Simulation Study for Electron Beam in the Main Linac of the Linear Collider

The University of Tokyo
Taiki Yamamura

Introduction

In the main linac, emittance-blowup can be produced by

- non-linear magnetic field in quadratic magnets
- beam-scattering on residual gas and thermal photon

→ We have a simulation for the GLC main linac and estimate the effect in detail.

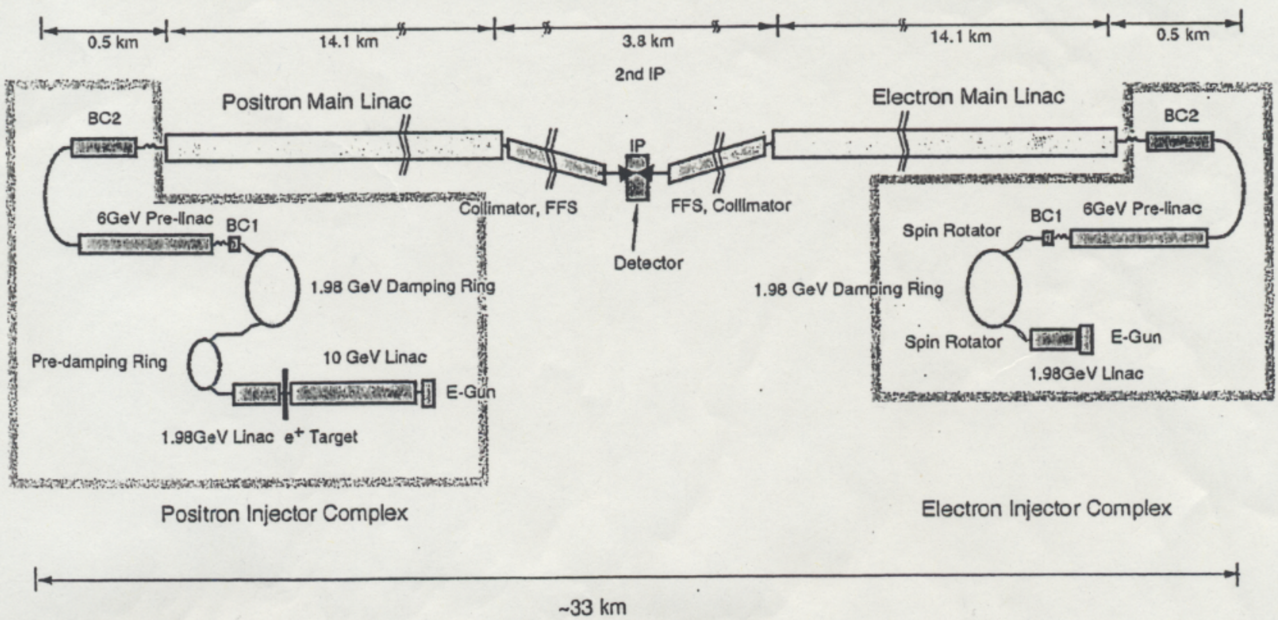


Fig. Schematic view of GLC

Tracking Simulation

◎Tracking code

★SAD(Strategic Accelerator Design)

- It calculates the orbit of electrons by matrix method.
- Lattice design is to be input.
- It has been developed by KEK.

◎Monte Carlo simulation for the beam-scattering is performed by our own program.

Tracking Simulation(2)

◎ Parameters for accelerator and beam

We mainly use the parameters designed for the stage-Ⅱ of GLC ($\sqrt{s}$ = 1TeV) .

◎ Core or Halo ?

Only core is studied in this simulation.

