

From micro to macro (m2m) cosmos

MY @icepp04 lectures

Content of lectures

1. Fundamentals on cosmology
2. Thermalization and decoupling: application to dark matter abundance
3. Generation of baryon asymmetry
4. Inflation
5. Dark energy problem and towards a possible resolution

Fundamentals on cosmology

For discussion of the universe as a whole, one needs the law of gravity in the large scale.

Law of gravity = Einstein's general relativity

Basic ideas of general relativity

1. Equivalence principle

At each spacetime point the effect of gravity may be eliminated and microphysical laws based on special relativity and quantum mechanics hold.

2. **Geometry** = $8\pi G \times$ Energy-momentum of matter formulated by Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} \quad (1)$$

with the **spacetime metric** and the **curvature tensor**,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \quad (2)$$

$$R^\mu_{\nu\rho\sigma} = \frac{\partial\Gamma^\mu_{\nu\rho}}{\partial x^\sigma} - \frac{\partial\Gamma^\mu_{\nu\sigma}}{\partial x^\rho} + \Gamma^\eta_{\nu\rho}\Gamma^\mu_{\eta\sigma} - \Gamma^\mu_{\rho\eta}\Gamma^\eta_{\nu\sigma}, \quad R_{\mu\nu} = R^\rho_{\mu\rho\nu} \quad (3)$$

(Replacing the **Newtonian** Poisson equation to determine the gravitational potential)

3. Test particle following **geodesics**

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = 0, \quad (4)$$

$$\Gamma_{\rho\sigma}^\mu = \frac{1}{2} g^{\mu\lambda} \left[\frac{\partial g_{\sigma\lambda}}{\partial x^\rho} + \frac{\partial g_{\rho\lambda}}{\partial x^\sigma} - \frac{\partial g_{\rho\sigma}}{\partial x^\lambda} \right] \quad (5)$$

(Replacing Newton's equation of motion for point particle)

Cosmological principle

Assumption;

Our 3-space is close to a **maximally symmetric space**, with

- Homogeneity (translational invariance)
- Isotropy (rotational invariance)

In other words, every space point has an equal footing, like a democratic society. But of course, with presence of (quantum ?) fluctuation the local part of universe may differ from each other, due to the gravitational attraction at later epochs (basis for the structure formation).

If the 3-space is flat (spatial curvature = 0), it should be described by the Euclidian geometry. More generally, the **Robertson-Walker metric** after using an appropriate coordinate system,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right], \quad k = \pm 1, 0 \quad (6)$$

corresponding to a flat Euclidian ($k = 0$), a positively curved ($k = 1$), and a negatively curved ($k = -1$) 3-space. $a(t)$ is called **the cosmic scale factor**.

The maximal symmetry leads to a form of the energy-momentum tensor of an ideal perfect fluid,

$$(T_{\mu}^{\nu}) = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \quad (7)$$

with ρ the energy density, and p the pressure.

Functional relation $\rho(p)$ is given by a state of the many-body system representing the universe. For an ensemble of massless particles, or extremely relativistic particles $\rho = 3p$, or traceless $T_{\rho}^{\rho} = 0$. For an ensemble of massive particles, or non-relativistic particles, $\rho \gg p$

Using the **Hubble rate** $H(t)$, $H = \frac{\dot{a}}{a}$ the Einsteint equation is reduced to

$$\dot{H} = -4\pi G(\rho + p), \quad (8)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho. \quad (9)$$

Dual significance of the k -term:

1. Spatial curvature $R^{(3)} = \frac{k}{a^2}$

2. Evolution

$k = 0$: expansion foreever

$k > 0$: collapse later

$k < 0$: expansion foreever

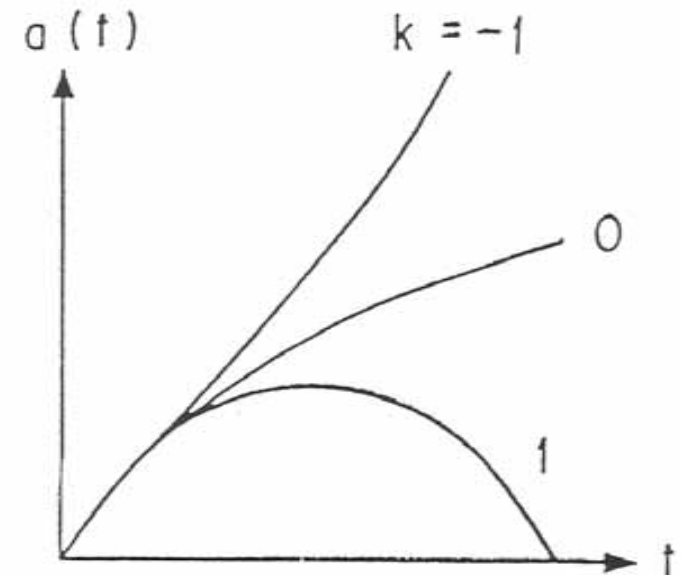


Fig. 1 SCALE FACTOR

Introduction of Ω parameters

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} \approx 1.9 \times 10^{-29} h^2 \text{ gcm}^{-3}, \quad h \equiv \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}} \quad (10)$$

$$\Omega_m + \Omega_\lambda = 1 + \frac{k}{\dot{a}^2}, \quad \Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_\lambda = \frac{V_0}{\rho_c} \quad (11)$$

The cosmological principle is later challenged by causality argument, and explained by the inflationary scenario.

Newtonian interpretation

Consider a test particle at distance a from a center which can be taken at any point in the homogeneous and isotropic space filled by a gas of uniform mass density $\rho \propto a^{-3}$ (no creation or destruction). The Newton equation gives

$$\ddot{a} = -G \frac{4\pi a^3 \rho}{3} \frac{1}{a^2} \quad (12)$$

leading to an integrated form,

$$\frac{\dot{a}^2}{2} - G \frac{4\pi a^2 \rho}{3} = -k \quad (13)$$

with k a constant of motion. This is identical to the H^2 equation (the Newton equation also being consistent with $\dot{H} = \frac{\ddot{a}}{a} - (\frac{\dot{a}}{a})^2$ equation for the pressureless gas), the difference being the geometrical meaning of a .

Key observations

3 pillars of big bang cosmology:

- Hubble expansion
- 3 K microwave background radiation
- Nucleosynthesis

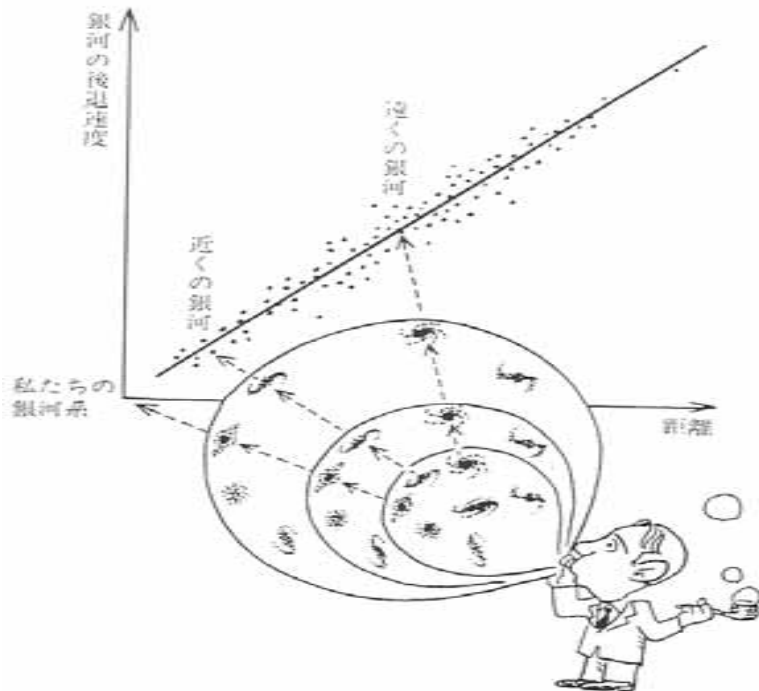


図 4.6 ハッブルの法則。遠くの銀河ほど速い速度で私たちから遠ざかっている。

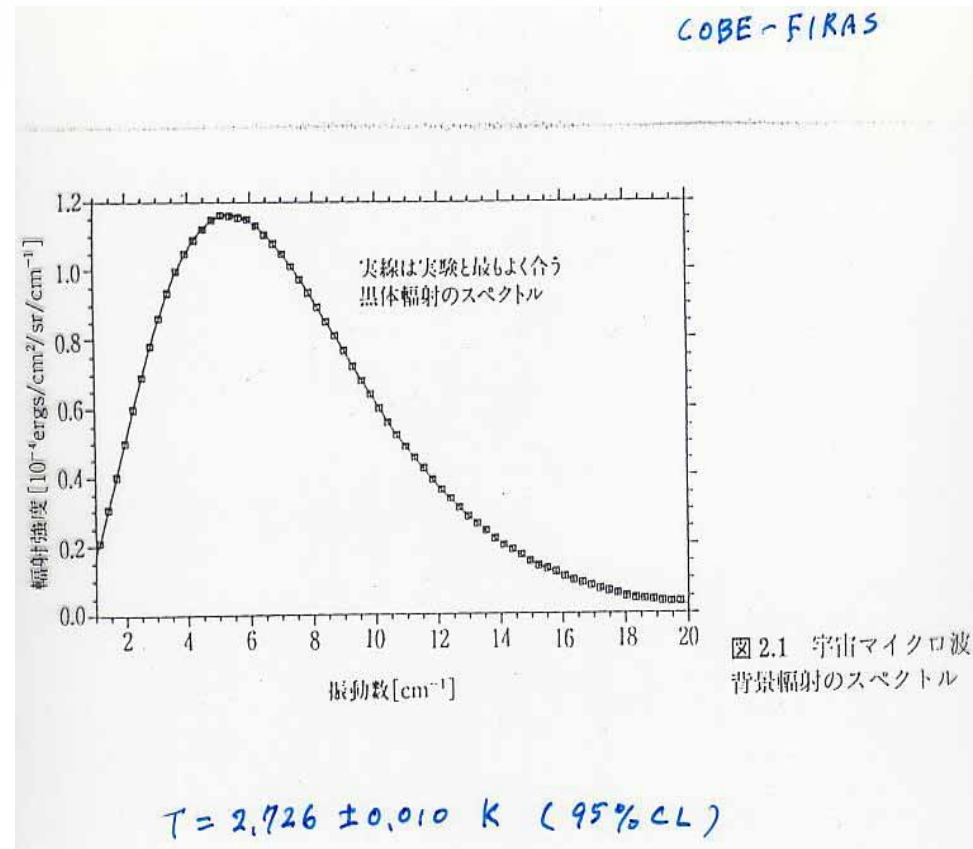


図 2.1 宇宙マイクロ波背景輻射のスペクトル

3 pillars of big bang cosmology

- Hubble law

$$H_0 = 72 \pm 8 \text{ km sec}^{-1} \text{ Mpc}^{-1}$$

- Microwave background radiation

$$2.726 \pm 0.01 \text{ K}$$

almost perfect Planck, with small dipole and
very small multipoles

- Nucleosynthesis
consistent with

$$\Omega_B h^2 = 0.019 \pm 0.0024 (95\%)$$

CMB、密度揺らぎと基本パラメータ

- 大規模構造(銀河、銀河団など)形成の種は、3Kのマイクロ波の揺らぎに反映される
- 晴れ上がり時の因果領域(ホライズン)のスケールが重要
- 特に、初期揺らぎの大きさのサイズ依存性から、宇宙進化を支配するパラメータ(重力源の量、バリオン物質の量、宇宙定数など)が決定可能

観測の進歩

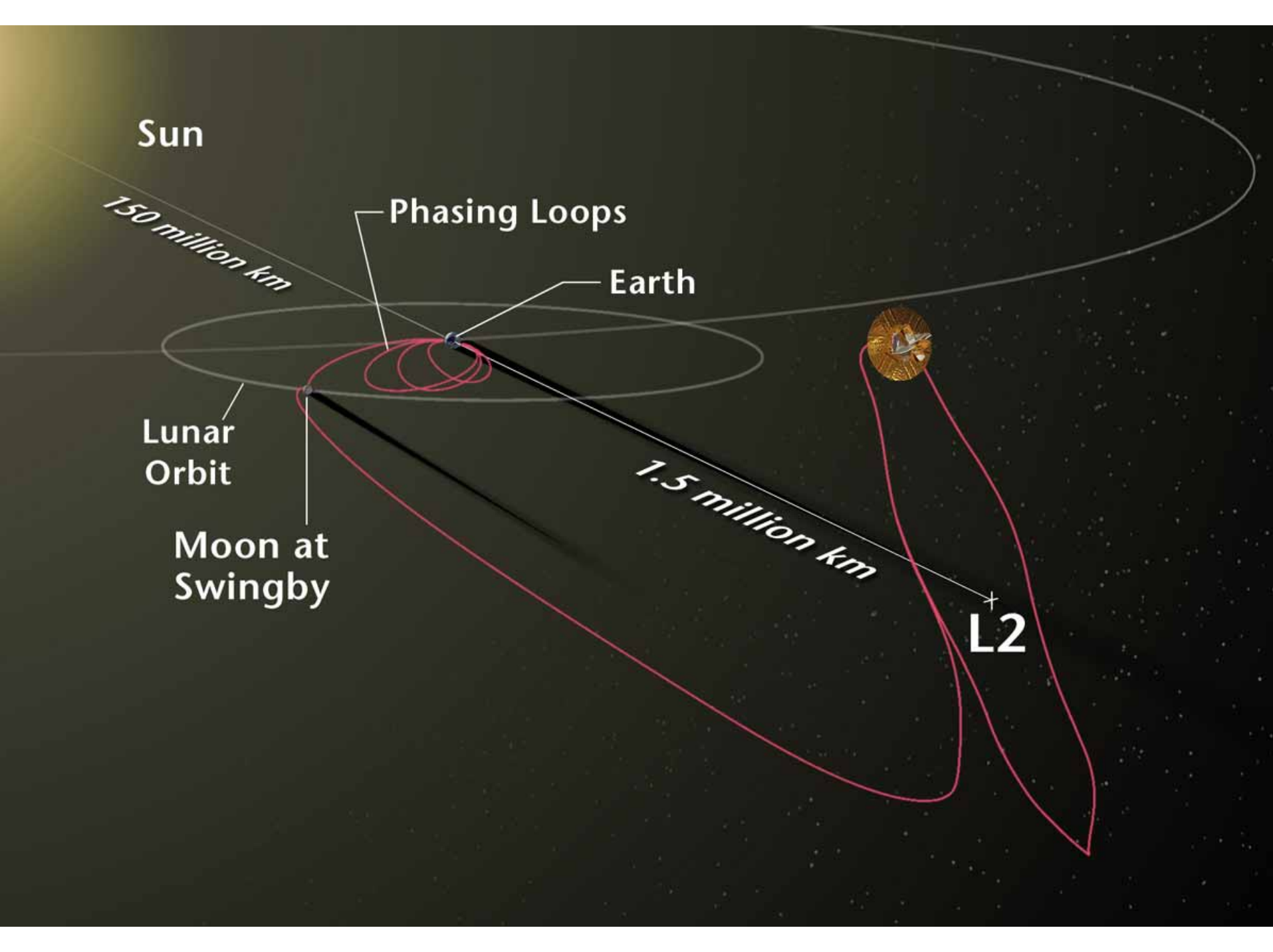
First Results of

Wilkinson Microwave Anisotropy Probe:

