおさらい

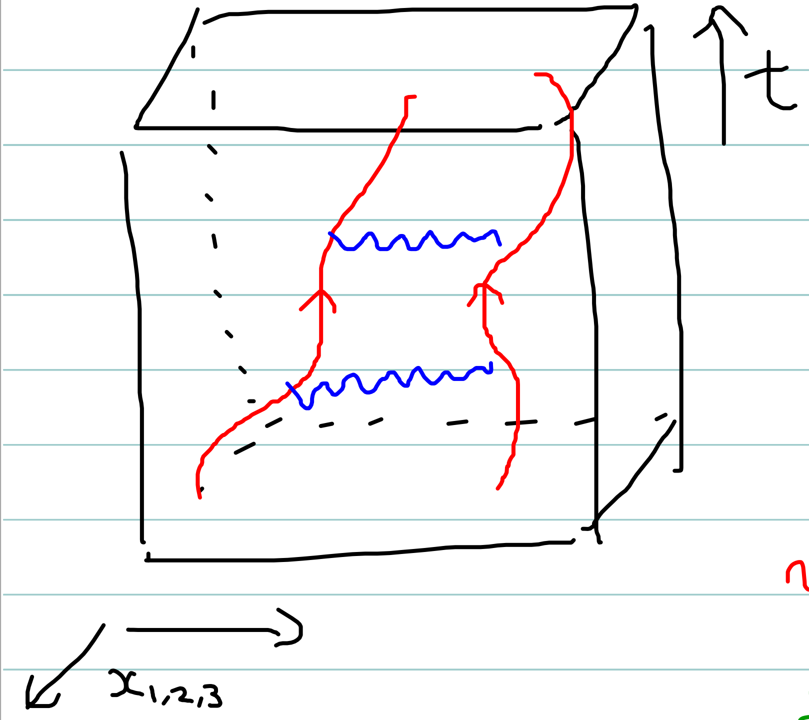
  粒子じゃなくて弦にすると？

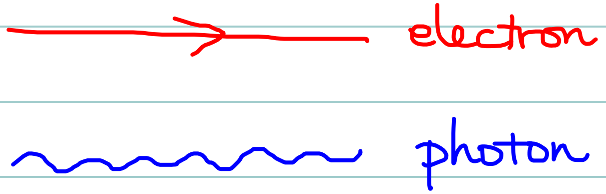
- 明日午前

  弦理論は膜理論を含んでいる

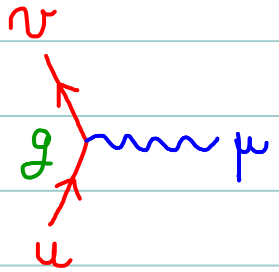
- 明日午後

  弦理論から標準模型を出すには？





Need to sum over all possible paths of particles.



$$g \, \bar{v} \, \gamma^\mu \, u \, e_\mu$$

wavefunc. of fermions

coupling const.

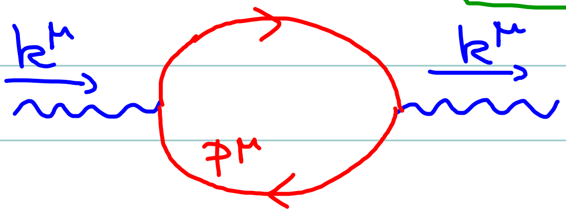
polarization of photon

What was the problem back then?

- Compatibility of Lorentz invariance & Positivity of probability.

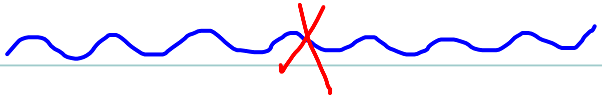
$\rightsquigarrow$ $\boxed{\text{Gauge principle}}$

- Treatment of infinities $\rightsquigarrow$ $\boxed{\text{Renormalization}}$



$$\sim \int d^4p \, \frac{k_\mu k_\nu}{p^4} \sim \log(\text{cutoff}) + \underbrace{f(|k|)}_{\text{measurable}}$$

Counterterm



cancel

$$\sim -\log(\text{cutoff})$$

Positivity of Probability ~ Unitarity

State $|\psi\rangle = c_1|\uparrow\rangle + c_2|\downarrow\rangle$, $\boxed{\langle\psi|\psi\rangle = \langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1}$

$\rightsquigarrow |\uparrow\rangle$ "happens" with prob. $|c_1|^2$, $|\downarrow\rangle^2$ with prob. $|c_2|^2$.