Parameters

◎Parameters for GLC

intial energy ··· 8GeV

final energy ··· 500GeV

total length ··· 12.8km

number of electrons ··· 7.5×10^{9} /bunch

RF frequency ··· 11.424GHz (X-band)

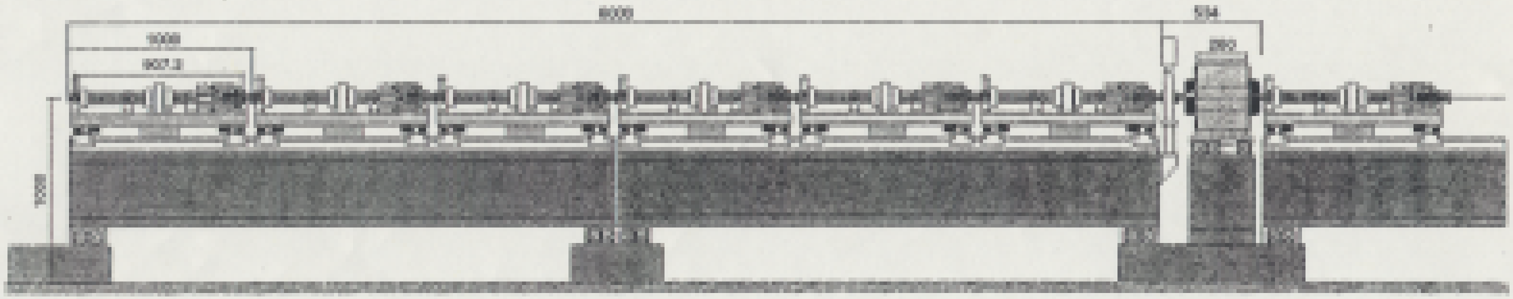


Fig: GLC main linac

Parameters(2)

◎Other assumptions

Pressure in the linac ··· 10nTorr
Temperature in the linac ··· 300K

◎Misalignment and wakefield are not considered in this study.

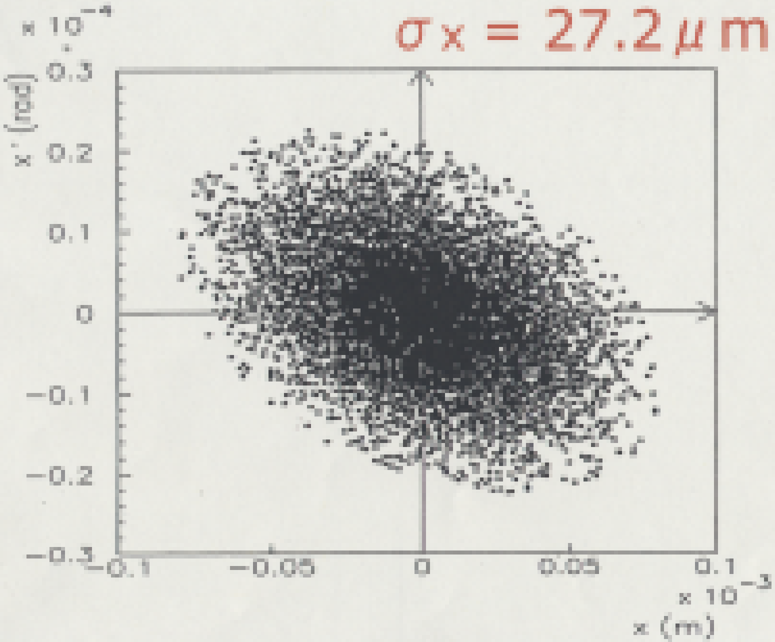
Initial State of the Beam Core

Distribution in phase space

… elliptic in x-x′(y-y′) plane

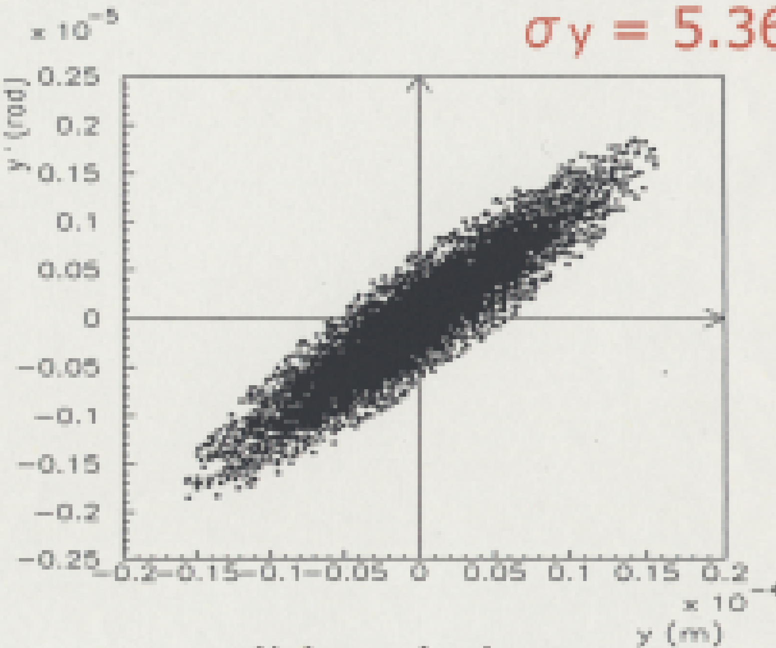
($x' = dx/ds$, $y' = dy/ds$
s ; coordinate along the beam line)