WMAP

Wilkinsonは昨年亡くなったチームの精神的リーダー





Sun

150 million km

Phasing Loops

Earth

Lunar
Orbit

Moon at
Swingby

1.5 million km

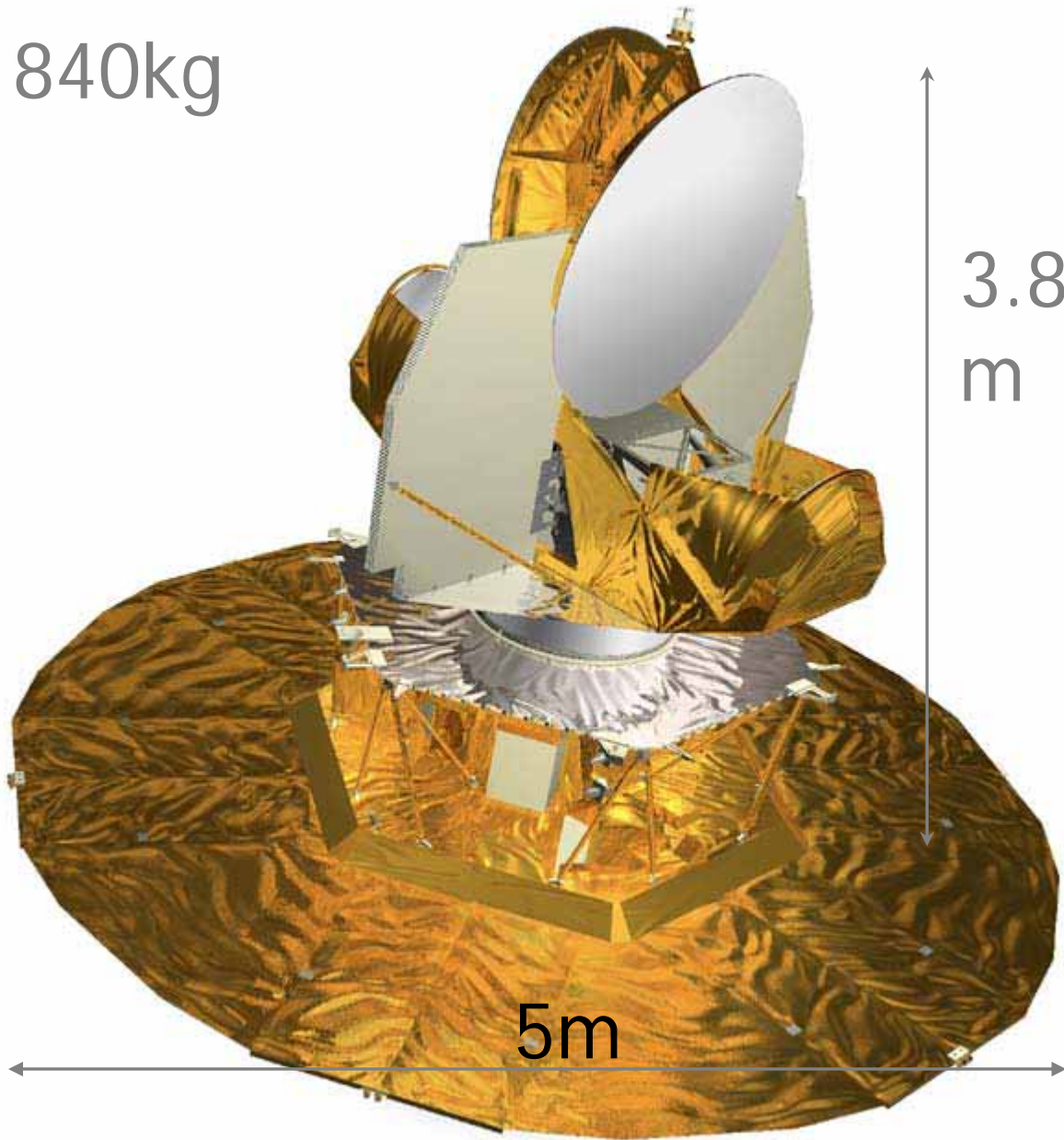
L2

840kg

4年の観測計画

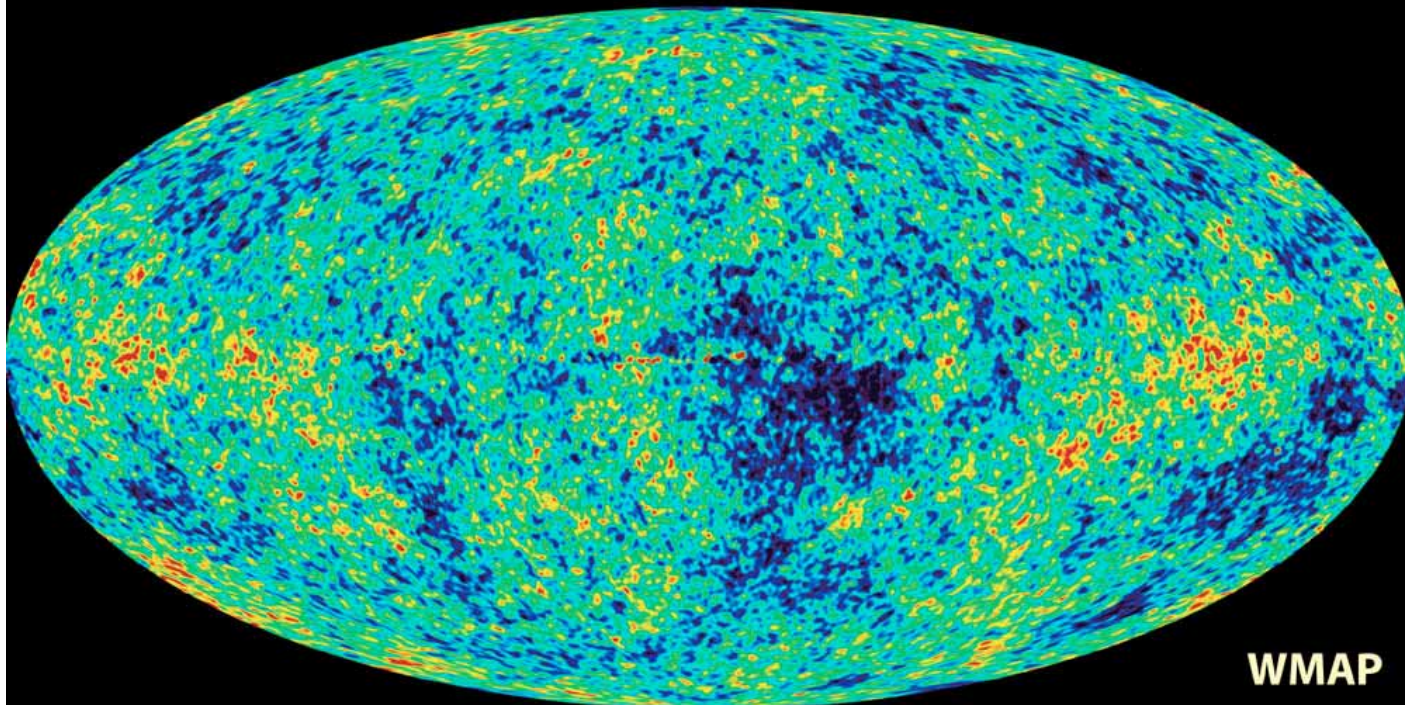
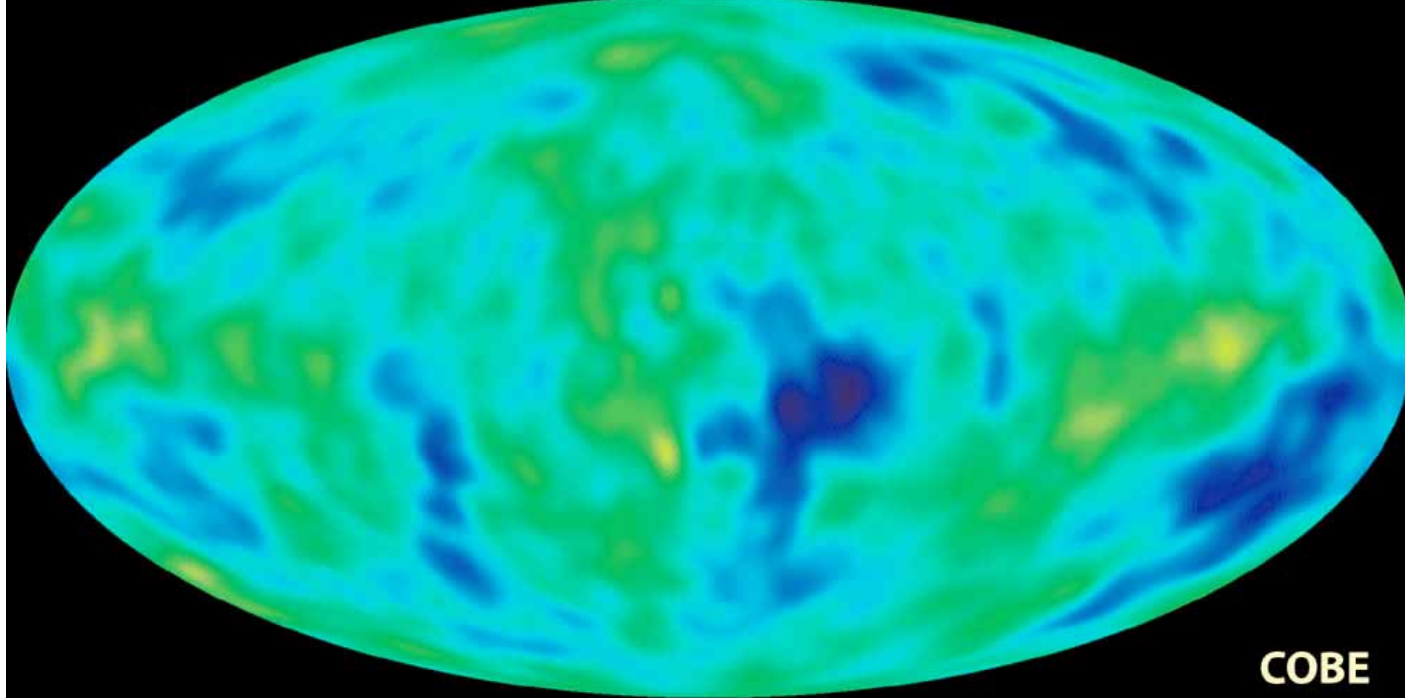
3.8
m

5m

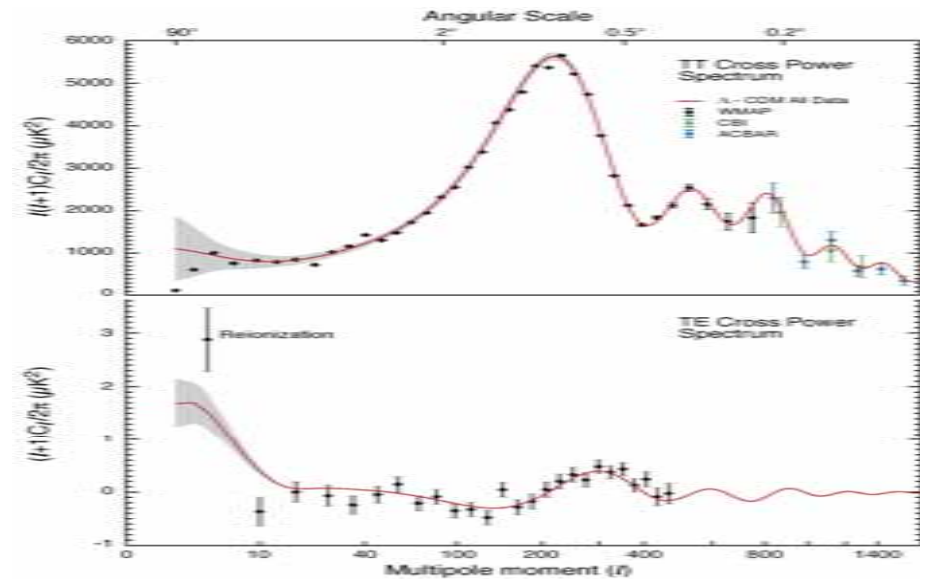
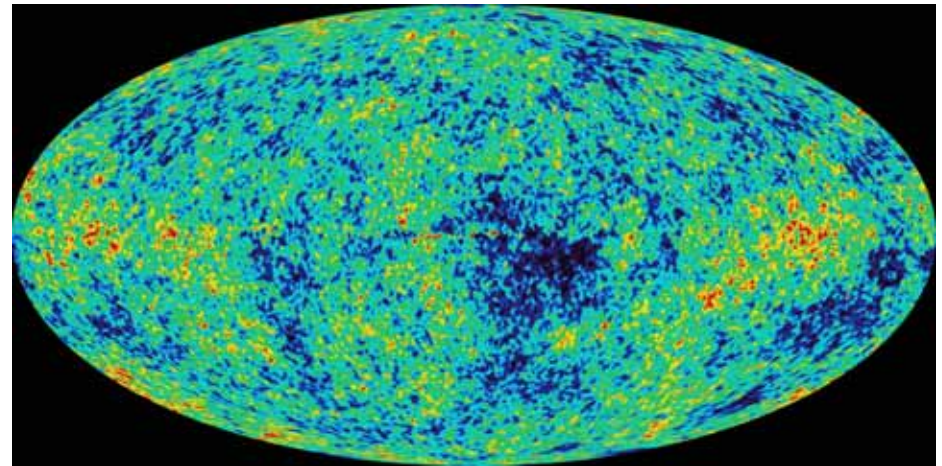
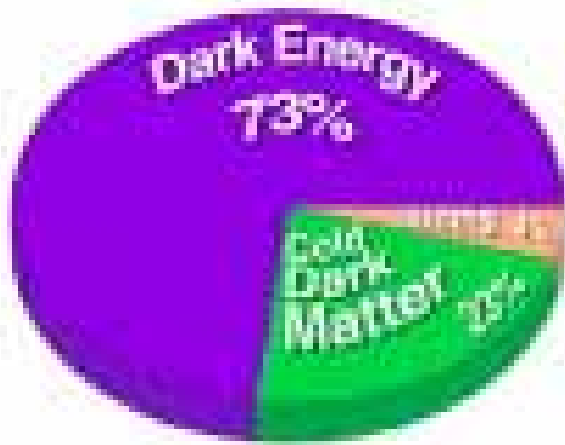


- 人工衛星、ラグランジュポイント [L2](#)、地球から 150 万キロ太陽とは逆側に。
- COBE 以後、最初のスペース、全天観測
- COBE の 10 倍細かく分解し、10 倍感度のよい観測 ($l=900$ まで)
- 多波長で観測: 5-bands, 23, 33, 41, 61, 94 GHz
- 宇宙マイクロ波背景放射の温度の分布を 100 万分の 1 の精度で観測
- 偏光成分も詳細に測定
- 誕生からおおよそ 40 万年 (38 万年) 後の宇宙の様子