WE NEED $\langle\Psi|\Psi\rangle \geqslant 0$ for all $|\Psi\rangle$.

Let $|0\rangle$: vacuum. $\langle 0|0\rangle = 1$.

Create a photon

$$A^\mu|0\rangle \qquad \langle 0|A^{\mu\dagger} A^\nu|0\rangle \sim -\eta^{\mu\nu} = \begin{pmatrix} \boxed{-1} & & & \\ & +1 & & \\ & & +1 & \\ & & & +1 \end{pmatrix}$$

Lorentz inv.

$$|\psi\rangle = c_t|e_t\rangle + c_x|e_x\rangle + c_y|e_y\rangle + c_z|e_z\rangle$$

$$p^\mu = (\overset{e_t}{E}, \overset{e_x}{E}, \overset{e_y}{0}, \overset{e_z}{0})$$

$$\langle\psi|\psi\rangle = \underline{-|c_t|^2} + |c_x|^2 + |c_y|^2 + |c_z|^2$$

If always $\boxed{c_t = c_x}$

$$\langle\psi|\psi\rangle = |c_y|^2 + |c_z|^2$$

$\rightsquigarrow$ $|e_y\rangle$ "happens" $\frac{|c_y|^2}{|c_y|^2+|c_z|^2}$, $|e_z\rangle$ with $\frac{|c_z|^2}{|c_y|^2+|c_z|^2}$.

$$|\psi\rangle = c_t\underbrace{(|e_t\rangle + |e_x\rangle)}_{\text{unphysical mode}} + c_y|e_y\rangle + c_z|e_z\rangle$$

Mode with polarization $\sim p_\mu$ is unphysical.

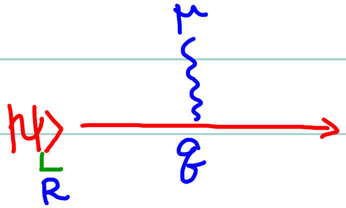
$A_\mu(x)$ and $A_\mu(x) + \partial_\mu \lambda(x)$

are physically THE SAME.

Gauge Symmetry

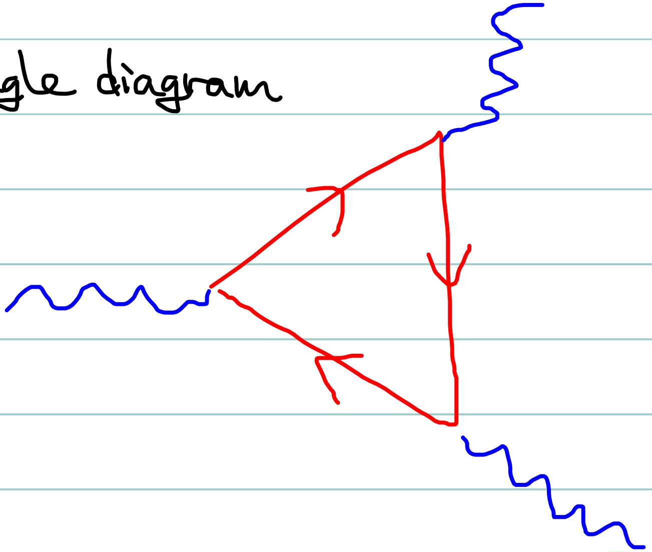
This settled the problem for 20 years.