$\sigma_x = 27.2\,\mu m$



(a)x-x′ plane

$\sigma_y = 5.36\,\mu m$



(b)y-y′ plane

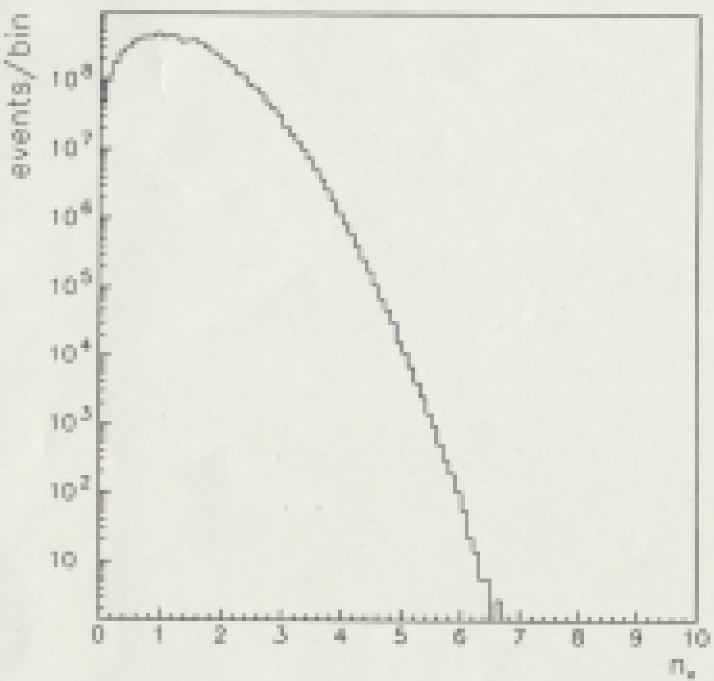
Fig: Phase space distribution at the entrance of the linac

(※ Only electrons within 3σ are plotted here.)

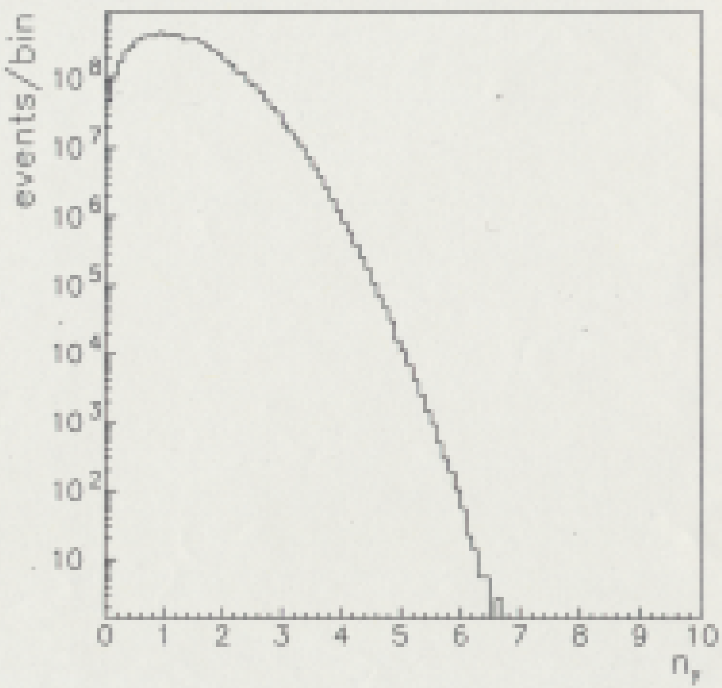
Initial State of the Beam Core (2)

Density distribution in phase space

- We assume Gaussian distribution.
- Tail : Up to 6-7σ is considered.