COBEと の比較



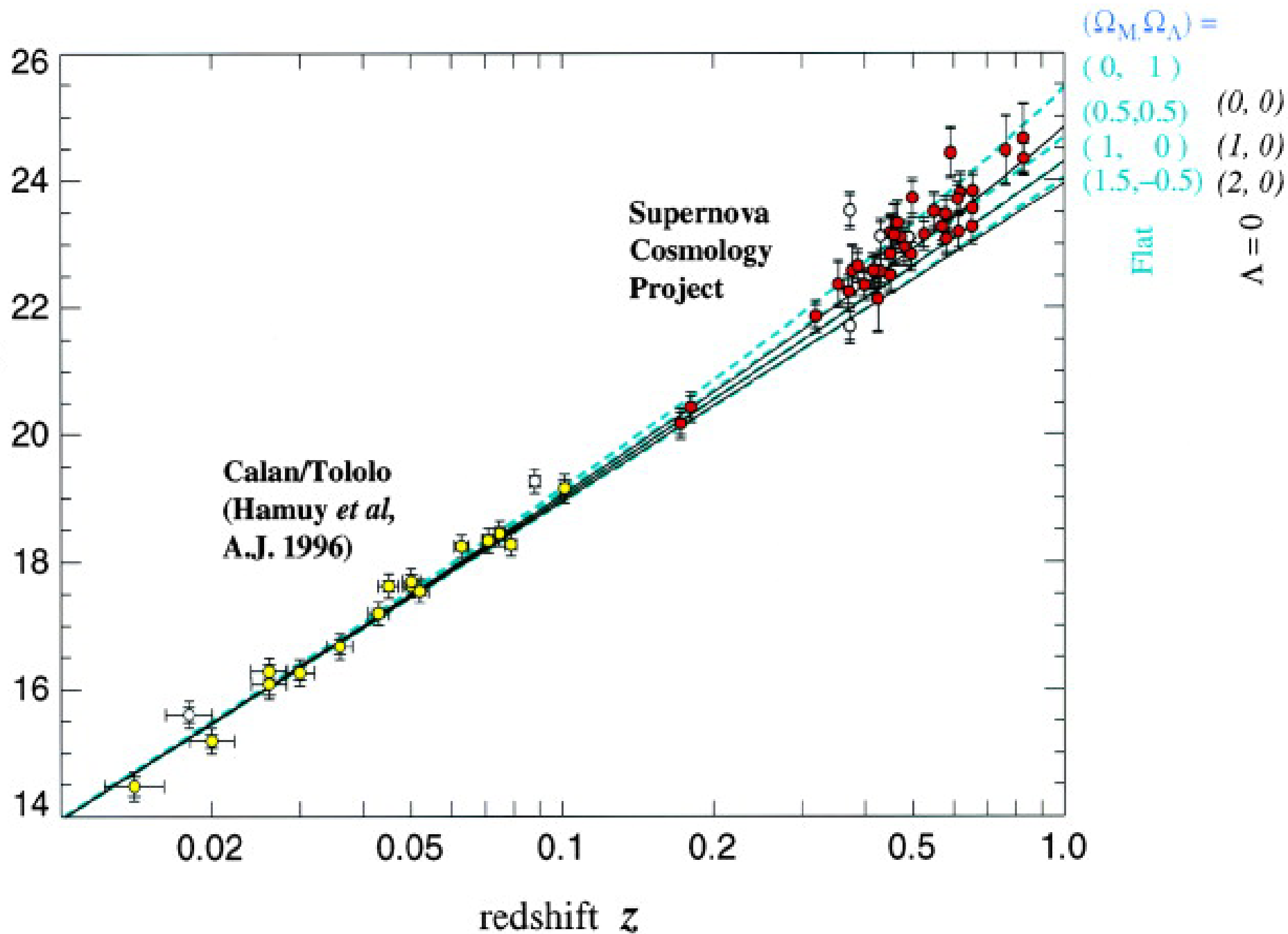
WMAP results

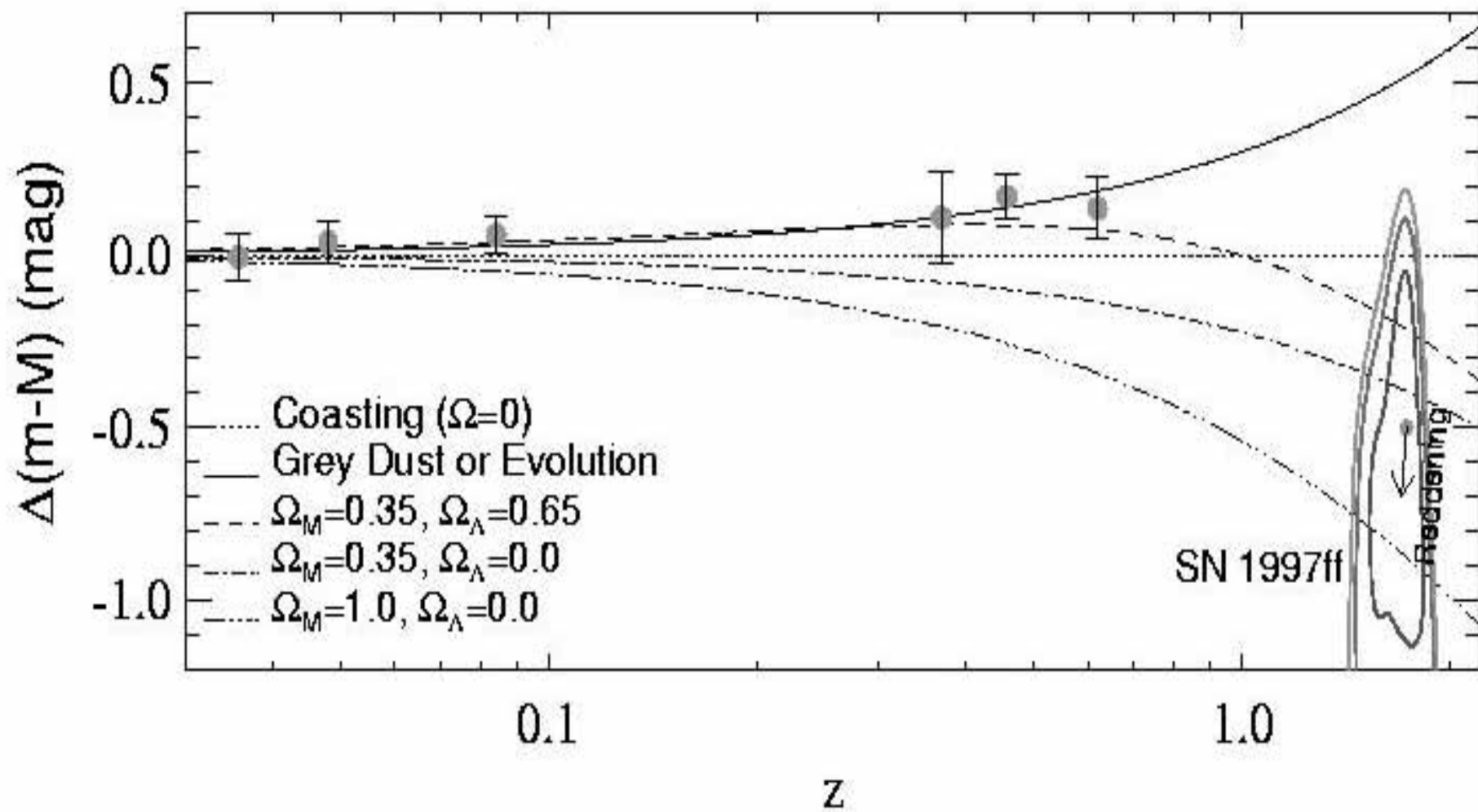


宇宙項 Λ の測定

- ・ 遠方の距離 — z (赤方変位) 関係を変化させる
- 遠方の(標準光源としての)超新星光度を距離測定に使う
- 物質(主として暗黒物質)項の約2倍の大きさ

effective m_B





Cosmological Parameters

$$h = 0.72 \pm 0.05(\text{WMAP})$$

$$= 0.71^{+0.04}_{-0.03}(\text{all})$$

$$\Omega_M h^2 = 0.14 \pm 0.02(\text{WMAP})$$

$$= 0.135^{+0.008}_{-0.009}(\text{all})$$

$$\Omega_B h^2 = 0.024 \pm 0.001(\text{WMAP})$$

$$= 0.0224 \pm 0.0009(\text{all})$$

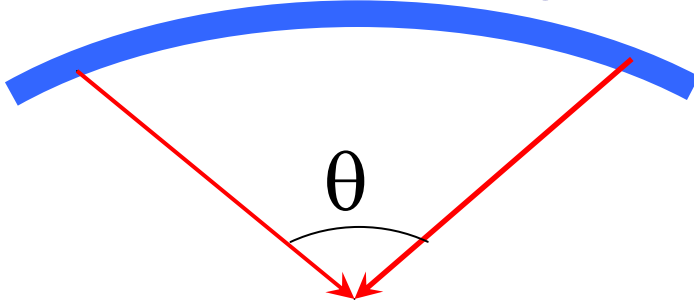
$$\Omega_{tot} = 1.02 \pm 0.02(\text{WMAP} + \text{SN, or, HST, 2DF})$$

宇宙の幾何学

- 一般に、非ユークリッド幾何学
- ユークリッドの平坦な宇宙は、物質、宇宙項エネルギーの総和が臨海値 ($\Omega_0 = 1?$) に等しいかどうかで決まる
- 宇宙の膨張則とも関連

Flat Universe

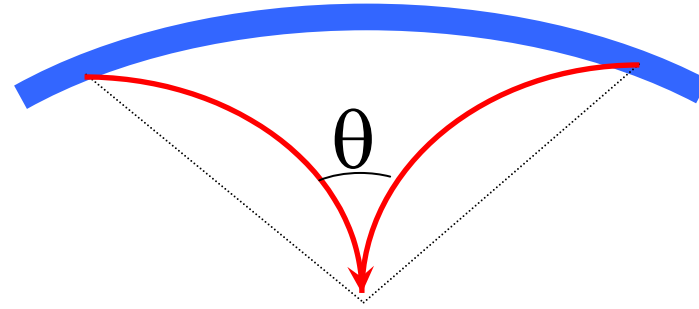
Angle sum = 180
Last Scattering



Observer

Open Universe

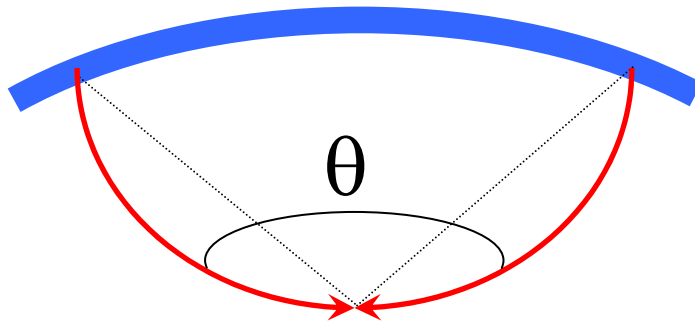
Angle sum < 180



Observer

Closed Universe

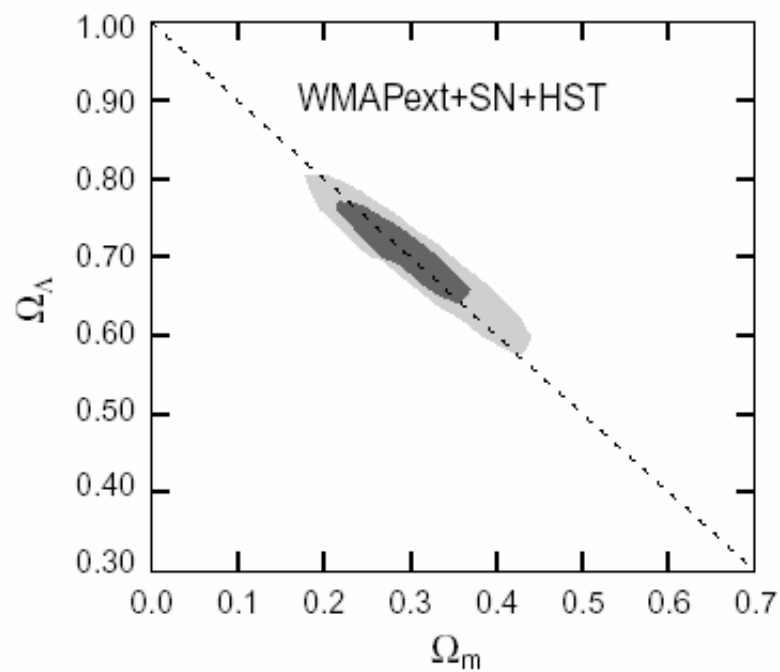
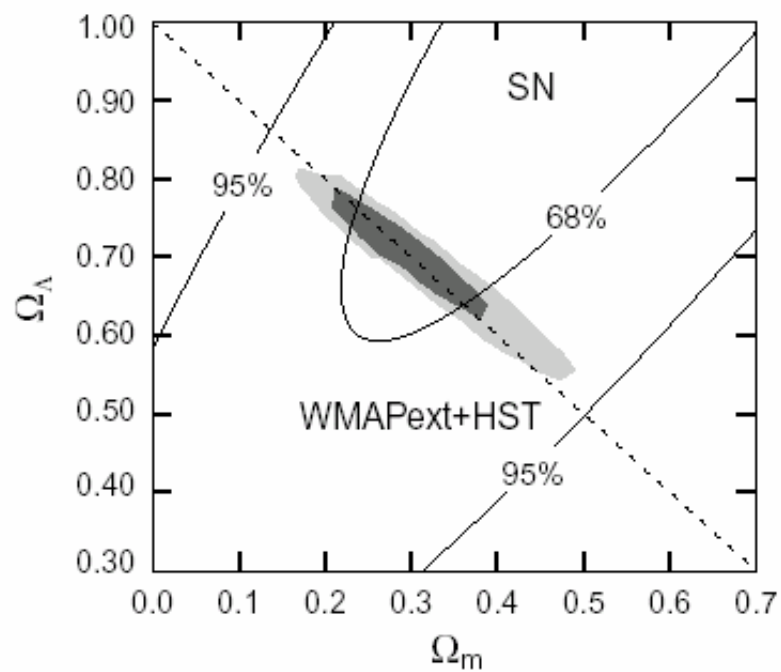
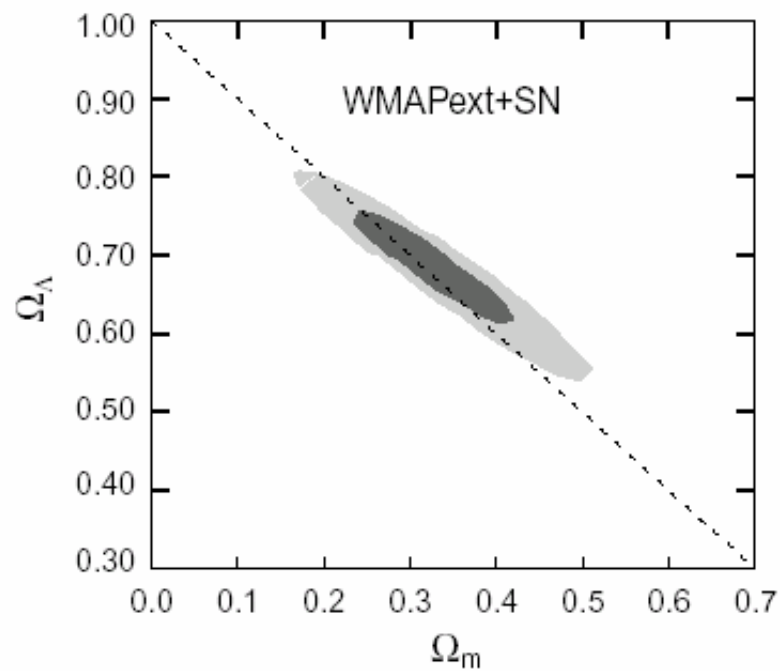
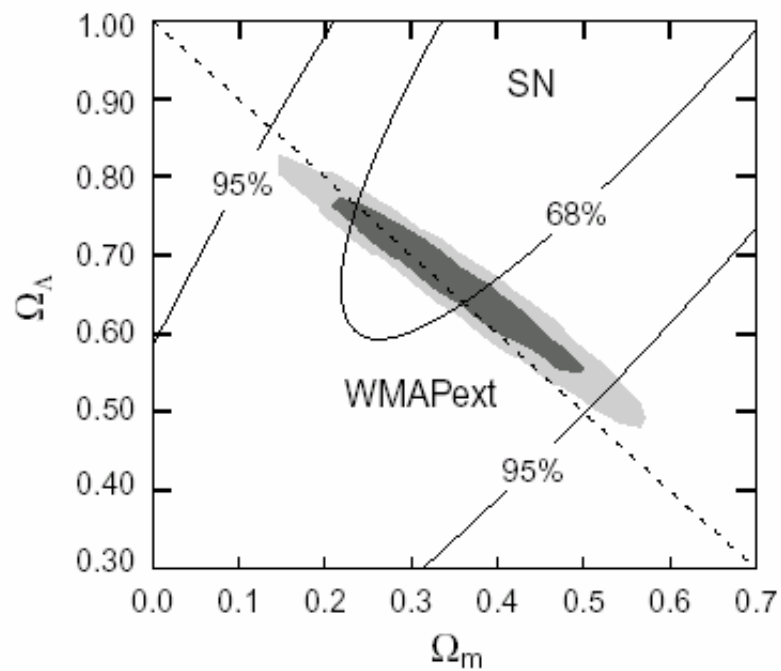
Angle sum > 180



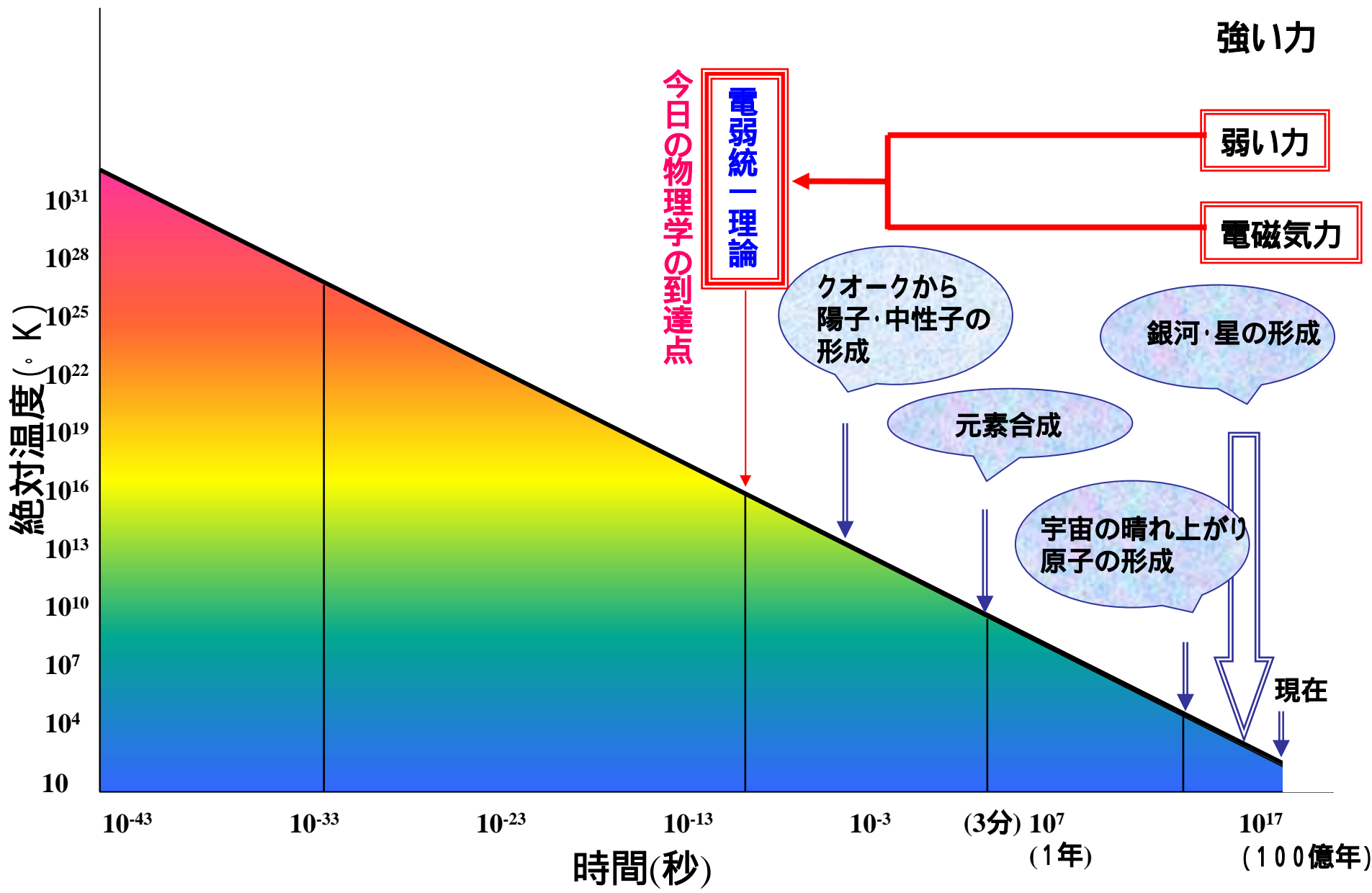
Observer

$$l \propto 1/\theta$$

- 宇宙はやはり平坦だった
- 宇宙の全エネルギーのうち、4 %がbaryon、23 %がnon-baryonic dark matter、残り73%がdark energy
- 現在の宇宙年齢は
 - 134 \pm 3億歳 (WMAPのみ)
 - 137 \pm 2億歳 (all)
- Recombination epochは
 - $z = 1089 \pm 1$
 - $\Delta z = 195 \pm 2$
 - $t = 379 \pm 8 - 7 \text{ kyr.}$



宇宙の熱的歴史



Important concept: decoupling

- Fast (rate \gg Hubble) processes
in equilibrium

$$a + b + \cdots \leftrightarrow x + y + \cdots$$

When rate = Hubble with cosmic expansion,
processes decouple and **freeze-out**

Stable remnant remains at later epoch

Physics involved at decoupling

Microwave background radiation

Ionization and Thomson scattering

$$\gamma + e \rightarrow \gamma + e$$

thermal 3000 K at decoupling \rightarrow 3 K now

Nucleosynthesis

Weak interaction for $p \leftrightarrow n$

$$p + e \leftrightarrow n + \nu_e \quad \text{etc}$$

giving the initial value $\frac{n_n}{n_p} = e^{-\Delta M / T}$

Proton-Neutron Concentration Ratio in the Expanding Universe at the Stages preceding the Formation of the Elements.