But it was realized that weak interaction is chiral.



$$\gamma^5 |\psi\rangle_{\substack{L\\R}} = \substack{+\\-} |\psi\rangle_{\substack{L\\R}}$$

Triangle diagram



$$\sim \underbrace{\sum q_i^3}_{\text{Left handed fermions}} - \overbrace{\sum q_i'^3}^{\text{Right handed fermions}}$$

$\boxed{\text{BAD}}$

produces modes with $\boxed{C_t \mp C_x}$! ⇝ Negative Prob.

Summary

- For vector bosons, it's hard to have Unitarity & Lorentz invariance.

- Gauge symmetry saves the day.

- If chiral, it's necessary that

$$\sum q_L^3 - \sum q_R^3 = 0 \quad \cdots\cdots (\#)$$

called ANOMALY FREE CONDITION.

Question

the Standard Model is chiral.

Is (#) satisfied ?

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
(u_L, d_L)	3	2	$1/3$
$\overline{u}_R$	$\overline{3}$	1	$-4/3$
$\overline{d}_R$	$\overline{3}$	1	$2/3$
(e_L, ν_L)	1	2	-1
$\overline{e}_R$	1	1	$+2$

$$\sum q_L^3 - \sum q_R^3 = 3\times 2\times \frac{1}{27} + 3\times 1\times \frac{-64}{27} + 3\times 1\times \frac{8}{27} + 1\times 2\times (-1)^3 + 1\times 1\times 2^3$$

$$= \frac{1}{9}(2-64+8-18+72)$$

$$= 0 \quad !!$$

That looks totally accidental.

GUT :

$$\begin{array}{ll} SU(3)\times SU(2)\times U(1) & \text{one generation} \\ \downarrow & \\ SU(5) & \bar{5}+10 \\ \downarrow & \\ SO(10) & 16 \end{array}$$

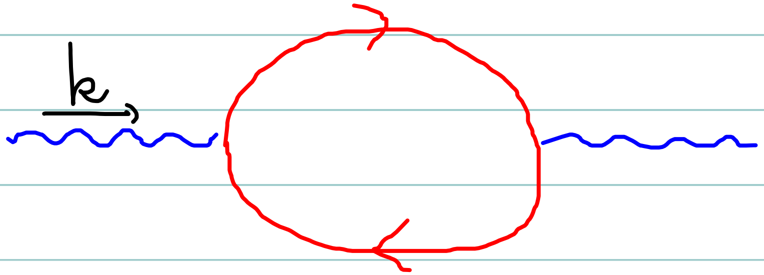
anomaly is cancelled between them.

no anomaly.

SO(10) doesn't have anomalies.

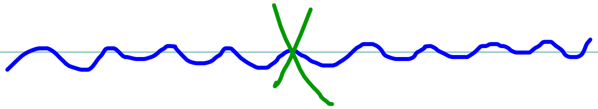
But why are there 3 generations?

Renormalizability



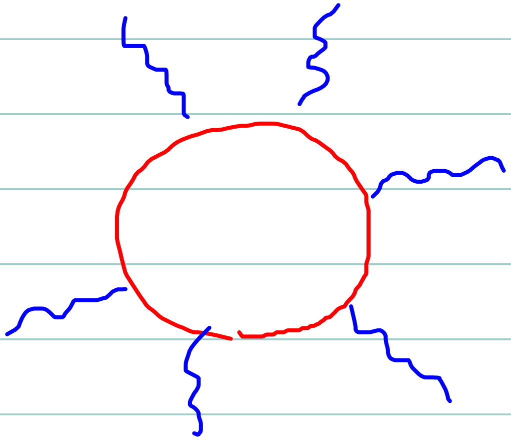
$\sim \log(\text{cutoff}) + \boxed{f(|k|)}$

counter term

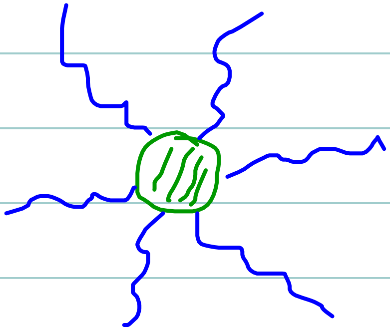


$\sim -\log(\text{cutoff})$

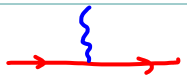
How about

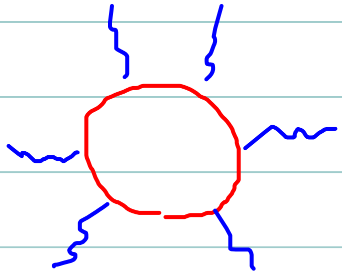


do we need counterterm for this?