Fig(a): n_x distribution



Fig(b): n_y distribution

(n_x [n_y]: number of sigma in x-x′[y-y′] plane)

Emittance $\varepsilon_x, \varepsilon_y$ ··· parameters which show the spatial spread of the beam

$$\varepsilon_x = (\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2)^{1/2}$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\sigma_{x'}^2 = \langle x'^2 \rangle - \langle x' \rangle^2$$

$$\sigma_{xx'} = \langle (x - \langle x \rangle)(x' - \langle x' \rangle) \rangle$$

normalized emittance ··· expected to be conserved

$$\varepsilon_N = \gamma \beta \varepsilon \qquad (\varepsilon \sim 1/\gamma\beta)$$

Non-linear Magnetic Field

In each quadrupole magnet, non-linear magnetic field can be produced by manufacturing error of the magnet.

⬇

In the simulation, we investigate the effect of sextupole magnet .

(the largest contribution of all non-linear components)

Simulation for Sextupole Magnetic Field

- Sextupole magnet is installed (virtually) just before each quadrupole magnet.
- Strength: $\left|\frac{B_3}{B_2}\right| = 10^{-3}$
- Direction of the sextupole: random

General form of the magnetic field

$$B_x + i B_y = \sum_{n \geqq 2} B_n \cdot \left(\frac{x+iy}{a}\right)^{n-1} \sim \sum B_n \cdot \left(\frac{r}{a}\right)^{n-1}$$

$\left(\begin{array}{l} B_n : \text{complex constant} \\ a = 20\text{mm} : \text{aperture} \times 0.5 \end{array} \right)$

n=m ⟷ (2m)-pole field

(n=2 : quadrupole $n \geqq 3$: non-linear)

In KEKB and Tristan,

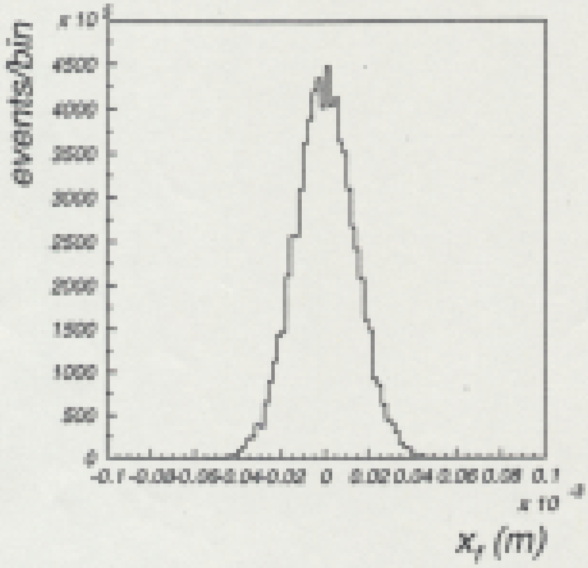
$$\frac{\sum_{n \geqq 3} |B_n|}{|B_2|} \lesssim 10^{-3}$$

was achieved. (K.Egawa)

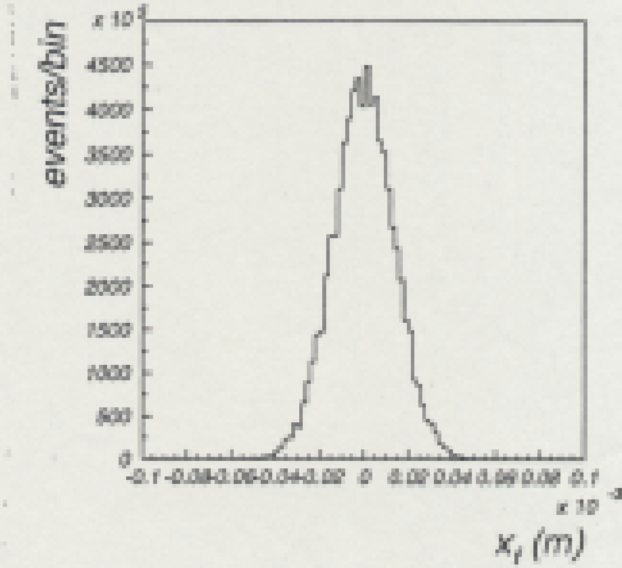
effect on x_f distribution

(x_f ; position in x at the exit)