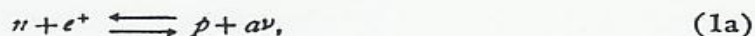
Chushiro HAYASHI.

Department of Physics, Naniwa University.

(Received January 12, 1950)

§ 1. Introduction.

In the theory of the origin of the elements by Gamow, Alpher, and collaborators¹⁾, primordial matter (ylem) of the universe, which afterwards has been cooled down owing to the expansion of the universe and has formed the elements through nuclear reactions such as radiative capture and beta-decays, is assumed to consist solely of neutrons. At early stages, however, of high temperatures ($kT \gtrsim mc^2$, m being the electron mass) in the expanding universe before the formation of the elements, induced beta-processes caused by energetic electrons, positrons, neutrinos and antineutrinos, in addition to the natural decay of neutrons, such as



must have proceeded, their rates being faster at higher temperatures, and had a effect on the proton-neutron concentration ratio. At still higher temperatures $kT \gtrsim \mu c^2$ (μ is the mesons' mass), where large number of mesons are expected to be in existence, n - p conversion process induced by mesons would have been much more rapid owing to their stronger interactions with nucleons than the processes induced by light particles. Consequently, the n - p ratio must have been determined by the rates of such processes and those of changes in temperature and density in the universe resulting from its expansion.

We shall be based on the relativistic theory of the expanding universe, which are shown by Gamow as having a possibility to explain the origins both of the elements and the galactic nebulae.^{1),5)} Then, the expansion and contraction rates of the universe at the stages of high compression are given by²⁾

$$\frac{1}{l} \frac{dl}{dt} = \pm \left(\frac{8\pi}{3} G \rho \right)^{\frac{1}{2}}, \quad (2)$$

where l is an arbitrary proper length of a volume containing a given amount of

Thermalization and decoupling

Typical behavior of reaction rate, for instance, $N + \bar{N} \longrightarrow \pi's$, is $\langle \sigma v n \rangle$, per unit time with cosmic expansion, where σ is the microscopic cross section, and n the number density of participating species like \bar{N} , $\langle \dots \rangle$ denoting averaging over an (often thermal) ensemble.

Crucial comparison: how large or how small $\langle \sigma v n \rangle$ is **compared to the expansion rate**,

$$H = \sqrt{\frac{8\pi G \rho}{3}} \approx \sqrt{N} \frac{T^2}{m_{pl}} \quad (14)$$

with $m_{pl} \approx 10^{18} GeV$ the Planck mass scale, and N the number of species of participating ER particles. If $\langle \sigma v n \rangle \gg H$, the process is thermalized and statistical equilibrium is realized. If $\langle \sigma v n \rangle \ll H$, there is no time for the process taking place.

Example: Weak reaction among leptons

$$\sigma \approx \frac{G_F^2 s}{(1 + 2s/m_W^2)^2}, \quad (15)$$

$$\Rightarrow \langle \sigma v n \rangle \approx \frac{G_F^2 T^5}{(1 + T^2/m_W^2)^2}, \quad \text{for } T \gg m_e \quad (16)$$

behaving $\approx \alpha^2 T$ for $T \gg m_W$ and $\approx G_F^2 T^5$ for $T \ll m_W$.

The maximum peak value of

$$\left(\frac{\langle \sigma v n \rangle}{H} \right)_{max} \approx \frac{\alpha^2 m_{pl}}{\sqrt{N} m_W} \gg 1 \quad (17)$$

Thus, even the weak process is thermalized at

$$T \gg \left(\frac{\sqrt{N}}{G_F^2 m_{pl}} \right)^{1/3} \approx 1 MeV \quad (18)$$

Thermal history of universe

Assuming thermal equilibrium of N_b boson and N_f fermion species of massless (or extremely relativistic) particles, and entropy conservation, $aT = \text{const.}$,

$$\rho = \sum_f N_f \frac{\pi^2}{30} T^4 + \sum_b N_b \frac{7\pi^2}{240} T^4 \equiv N_{eff} \frac{\pi^2}{30} T^4 \quad (19)$$

giving

$$\frac{\dot{T}}{T} = -\frac{\dot{a}}{a} = -\sqrt{\frac{8N_{eff}\pi^3}{90}} G T^2, \quad (20)$$

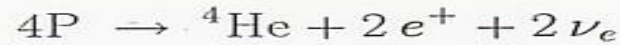
$$\Rightarrow t \approx 2 \times 10^{-6} \text{sec} \frac{\sqrt{N_{eff}}}{(T/\text{GeV})^2} \quad (21)$$

Epoch of (atomic) recombination, nucleosynthesis, quark-hadron transition, electroweak phase transition, GUT, and Planck may be estimated.

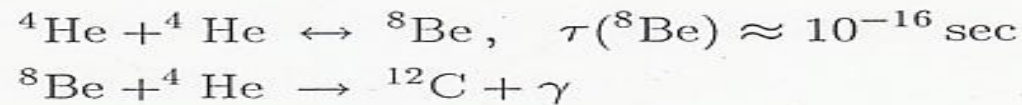
元素の起源

- 星内合成 He より重い元素
- 宇宙初期 ${}^4\text{He}$, ${}^2\text{H}$ (d), ${}^3\text{He}$, ${}^7\text{Li}$

太陽などの主系列星



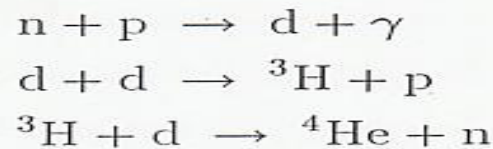
宇宙の～4% のヘリウムしか説明できない
赤色巨星



超新星爆発

中性子の多い重い原子核

宇宙初期の軽い元素合成



重陽子 d は結合エネルギーが小さく,
温度が 10 億度 (エネルギー 0.1MeV) 以上では
光分解しやすく蓄積しない.

計算値 ヘリウム重量比 ～ 25%: 観測とよく合う.

$n \leftrightarrow p$ 転換とニュートリノ平衡

- ニュートリノとは,

電氣的に中性で物質とは弱く相互作用するのみ.

例えば, 太陽ニュートリノフラックス $10^{11} \text{ cm}^{-2} \text{ sec}^{-1}$

地球と反応するのは 10^{-8}

- 宇宙ニュートリノ

$k_B T \gg 1 \text{ MeV}$ (温度 100 億度以上)

電子, 陽電子 (電子の反粒子), ニュートリノ ν_e の平衡

$$\gamma + \gamma \leftrightarrow e^- + e^+$$

$$e^- + e^+ \leftrightarrow \nu_e + \bar{\nu}_e$$

電子の数密度 $\sim 10^{32} \text{ cm}^{-3} (k_B T / \text{MeV})^3$

ニュートリノと電子の反応時間 $\sim 10 \text{ sec} (k_B T / \text{MeV})^{-5}$

宇宙年令 $\sim 1 \text{ sec} (k_B T / \text{MeV})^{-2}$

初期 $T \geq 0.5 \text{ MeV}$ には

熱平衡状態のニュートリノが大量にあった.

$n \leftrightarrow p$ 平衡

$$e + p \leftrightarrow \nu_e + n$$

$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$

$$n \leftrightarrow p + e + \bar{\nu}_e$$

従って

$$\frac{\text{中性子の数}}{\text{陽子の数}} = \exp\left[-\frac{1.3 \text{ MeV}}{k_B T}\right]$$

平衡が存在した結果として,

ヘリウム量その他があいまいさなく決まる.

重要なパラメータ

$$\frac{B}{\gamma} = \frac{\text{核子数}}{\text{光子数}} \approx 10^{-10}$$

- ニュートリノの種類数

$$\text{膨張時間} \propto \frac{1}{\sqrt{N_\nu}}$$

$$\text{中性子の崩壊} \sim \exp\left[-\frac{t}{15 \text{分}}\right]$$

N_ν の増大 \Rightarrow 中性子量の増大 \Rightarrow ヘリウム量増大

これより, $N_\nu \leq 3$.

電子・陽電子衝突実験 (1990)

$$N_\nu = 3, \quad \nu_e, \nu_\mu, \nu_\tau \text{ のみ}$$

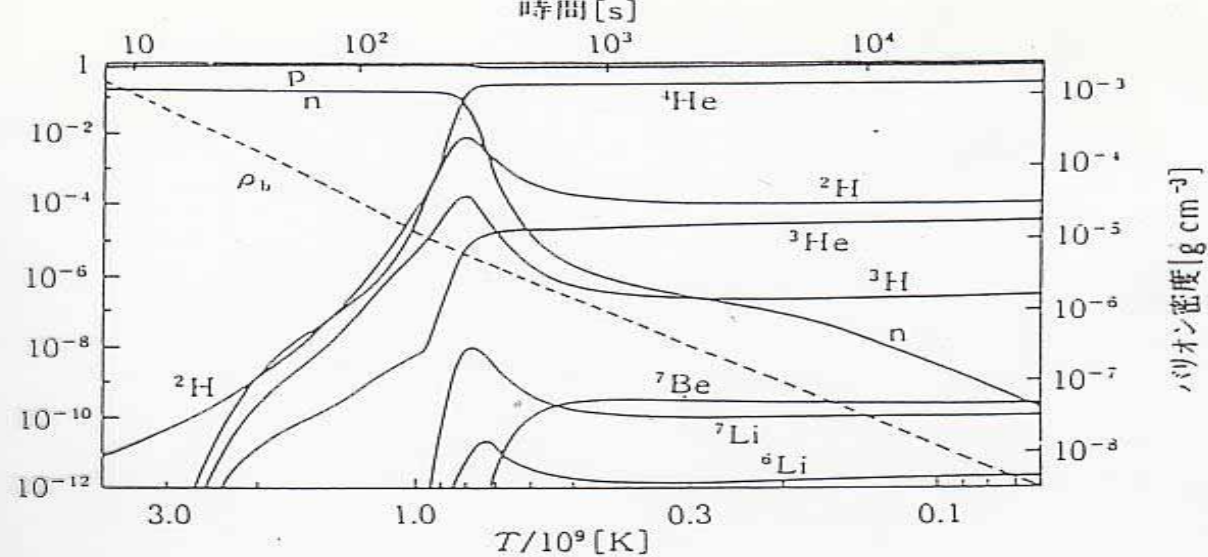


図 2.14 各原子核の生成量の時間変化

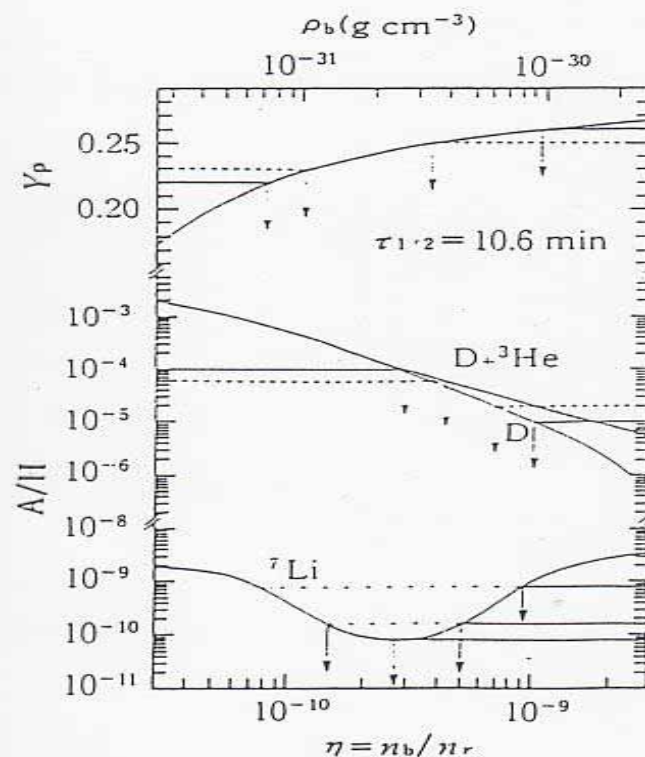
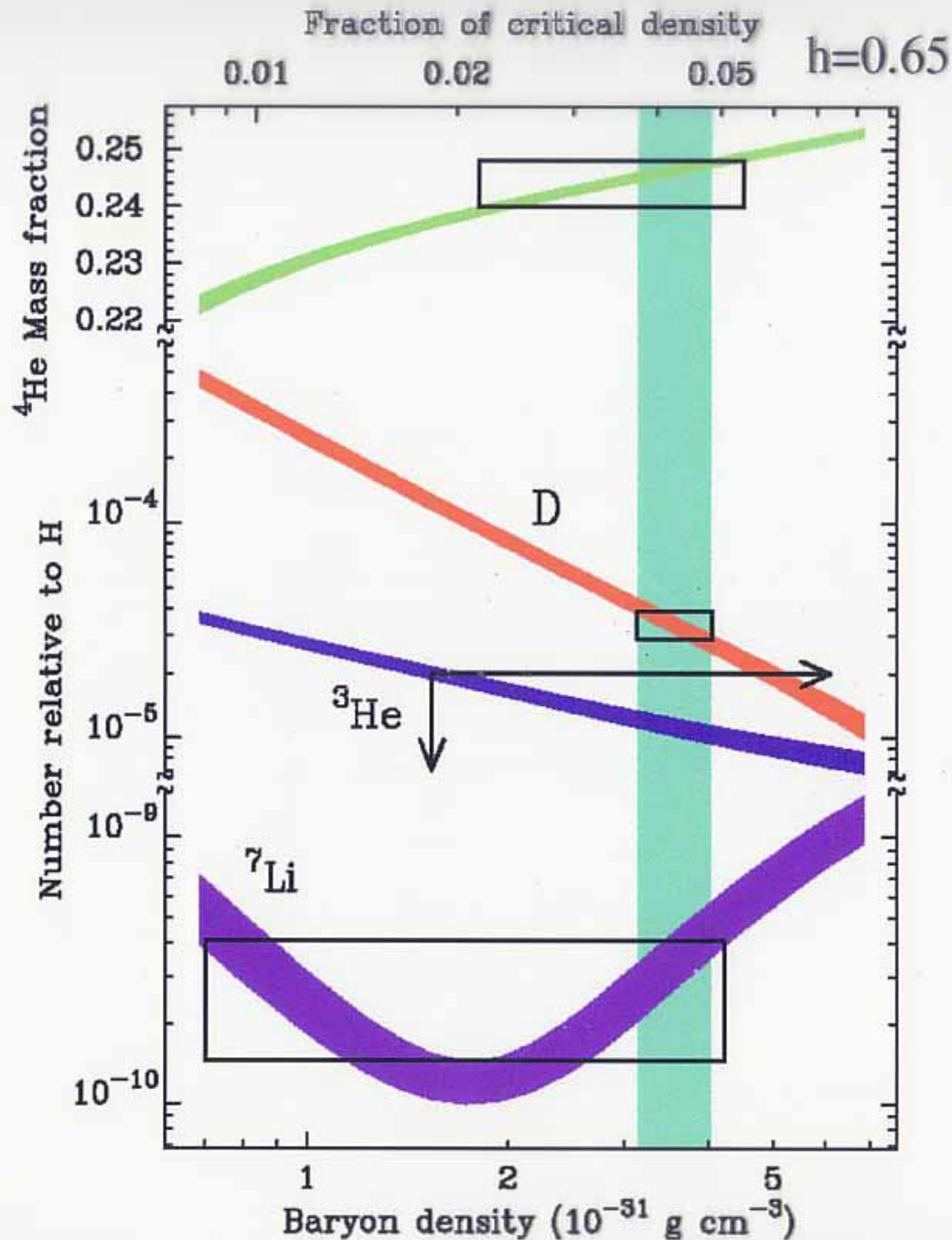


図 2.15 最終的な生成量



$$\Omega_B h^2 = 0.019 \pm$$

$$0.0024(95\%)$$

理論値と

観測値

残留ニュートリノ

宇宙が 1 MeV ($\approx 10^{10}$ °K) の頃の名残り.
熱平衡分布

$$\frac{1}{\exp(\frac{p_\nu c}{k_B T_\nu}) + 1}$$

$$T_\nu = (\frac{4}{11})^{1/3} T_\gamma \sim 1.9^\circ\text{K}$$

$$\text{個数密度} \approx 150\text{cm}^{-3}$$

$$\text{エネルギー} \approx 10^{-4} \text{ eV}$$

測定方法を知らない.

次世紀に残された実験的挑戦

Simple application: Dark matter abundance

(Still unknown) stable elementary particle X of mass \tilde{M} , first existing with the **thermal abundance** of $\approx T^3$, then disappearing via pair annihilation into ordinary particles of standard model according to the Boltzmann factor $\propto e^{-\tilde{M}/T}$, and finally pair annihilation being frozen (namely, being unable to pair-annihilate due to fast expansion). After the **freeze-out**, no change of density ratio, n_X/n_γ , occurs.