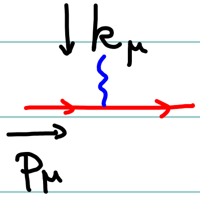
For gauge theories, <u>NO</u>.

 $\sim$ const.

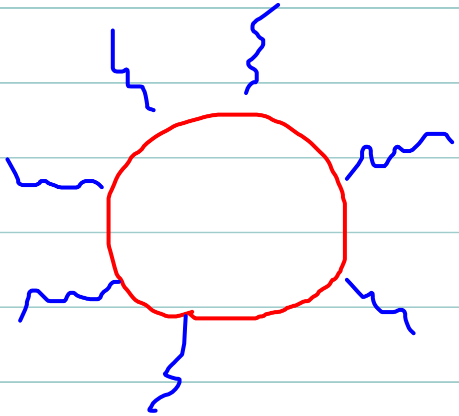
 $\sim \int d^4p \frac{1}{(p)^{\#\text{足の数}}} \sim$ finite.

Only a few types of counterterms are necessary.

For gravity, it's much <u>worse</u>.



$\sim p_\mu k_\nu$



$$\sim \int d^4p \left(\frac{p\cdot k}{\not{p}}\right)^{\#\text{足の数}} \sim \text{infinite!}$$

Need infinite types of counterterms. NONRENORMALIZABLE.

<u>ASIDE</u> Do we really need to quantize gravity ?

Is there anything wrong with

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle$$

Classical equation

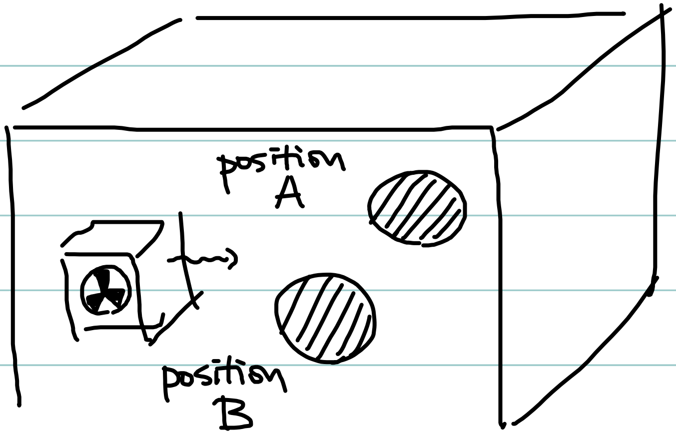
Quantum mechanical average

$$\Delta \phi(x) = \langle \rho(x) \rangle$$

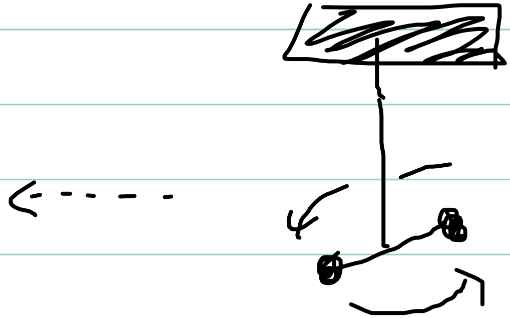
But this leads to various absurdities.

Page & Geilker, PRL 47 (1981) 979

Consider the following Schrödinger's cat-like setup



Position of a heavy obj.
is chosen QM'ally.



Measure its
position using torsion
balance from the outside.

If $\Delta\phi(x) = \langle \rho(x) \rangle$, the $\langle \rho(x) \rangle$

is the average value of the two positions of the heavy object.