- linear scale for vertical axis

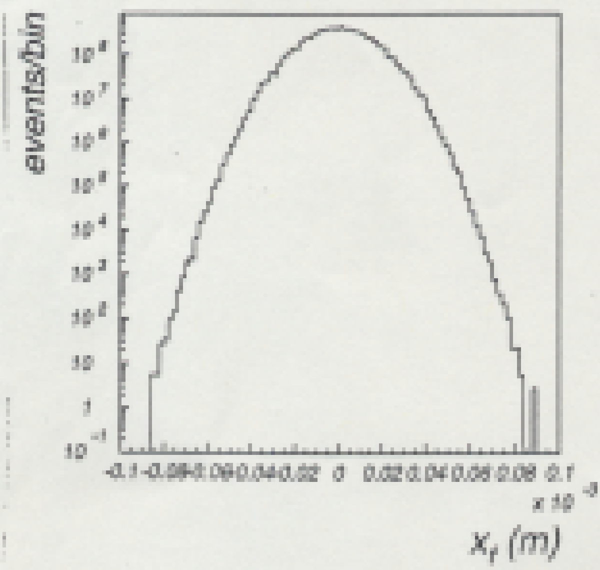


$|B_3/B_2|=0$

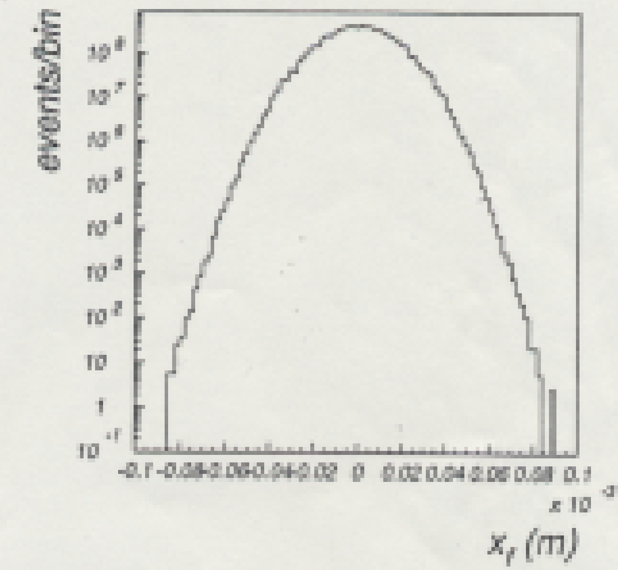


$|B_3/B_2|=10^{-3}$

- log scale for vertical axis



$|B_3/B_2|=0$

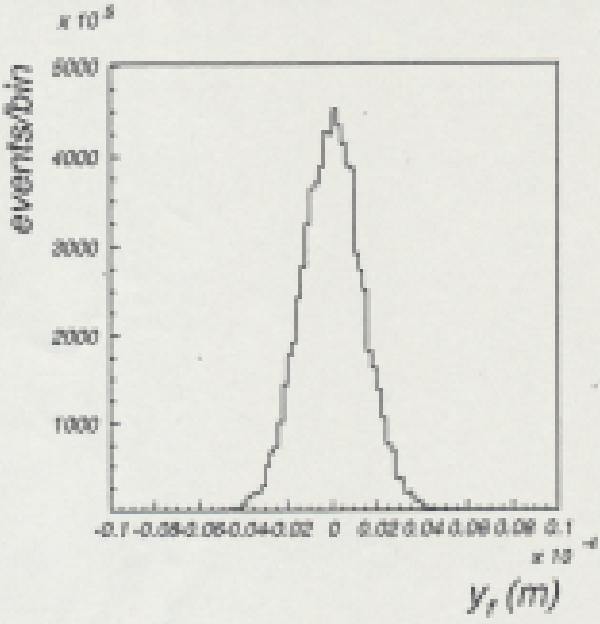


$|B_3/B_2|=10^{-3}$