In the case of WIMP, the cross section is weak, and for SUSY LSP (Lightest Super Particle) $\sigma \approx 1/\tilde{M}^2$ (roughly constant when they freeze-out), giving

$$\langle \sigma v n_X \rangle \approx \frac{1}{\tilde{M}^2} (2\pi \tilde{M} T)^{3/2} e^{-\tilde{M}/T} \quad (22)$$

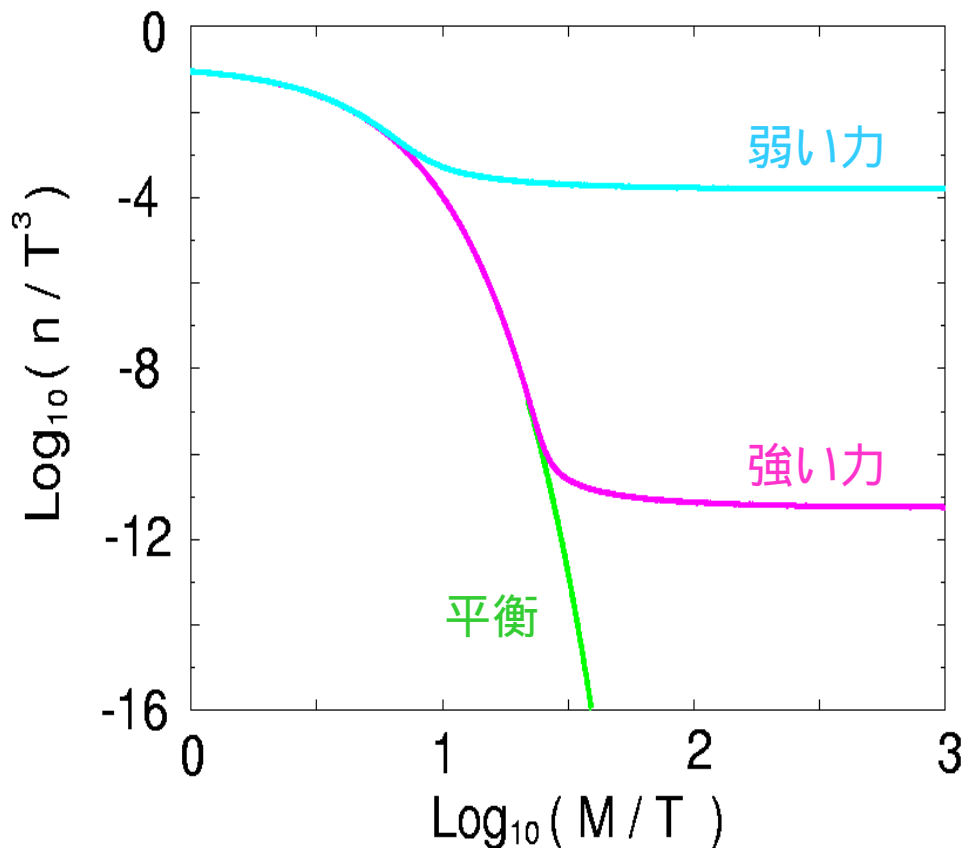
hence

$$\frac{n_X}{n_\gamma} \approx \frac{1}{\langle \sigma v \rangle \tilde{M} m_{pl}} \approx \frac{\tilde{M}}{m_{pl}} \quad (23)$$

where the expansion rate was equated to estimate the freeze-out temperature T_d .

Decoupling の応用

Stable remnants



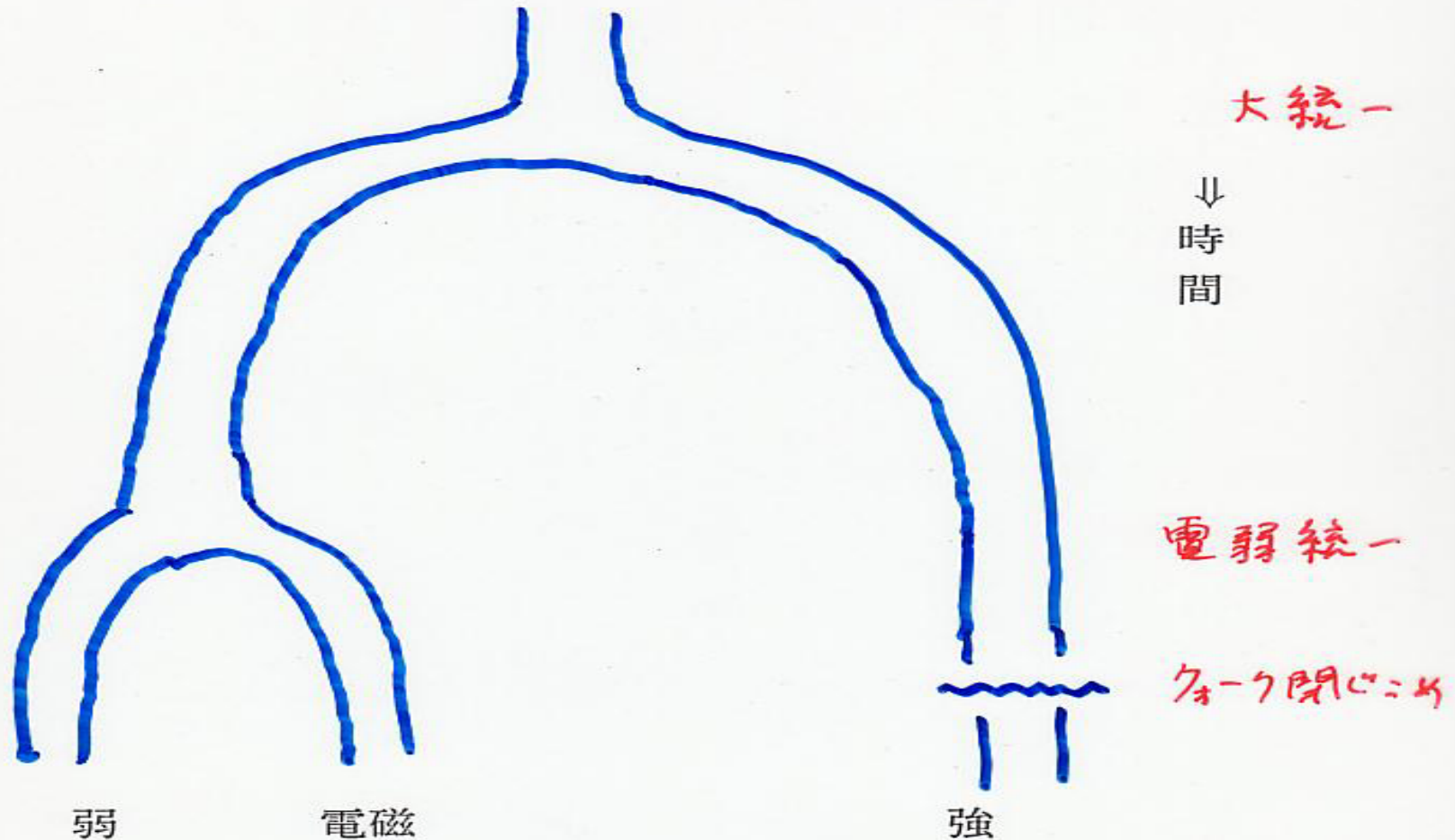
- SUSY ダークマターの残存量
- ビックバン名残のニュートリノ

Cosmic connection between TeV and meV : Very, very roughly the dark matter mass density is given by

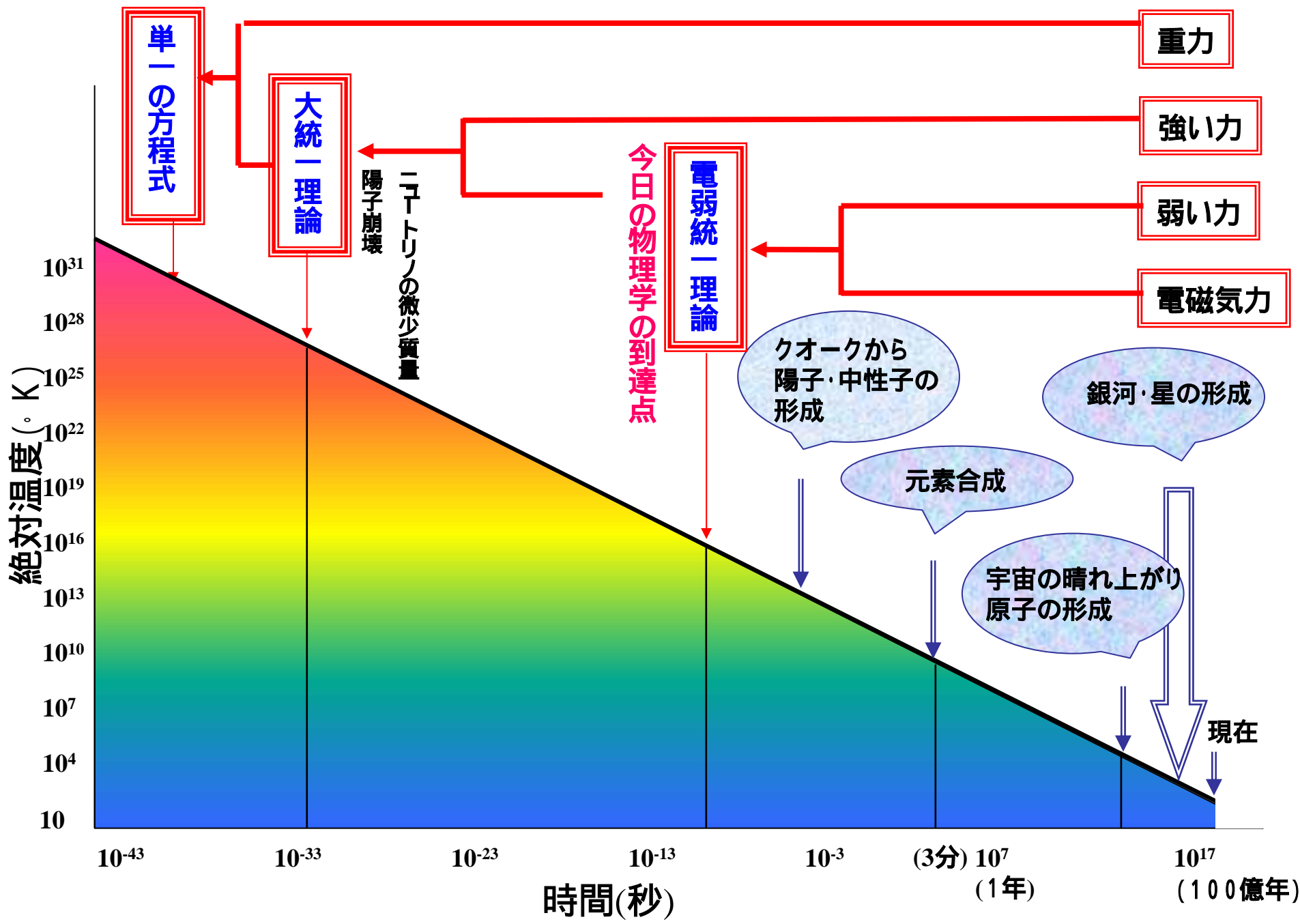
$$m_X n_X \approx \frac{\tilde{M}^2}{m_{pl}} n_\gamma \approx meV n_\gamma \left(\frac{\tilde{M}}{TeV} \right)^2 \approx 10^{-4} (meV)^4 \left(\frac{\tilde{M}}{TeV} \right)^2 \quad (24)$$

宇宙の相転移

- 宇宙初期には、低温では自発的に破れる対称性が、高温のために回復.
- 力の分化とクォーク閉じ込めの開放.
- 時間の経過とともに,
大統一 \Rightarrow 電弱統一 \Rightarrow クォーク閉じ込め



宇宙の熱的歴史



Generation of baryon asymmetry

The universe appears baryon-antibaryon asymmetric. This is strange due to that the microphysical law is nearly matter-antimatter symmetric, and that the universe has been hot enough to create antimatter energetically.

Quantitative measure of the baryon asymmetry,

$$\left(\frac{n_B}{n_\gamma}\right)_{\text{after annihilation}} \approx O(1) \left(\frac{B - \bar{B}}{B + \bar{B}}\right)_{\text{before annihilation}} \approx 10^{-10} \quad (25)$$

implying that one excess of B over $10^{10} B - \bar{B}$ pairs created the baryon-dominated universe.

How to produce the asymmetry: 3 conditions

in the early universe

Necessary ingredients

\mathcal{B} \mathcal{CP} *out of equilibrium*

Need of arrow of time

without suppression of inverse process,

$$\Delta B = (\Delta B)_{\rightarrow} + (\Delta B)_{\leftarrow} = 0$$

Sources of B nonconservation

- GUT
- Electroweak at high T
- SUSY (Affleck-Dine mechanism)
- Black hole evaporation

G R A N D U N I F I E D T H E O R I E S

UNIFICATION OF FORCES

QUANTUM CHROMODYNAMICS

ELECTROWEAK THEORY

GRAVITY

$SU(3) \times SU(2) \times U(1)$

ASYMPTOTIC FREEDOM OF QCD CRUCIAL.

GUT

SUPERGRAVITY

SUPERSTRING

COUPLING UNIFICATION

UNIFICATION OF MATTER

$SU(5)$

$(\underline{10} + \underline{5}^*)_L$

FOR 1 FAMILY

$$\begin{pmatrix} 0 & \bar{u} & \bar{u} & u & d \\ & 0 & \bar{u} & u & d \\ & & 0 & u & d \\ -(*) & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_L$$

+

$$\begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ \nu_e \\ e \end{pmatrix}_L$$

$B - L$

CONSERVED

$SO(10)$

$\underline{16}$

$$\begin{pmatrix} u & u & u & \nu_e \\ d & d & d & e \\ \bar{u} & \bar{u} & \bar{u} & N_e \\ \bar{d} & \bar{d} & \bar{d} & e^+ \end{pmatrix}$$

$B - L$

VIOLATED

GOOD,

ELECTROWEAK
DISSIPATION

N_e : RIGHT-HANDED PARTNER
OF ν_e

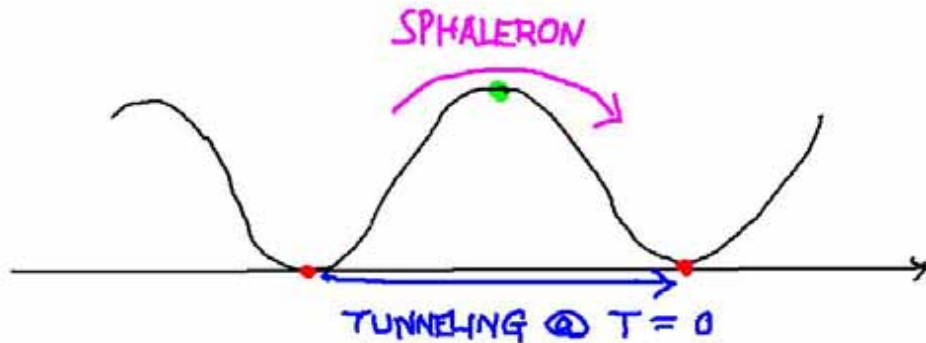
\Rightarrow NEUTRINO MASS

BARYON NON-CONSERVATION

$$\frac{g^2}{M_X^2} \bar{q} q q l$$



Electroweak baryon nonconservation



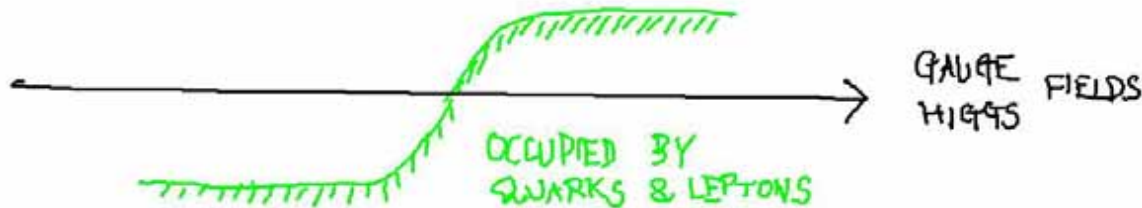
Gauge and Higgs

Electroweak baryon nonconservation

suppressed at $T=0$ by e^{-137}

enhanced at finite T by barrier crossing

Can destroy preexisting B and L while keeping $B-L$



Mechanism due to level crossing of fermions caused by nontrivial gauge and higgs configuration of sphaleron and alike

Electroweak redistribution of B and L

$$B = a \cdot \Delta(B - L), \quad a = \frac{8n_g + 4n_H}{22n_g + 13n_H} = \frac{28}{79}$$

For standard model of 3 generations

Damping effective @ $200\text{GeV} < T < 10^{12}\text{GeV}$

e.g. Luty

B-L conserved and never washed out.