$\longrightarrow$ $\phi(x)$ measures this.

But of course the torsion balance sees inside the box

$\phi(x)$ is either for position A or B, not the average.

Conclusion It's too naive to say $\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{classical}} = \underbrace{\langle T_{\mu\nu} \rangle}_{\text{Quantum}}$.

Summary

- Fermion content of the SM is fascinating.
- Quantization of gravity is tough in the standard QFT.

So, what happens if we consider

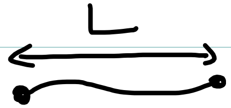
strings

instead of

particles ?

What changes if string is used ?

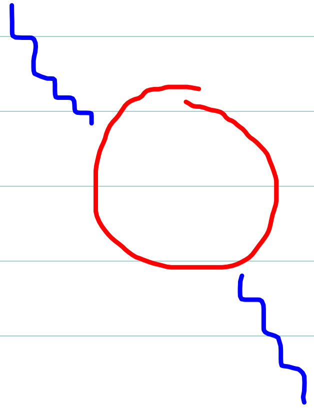
It has a length scale l_s s.t.

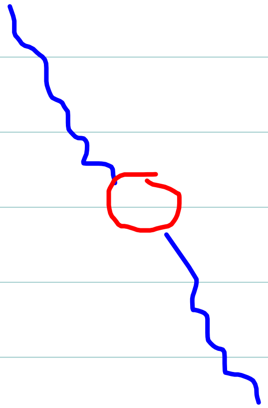


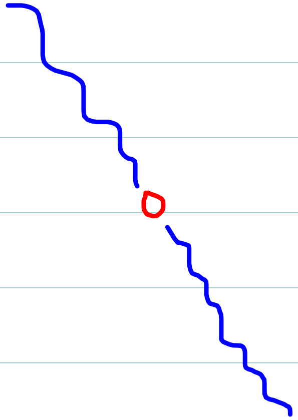
has mass L/l_s^2

The field theory divergence from UV

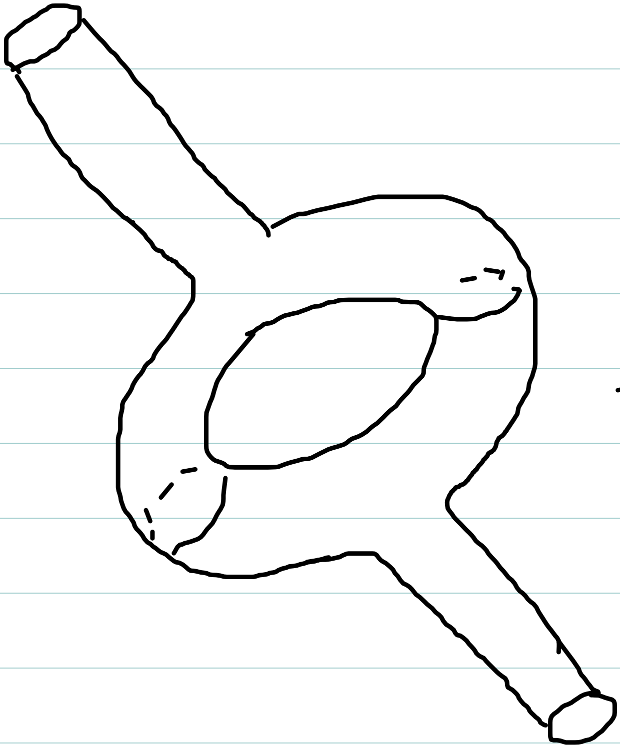
$\int d^4p \, \frac{1}{p^2}$, say, comes from