L-genesis and B-conversion

- L-genesis of amount ΔL first and electroweak conversion into B, via

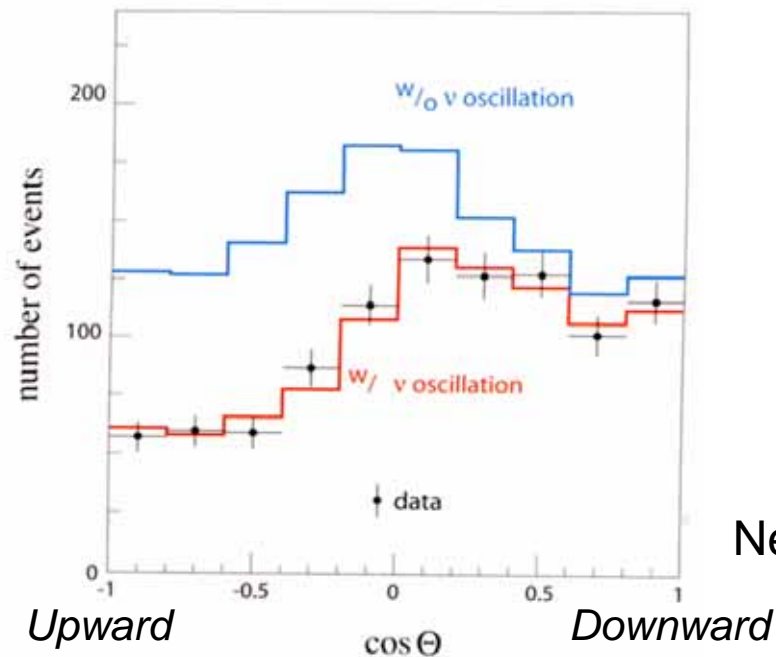
$$B = -\frac{28}{79} \Delta L$$

For standard model of 3 generations

Interesting in view of possible connection to observed neutrino masses by oscillation experiments

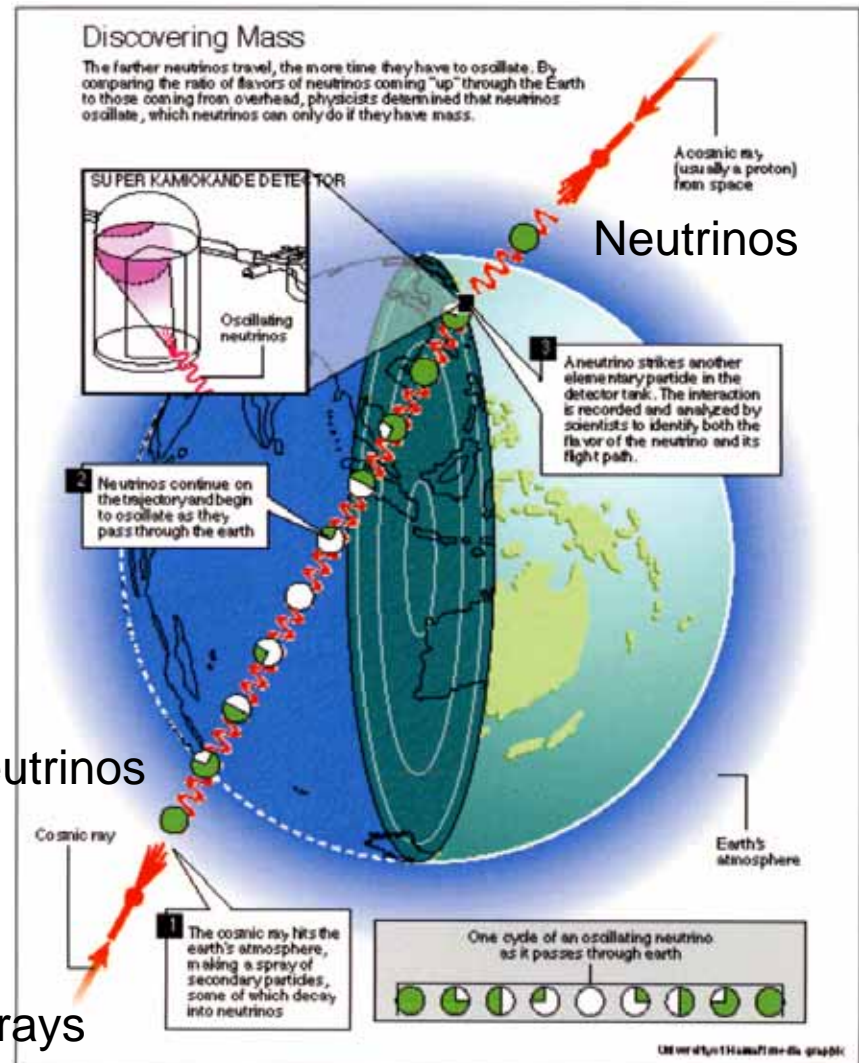
Neutrino physics

- Neutrino oscillation

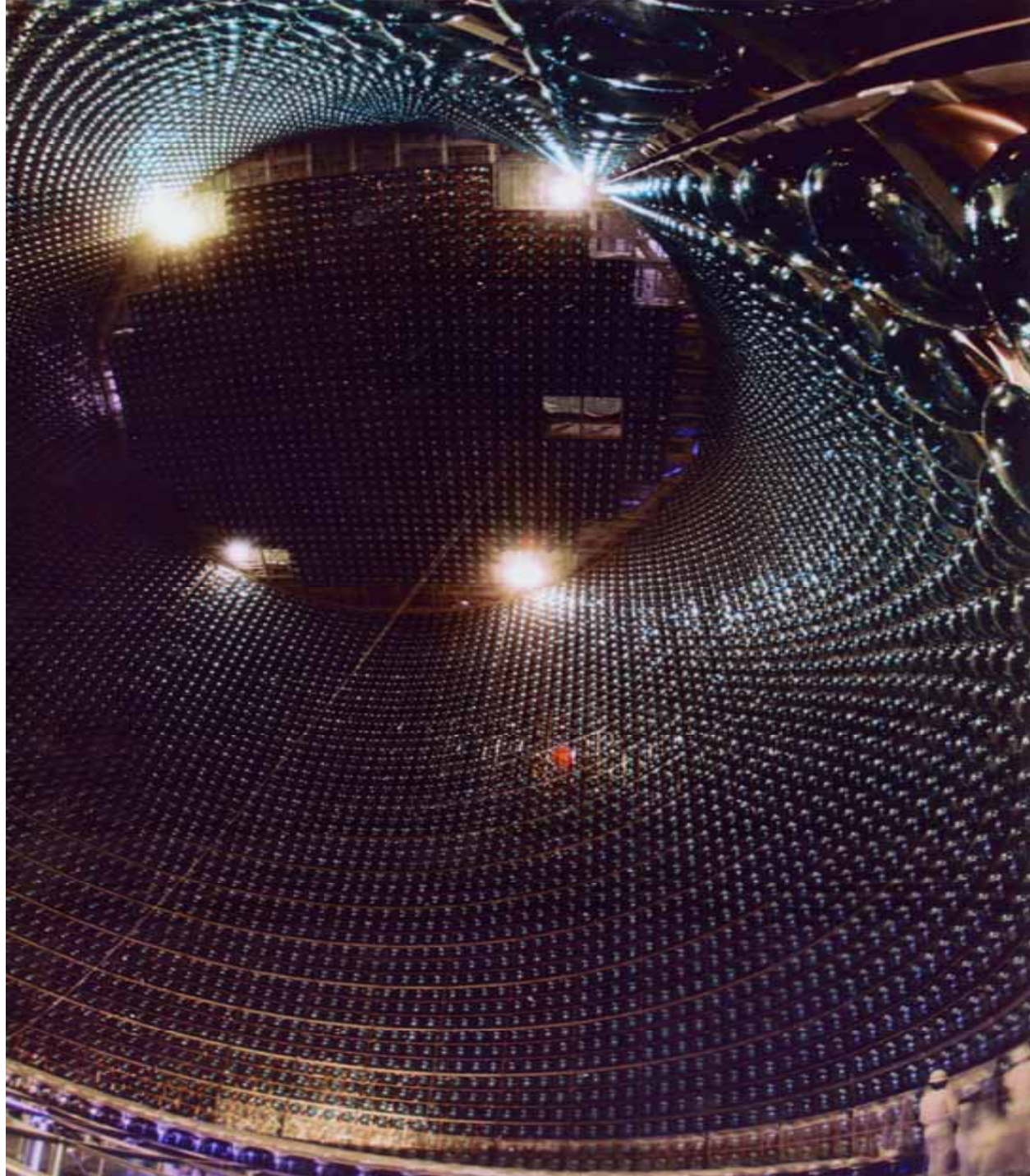


→ Evidence of
neutrino mass!

Cosmic rays



Cosmic rays



Likely mechanism of small neutrino mass generation

- **Seesaw mechanism**

Heavy Majorana type of masses of neutrino partner N_R , indicating physics beyond standard theory of particle physics, generates a tiny left-handed neutrino masses and mixing a la

Neutrino mass via seesaw

$$m_\nu = \frac{m_{q,l}^2}{M_{\text{new physics}}}$$

- **Necessarily violates lepton number conservation**
- Agent of L-asymmetry generation provided by N_R

Delicacy of CP: Quantum interference

Baryon excess from a pair of particle and antiparticle process, e.g. $X \quad \overline{X}$

$$\left| g_1 f_1 + g_2 f_2 + \dots \right|^2 - \left| g_1^* f_1 + g_2^* f_2 + \dots \right|^2$$

$$= -4 \operatorname{Im}(g_1 g_2^*) \operatorname{Im}(f_1 f_2^*) + \dots$$

$$\operatorname{Im}(g_1 g_2^*) \neq 0$$

CP violation

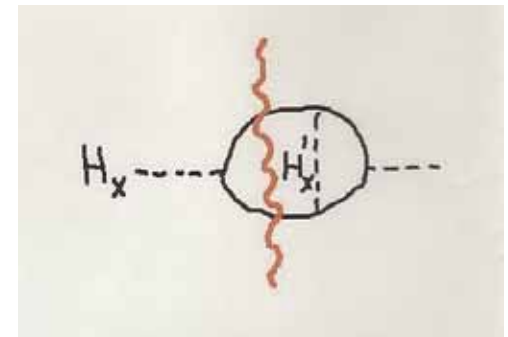
$$\operatorname{Im}(f_1 f_2^*) \neq 0$$

Rescattering phase

Interference computed by Landau-Cutkovsky rule

$$\sum_q \left(\text{Diagram 1} \right) \times \left(\text{Diagram 2} \right)^* = \text{Diagram 3}$$

Diagram 1: A vertex with an incoming dashed line from the left labeled H_X and two outgoing solid lines to the right labeled f_1 and f_2 .
 Diagram 2: A vertex with an incoming dashed line from the left labeled H_X and two outgoing solid lines to the right labeled f_1^* and f_2^* .
 Diagram 3: A circle with a dashed line entering from the left labeled H_X and a dashed line exiting to the right. Inside the circle, there is a vertical dashed line and a red wavy line.



Thermal L genesis

Fukugita-Yanagida

- Minimal extension of standard model with seesaw

$$N_R \rightarrow lH, \bar{l}\bar{H}$$

Right-handed Majorana decay

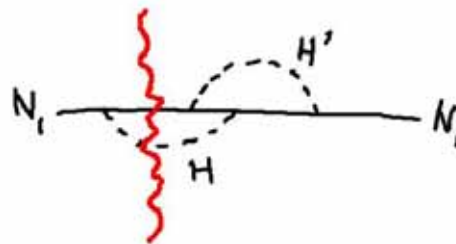
CP asymmetry with neutrino mass matrix $m_\nu = m_D M^{-1} m_D^T$

$$\varepsilon_1 = \frac{3}{16\pi} \frac{M_1}{v^2} \frac{\text{Im}(m_D^\dagger m_\nu m_D^*)_{11}}{(m_D^\dagger m_D)_{11}} = O\left[\frac{M_1 \overline{m_\nu}}{v^2} \delta\right]$$

$$M_1 \ll M_2 \ll M_3$$

For 3 R-Majoranas

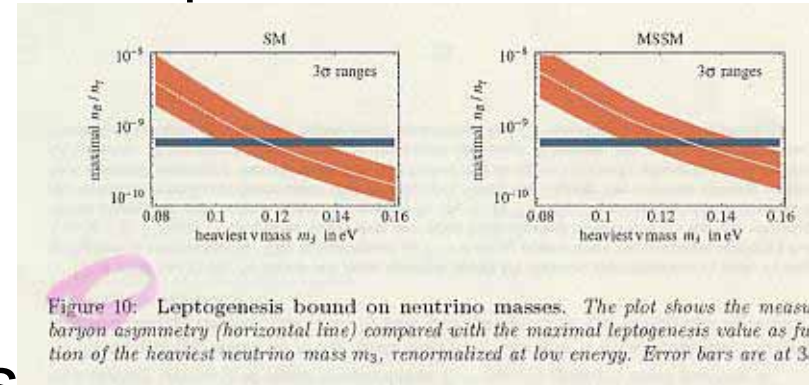
δ = CP phase



Great impacts on neutrino masses and thermal history of universe

With hierarchy of masses, dependence on 3 parameters Giudice et al

$$\varepsilon_1, M_1, \overline{m}_\nu = \frac{(hh^+)_{11}}{M_1}$$



- Connection to neutrino masses

$$m_3 < 0.13 \text{ eV}$$

heaviest neutrino (WMAP 0.23 eV)

$$M_1 > 5 \cdot 10^8 \text{ GeV}$$

lightest R-neutrino

- Reheat temperature

$$T_{RH} > M_1$$

Gravitino problem: a possible nightmare both for GUT B- and L-genesis

- Superpartner of graviton

mass $m_{3/2} = O[TeV]$

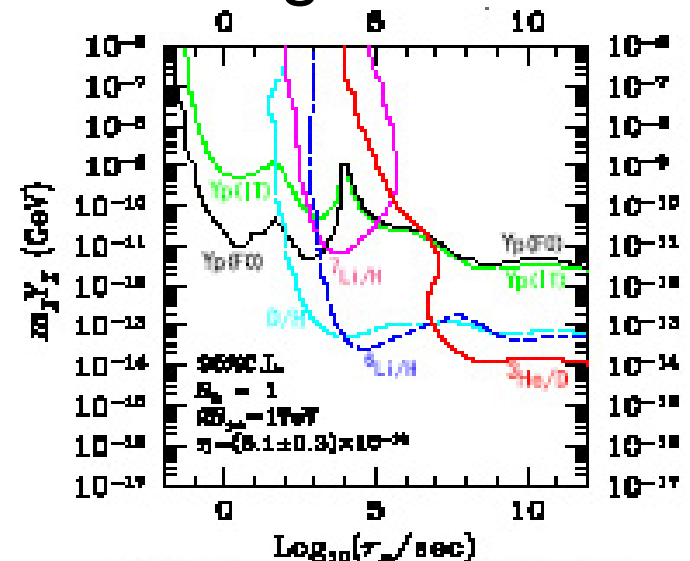
lifetime $\Gamma = O\left[\frac{m_{3/2}^3}{m_{pl}^2}\right] = O[(10^5 \text{ sec})^{-1} \left(\frac{m_{3/2}}{TeV}\right)^3]$

- Usual estimate of gravitino abundance and constraint from nucleosynthesis, including hadronic decay

$$\frac{n_{3/2}}{s} = O[10^{-2}] \frac{T_{RH}}{m_{pl}}$$

$$T_{RH} < 10^6 - 10^8 \text{ GeV}$$

Possible to produce N_R ?



Ways out

- EW baryogenesis
- Affleck-Dine mechanism
- Gauge mediation
- LSP = gravitino
- Preheating

Inflation

Horizon problem

Region of causal contact since the beginning of big bang is limited by light velocity,

$$r_H(t) = a(t) \int_0^t \frac{dt'}{a(t')} \quad (26)$$

which, for $a(t) \propto t^\gamma$ with $\gamma < 1$ ($\gamma = \frac{1}{2}, \frac{2}{3}$ for RD and MD), $r_H \approx \frac{t}{1-\gamma}$

This leads to $O(10^3)$ separated causal regions at the last scattering surface of 3K. Thus, the near equality of 3K should be regarded a mystery.

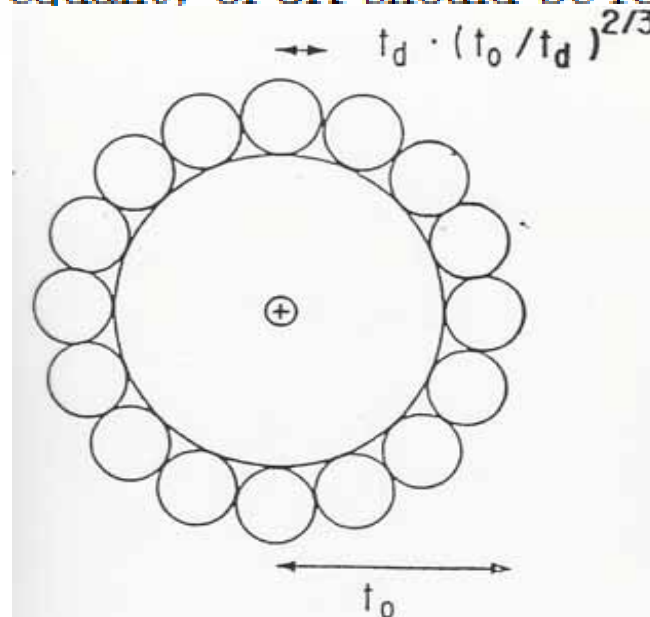


Fig. 2 LAST SCATTERING

Oldness or flatness problem

Typical time scale of cosmic evolution is $\frac{\Sigma^{2/3}}{m_{pl}}$ where $a^3 T^3 = \Sigma$ is entropy measure and is nearly constant, unless a drastic change of matter content occurs. Putting the cosmic age, this gives $\Sigma \geq 10^{87}$. If Σ is much less, one should observe the effect of curvature term k/a^2 .

Why Σ is so large to ensure a mature universe which can foster an intelligent life to think of the origin of universe.

Accelerating universe

Resolution of the above paradoxes may be sought using accelerated expansion like $\gamma > 1$ or the exponential expansion.

Introducing **scalar field** φ ,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}[\rho_r + \frac{1}{2}\dot{\varphi}^2 + V(\varphi)], \quad \rho_r \propto a^{-4} \quad (27)$$

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} \quad (28)$$

If

$$\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi) \quad (29)$$

inflaton field φ may stay at some value φ_0 for some time, and

$$a(t) \propto \exp(\sqrt{\frac{8\pi G V(\varphi_0)}{3}} t) \quad (30)$$

Implications from inflation

- $\Omega_{tot} = 1 \Rightarrow k = 0$
- all quantum numbers of universe $\longrightarrow 0$ after inflation
- density perturbation from quantum fluctuation

$$\frac{\delta\rho}{\rho} \approx \left(\frac{H^2}{\dot{\varphi}_0} \right)_{h\text{-crossing}} \quad (31)$$

Simple realization: chaotic inflation with

$$V(\varphi) = \frac{m^2}{2}\varphi^2, \quad V(\varphi_0) \approx m_{pl}^4 \quad (32)$$

giving the density perturbation

$$\frac{\delta\rho}{\rho} \approx \frac{m}{m_{pl}} \quad (33)$$

which suggests $m \approx 10^{-6}m_{pl}$ from the COBE observation.

New theory of entropy production

- Inflaton field oscillation given

$$\xi(t) = \xi_0 \cos(mt)$$

spatially homogeneous

Interaction by $\xi\phi^2$

Producing a pair of ϕ particles

For each momentum mode of massless particle

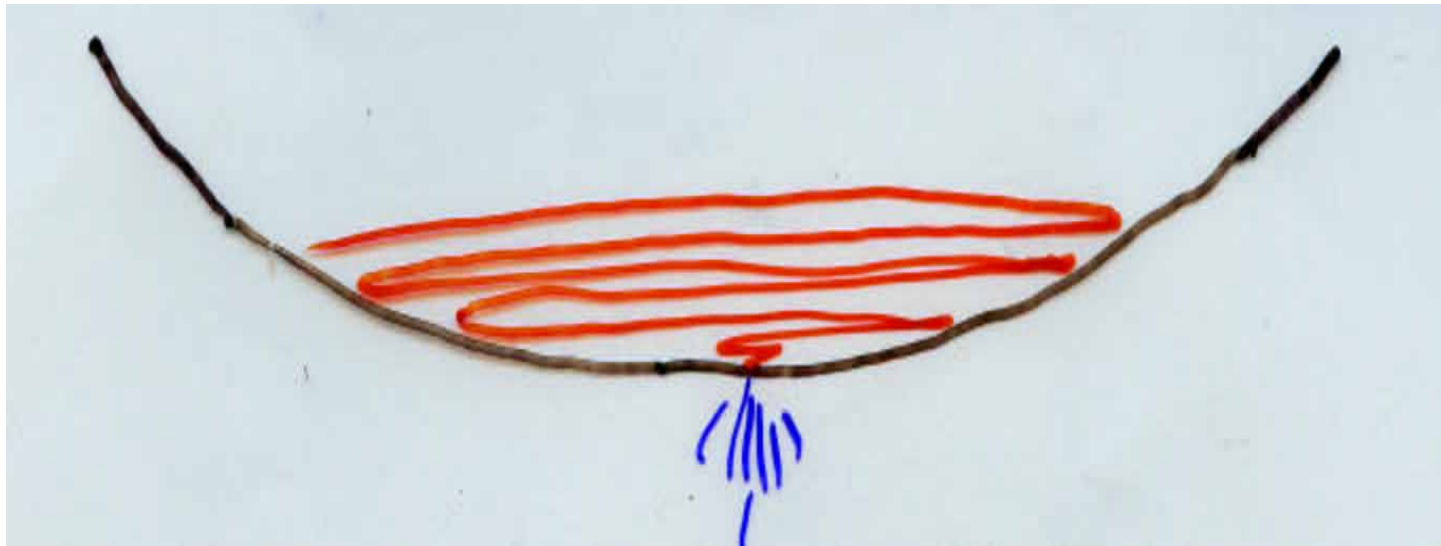
$$\ddot{\phi}_k + 3\frac{\dot{a}}{a}\dot{\phi}_k + (k^2 + g\xi_0 \cos(m_\xi t))\phi_k = 0$$

$$h = \frac{k^2 + g\xi_0}{m_\xi^2}$$

$$\theta = \frac{g\xi_0}{m_\xi^2}$$

Chaotic inflation under flat potential

Damped inflaton oscillation with $m_\xi = O[10^{13}] \text{ GeV}$
Produced particle $\xi_0 = O\left[\frac{m_{pl}^2}{m_\xi}\right]$

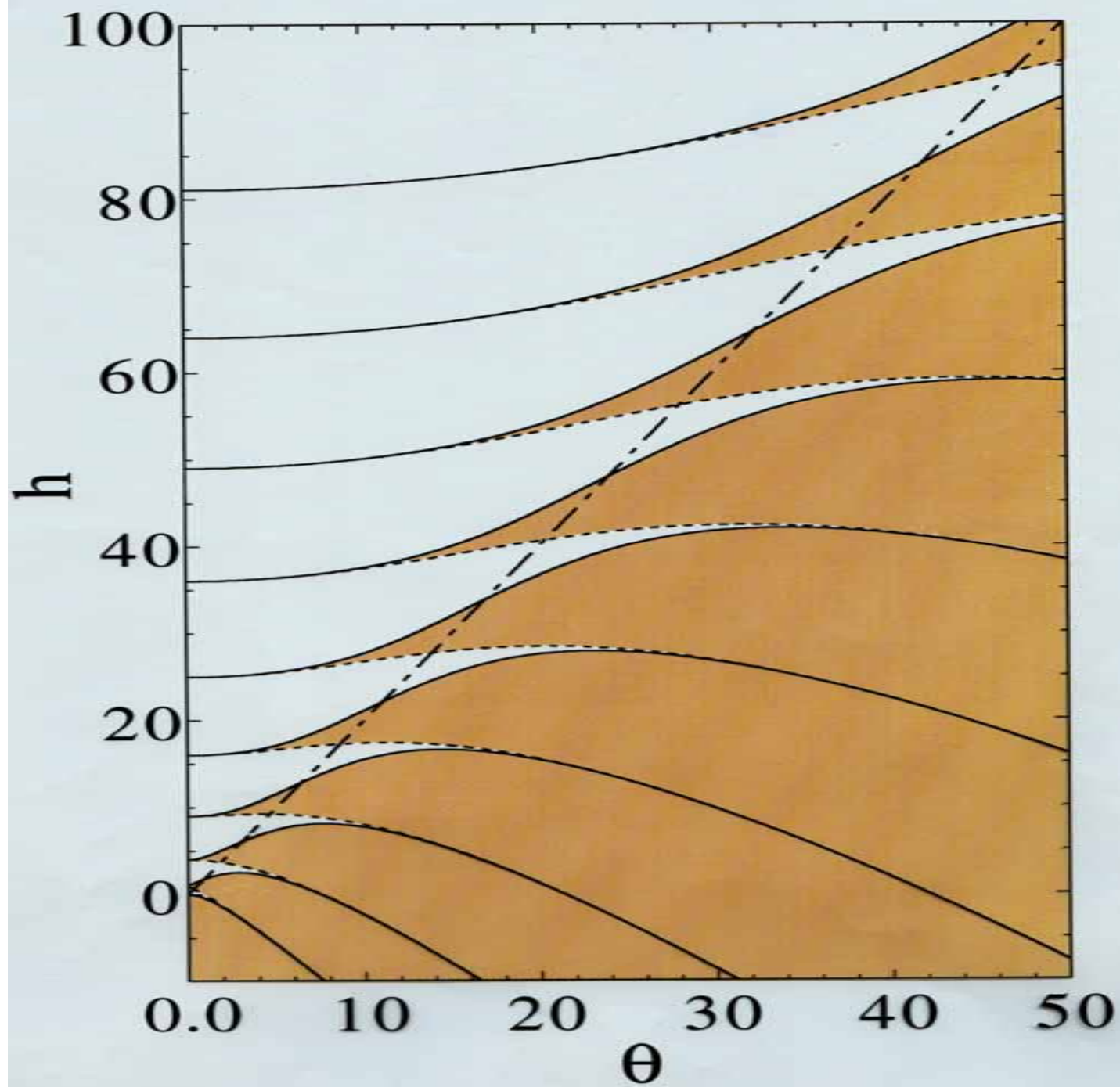


- 周期振動する振動数を持つ力学系の問題
- パラメータ共鳴が起こりえる
- 周期振動するインフラトンを外場、これと結合する量子場の解析から、
爆発的粒子生成の可能性

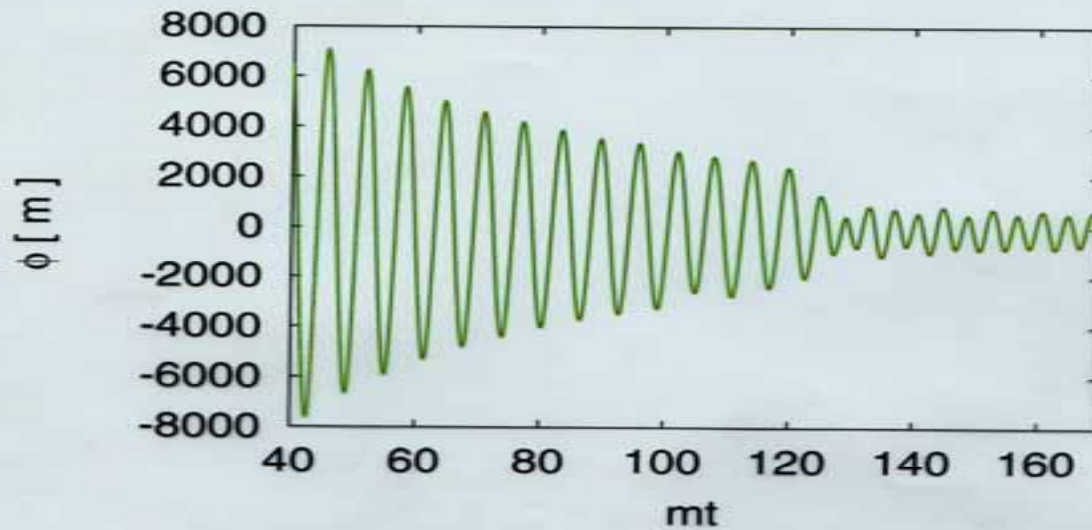
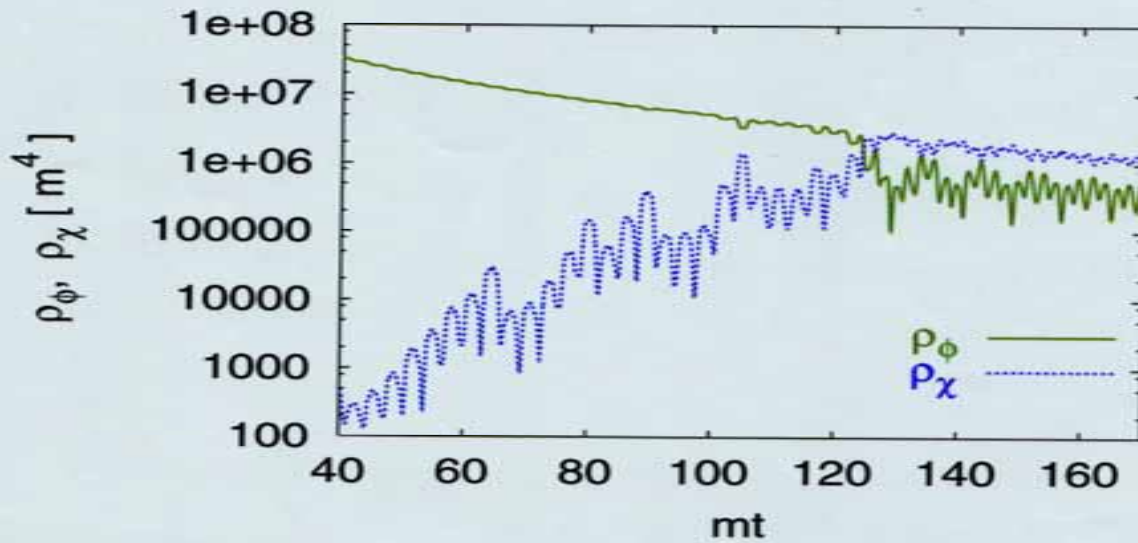
HOW TO SWING



NEED TO VARY CENTER OF YOUR BODY
PERIODICALLY



MODE - SUMMED METHOD



宇宙膨脹 + 反作用

田沢

New features : preheating

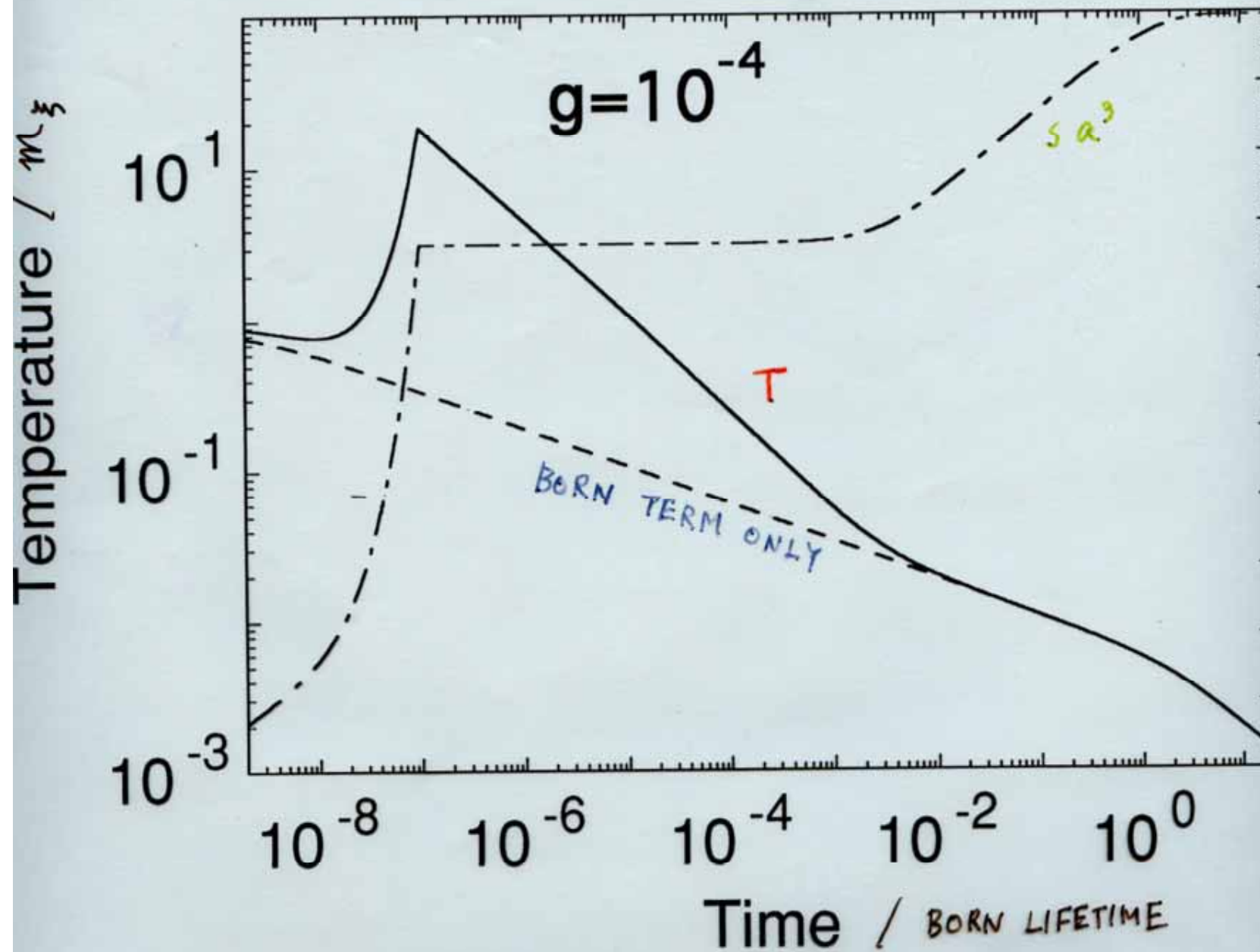
Violent process of particle production

after $O[10]$ oscillations

Initially highly **non-thermal**

Possibility of producing **GUT** scale particles

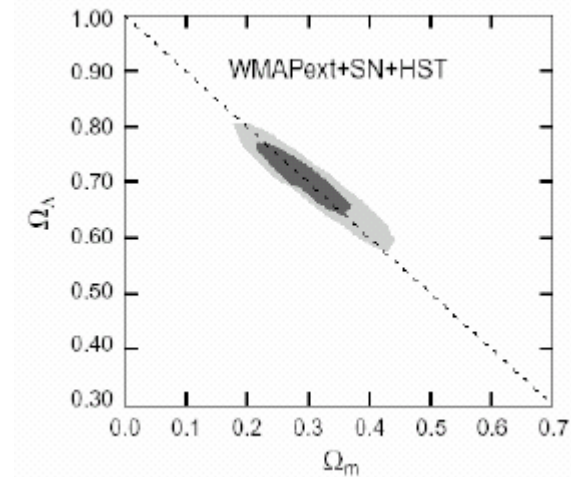
Estimate based on a single reheat
temperature doubtful