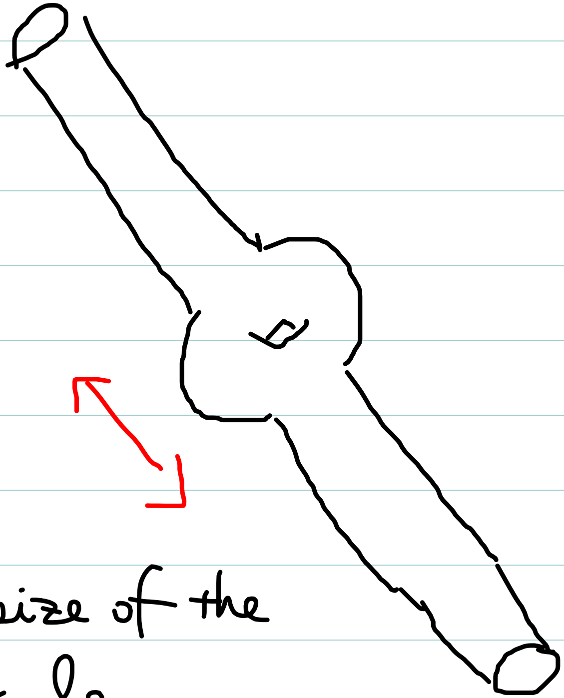






In string theory,





There's an intrinsic minimal size of the loop $\sim l_s$

$\rightarrow$ Every loop diagram is cut off around

$$|p| \sim 1/l_s$$

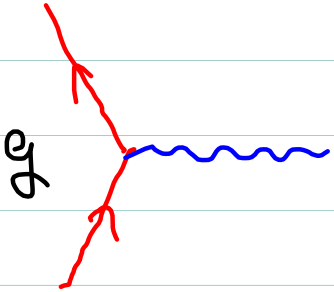
Another merit: in QFT, we start by specifying the Lagrangian ~ Gauge fields

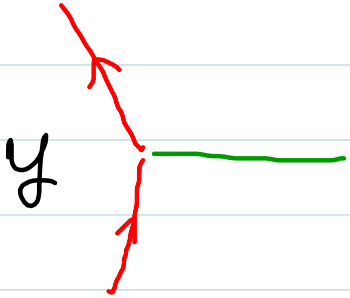
Fermions

Scalars, Yukawa couplings ...

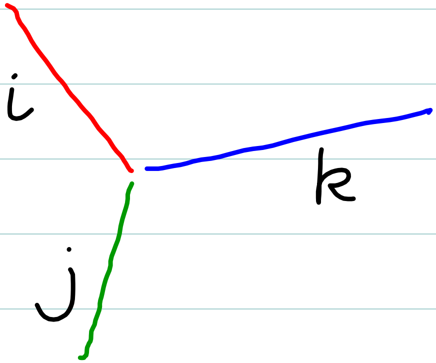
In terms of Feynman diagrams, we have different types of lines

and various interaction vertices



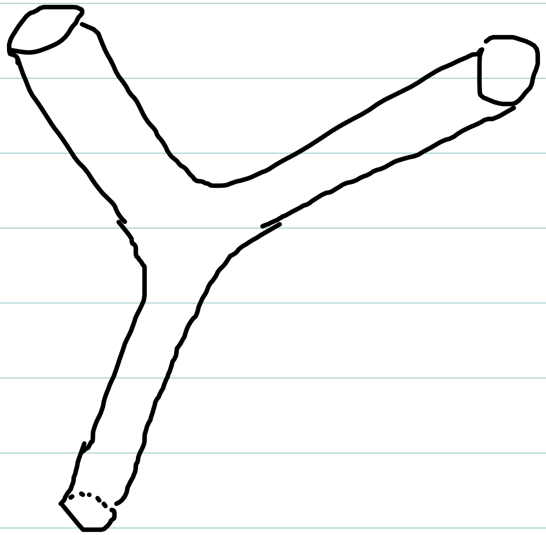


Note g & y needs to be chosen for each vertex.