Simple facts and great conundrums about our universe

- Spacetime structure:

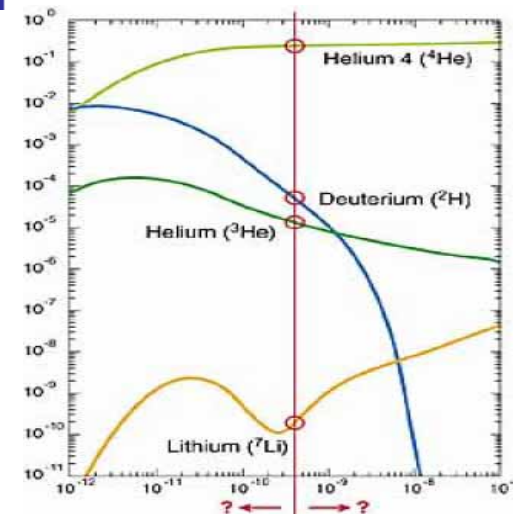
Euclidean 3-space
accelerating universe



$$\Omega_{tot} = 1.02 \pm 0.02 (\text{WMAP} + \text{SN, or, HST, 2DF})$$

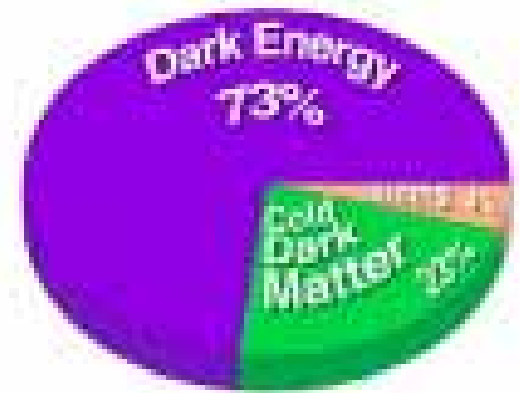
- Matter content:
baryon fraction with precision
better than nucleosynthesis

$$\Omega_B h^2 = 0.024 \pm 0.001 (\text{WMAP})$$



Towards resolution of hierarchy problems in cosmological context

$$10^{18} \text{GeV} = 10^{15} \times \text{electroweak}$$



$$\rho_{DE} = O[(\text{meV})^4] \ll (\text{TeV})^4$$

New problem either in cosmology or both in particle physics and cosmology

- Nonvanishing cosmological constant, or **dark energy** observed by WMAP, distant SN, large scale structure

$$\rho_{DE} = O[(meV)^4] \ll (TeV)^4$$


dominating the energy budget of our universe

SUSY lurking ?

$$\Lambda_{eff} \approx \left(\frac{TeV^2}{m_{pl}}\right)^4$$

Personal view

Inflation is a nice idea, solving the homogeneity and the flatness problem of old cosmology, but it relies on a temporary presence and subsequent disappearance of the cosmological constant

Ultimate theory should perhaps simultaneously solve the present accelerating universe

Our key ideas

- Brans-Dicke type of coupling $f(\varphi_i)R$
- Choice of the potential $V(\varphi_i)$; bounded, and having infinitely many, local minima and negative values. (Departure from Brans-Dicke)
- Non-static (cf. No-go theorem due to Weinberg)

Rotationary freedom in the dilaton kinetic term giving a possibility of cancelling a negative Λ in the Lagrangian.

Resurrection of Dirac's old idea in disguise

Framework

four dimensional Lagrangian model

$$\mathcal{L} = \sqrt{-g} \left[-f(\varphi_i) R + \frac{1}{2} (\partial \varphi_i)^2 - V(\varphi_i) + \mathcal{L}_m \right]$$

Gravitational strength $f(\varphi) = 1/16\pi G$ varying with scalar field φ_i

Choice of 2 functions

- Dilaton coupling

$$f(\varphi_1, \varphi_2) = \epsilon_1 \varphi_1^2 + \epsilon_2 \varphi_2^2 ,$$

- Dilaton potential: bounded and infinitely many local minima

$$V(\varphi_i) = V_0 \cos \frac{\sqrt{\sum_i \varphi_i^2}}{M} + \Lambda ,$$

- Negative values

$$V_0 > \Lambda > 0.$$

- No fine tuning

$$V_0 \approx \Lambda = O[M^4]$$

By definition, the standard model \mathcal{L}_m vanishes at its minimum, such that all constant terms are collected in Λ .

Time evolution

With Robertson-Walker metric $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$

$$\dot{H} = -\frac{\rho + p}{4f} - \frac{\dot{\varphi}_i^2}{4f} - \frac{\ddot{f} - H\dot{f}}{2f}$$

$$H^2 = \frac{1}{6f}(\rho + \frac{1}{2}\dot{\varphi}_i^2 + V) - H\frac{\dot{f}}{f}$$

$$\begin{aligned} \ddot{\varphi}_i + \frac{3f_{,i}f_{,k}}{f}\ddot{\varphi}_k + 3H(\dot{\varphi}_i + \frac{3f_{,i}f_{,k}}{f}\dot{\varphi}_k) + \frac{f_{,i}}{f}(\frac{1}{2}\dot{\varphi}_i^2 + 3f_{,k,l}\dot{\varphi}_k\dot{\varphi}_l) \\ = -V_{,i} + \frac{2f_{,i}}{f}V + \frac{f_{,i}}{2f}T \end{aligned} \quad ($$

Important features of dilaton dynamics

Effective potential for φ_i modified as seen from presence of f dependent terms, $2f_{,i}V/f$.

Induced matter coupling given by

$$\frac{T}{2} \ln f ,$$

with T the trace of the energy-momentum tensor.

In the large φ_i limit, coupling suppressed by powers of φ like

$$T\left(\frac{\delta f}{2f_0} - \frac{1}{4}\left(\frac{\delta f}{f_0}\right)^2 + \dots\right), \quad (13)$$

where δf is fluctuation around some fixed $f_0 \gg \delta f$.

Failure of 1 dilaton model and need for more freedom

Difficulty for resolving the cosmological constant problem and fine tuning

Stationary point of end point φ given by

$$-V_{,\varphi} + \frac{4V}{\varphi} = 0 \quad (17)$$

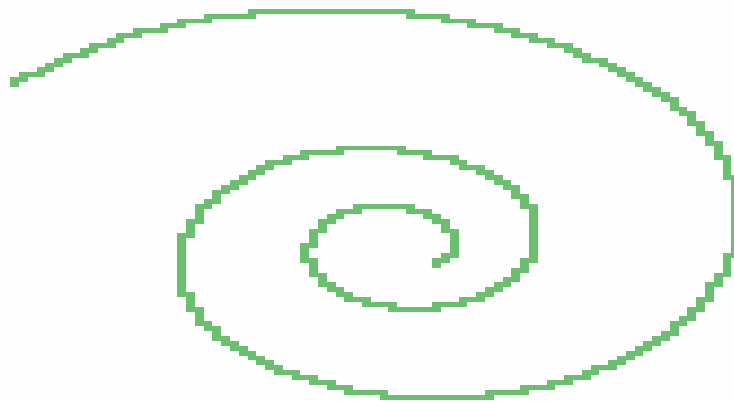
which generally gives $V(\varphi) = O[M^4]$

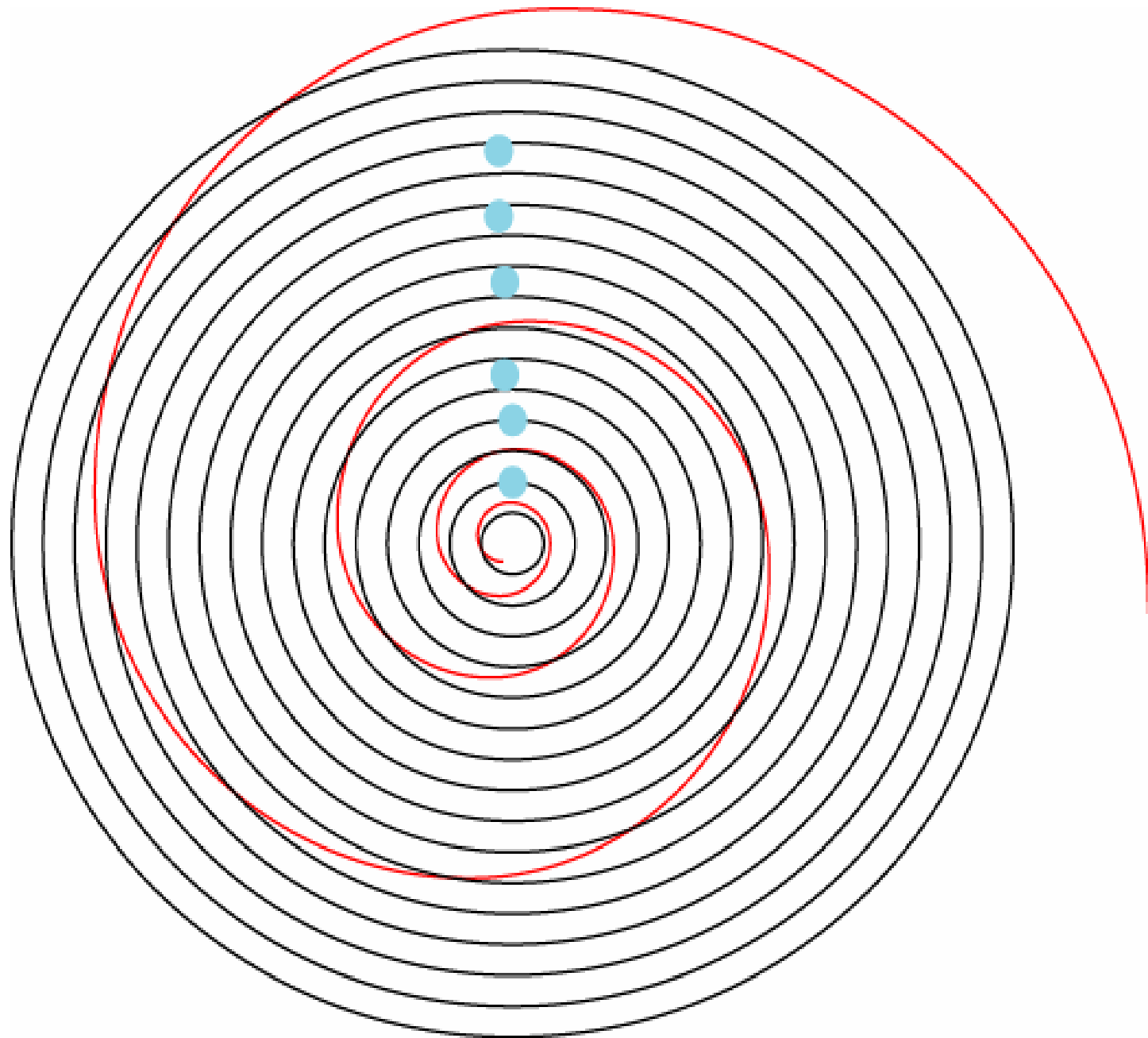
Fine tuning condition for the parameters; $V(\varphi) = 0$ at the stationary point. In our example of the potential, $V_0 = \Lambda$

2 dilaton model with angular momentum

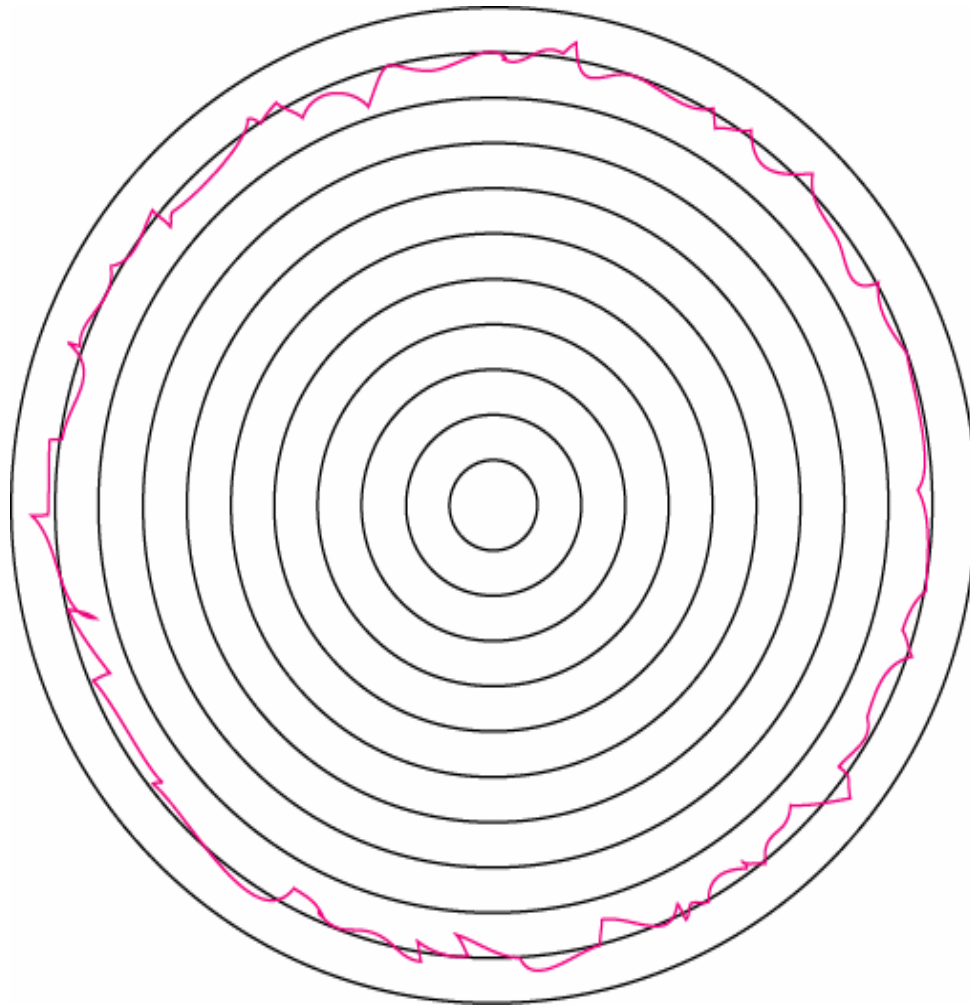
- 利点

ポテンシャルの極小値に安易に捕捉されない。
(保存しない)角運動量が導入され、回転エネルギーと負の宇宙項との相殺の可能性がある。





Dilaton oscillation around late orbit



Nucleosynthesis constraint

Reheat temperature T_R

$$T_R^4 = O(M^4) \left(\frac{t_{prod}}{t_{decay}} \right)^2 = O\left(\frac{M^6}{m_{pl}^2}\right) \approx (300keV \left(\frac{M}{1TeV}\right)^{3/2})^4 \quad (47)$$

Nucleosynthesis constraint

For success of nucleosynthesis, reheating temperature after the heavy dilaton must be higher than $O(MeV)$,

$$300keV \left(\frac{M}{1TeV}\right)^{3/2} > a \text{ few } MeV \Rightarrow M > \text{several } TeV, \quad (48)$$

Lambda Very small ?

Dynamical ending towards vanishing Λ_{eff} $\Lambda_{eff} \equiv \langle \frac{\dot{\varphi}_k^2}{2} + V \rangle$

Dynamics from $\Lambda_{eff} = O[(TeV)^4]$ towards $\Lambda_{eff} = O[(meV)^4]$ must be realized by some relaxation process.

Persistent angular momentum

Equation for angular motion

$$\ddot{\theta} + 3H\dot{\theta} + 2\frac{\dot{\varphi}_r}{\varphi_r}\dot{\theta} = (\epsilon_1 - \epsilon_2)\frac{\sin 2\theta}{f}(1 + 3\frac{\vec{f}^2}{f})^{-1}[\frac{L^2}{2\varphi_r^2} - 2(3\frac{\vec{f}^2}{f} - 1)|\Lambda|\cdots]$$

$$f = \epsilon_1\varphi_1^2 + \epsilon_2\varphi_2^2 \approx m_{\text{pl}}^2, \quad 3\frac{\vec{f}^2}{f} = 12\frac{\epsilon_1^2\varphi_1^2 + \epsilon_2^2\varphi_2^2}{\epsilon_1\varphi_1^2 + \epsilon_2\varphi_2^2} \approx 12\epsilon$$

like the pendulum under gravity.

When angular energy $\frac{L^2}{2\varphi_r^2} \approx |\Lambda| = O(M^4)$, the angular momentum may not decrease with cosmic evolution, if the angular velocity $\approx \frac{M^2}{m_{\text{pl}}} > H$.

Clearly much work to be done. Especially on points of

- (a) Transition from inflation to trap into the late-phase orbit
- (b) Relaxation mechanism of Λ_{eff} from $O(\frac{M^6}{m_{pl}^2})$ to $O(\frac{M^8}{m_{pl}^4})$
- (c) Constraint on initial condition to get the present state of universe

Summary

- Fruitful merger of particle physics and cosmology
- Still inviting new ideas for B-asymmetry generation, and
- Radical change of view needed for small dark energy density

Keys for successful cosmology

- Inflation;
what is inflaton, e-folding, density perturbation
Dilatonic inflaton
- Reheating; how hot
Violent inflaton decay
- Baryo- or lepto-genesis
L or B violation
- Dark matter
SUSY LSP
- Dark energy
Dilatonic inflaton