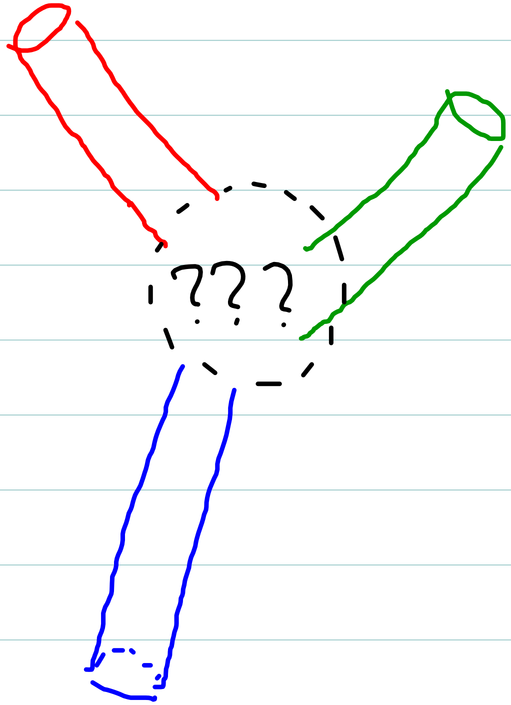


$\sim \lambda_{ijk}$

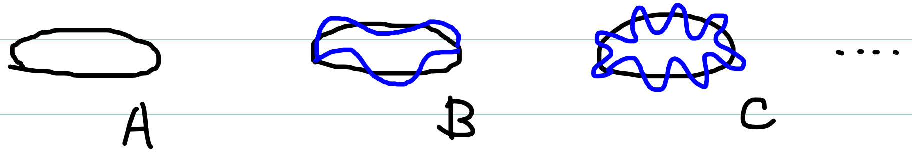
In string theory, you can't join two different types of strings consistently:



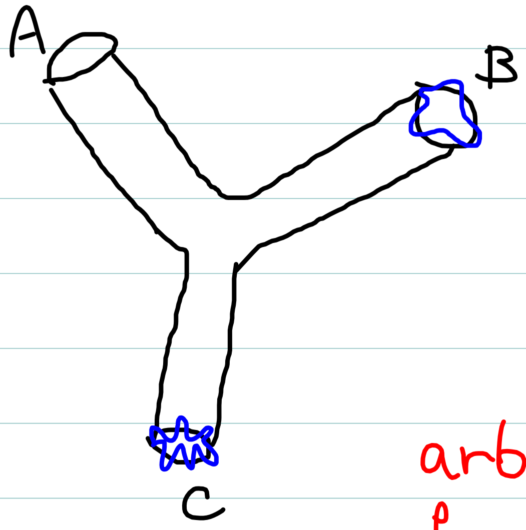
but



Instead, a string can vibrate in different ways:

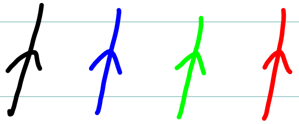
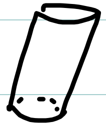
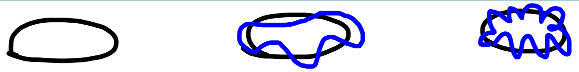
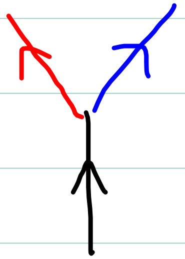
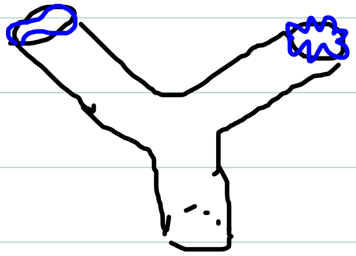


which correspond to various different particles

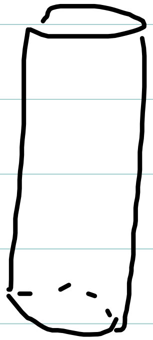


$$\sim \lambda_{ABC}$$

This is <u>no longer</u> an arbitrary numbers. Calculable from the dynamics of strings.

QFT	STRING
	
choose types of particles in a theory.	choose a type of string.
	study how they vibrate. 
choose how they interact.	interaction is then determined.
$\lambda \sim$ 	

There are two known consistent types of strings.



Type II string

Heterotic string

Both require 1+9 dimensional spacetime.
have supersymmetry.
Contrains quantized gravity.

All these follow from the consistency of
Lorentz inv. & Positivity of probability.