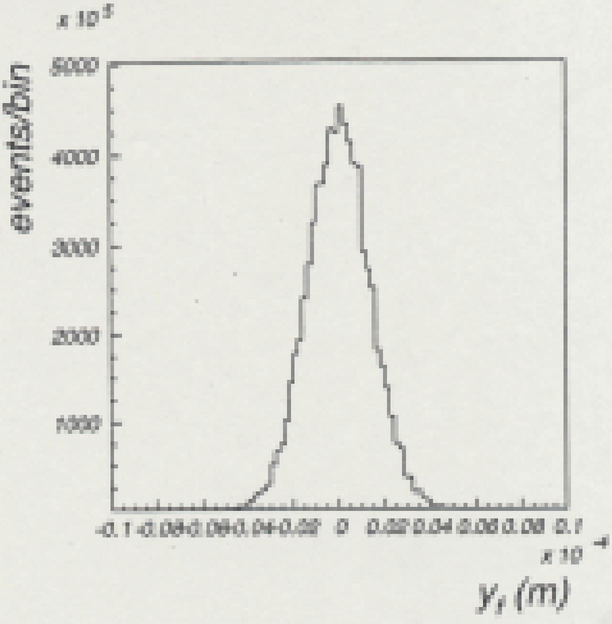
effect on y_f distribution

(y_f ; position in y at the exit)

- linear scale for vertical axis

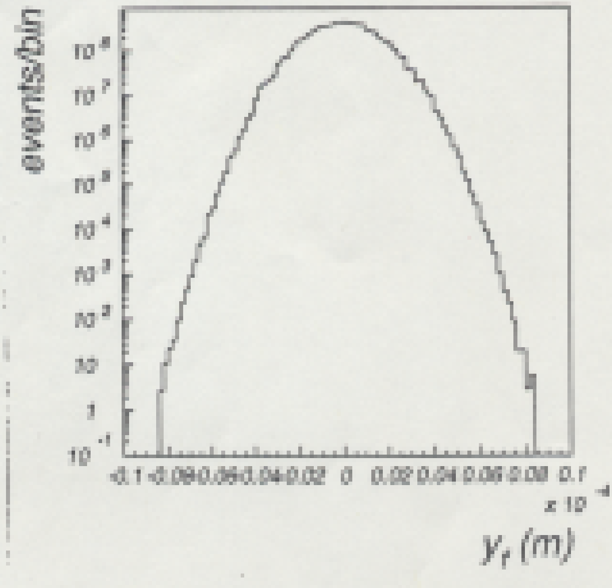


$|B_3/B_2|=0$

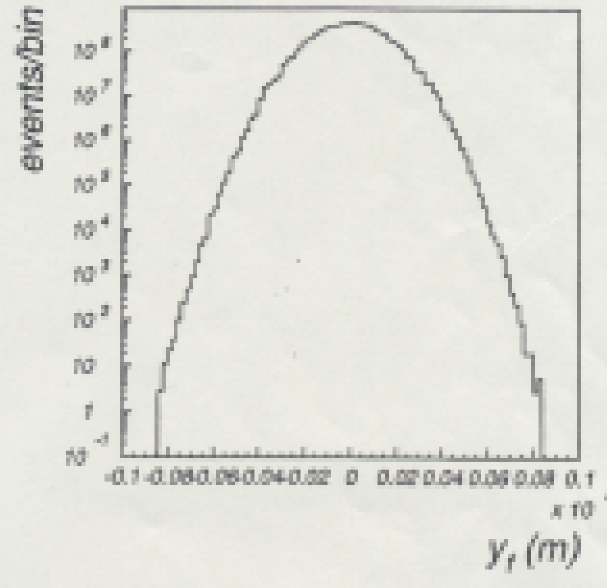


$|B_3/B_2|=10^{-3}$

- log scale for vertical axis



$|B_3/B_2|=0$

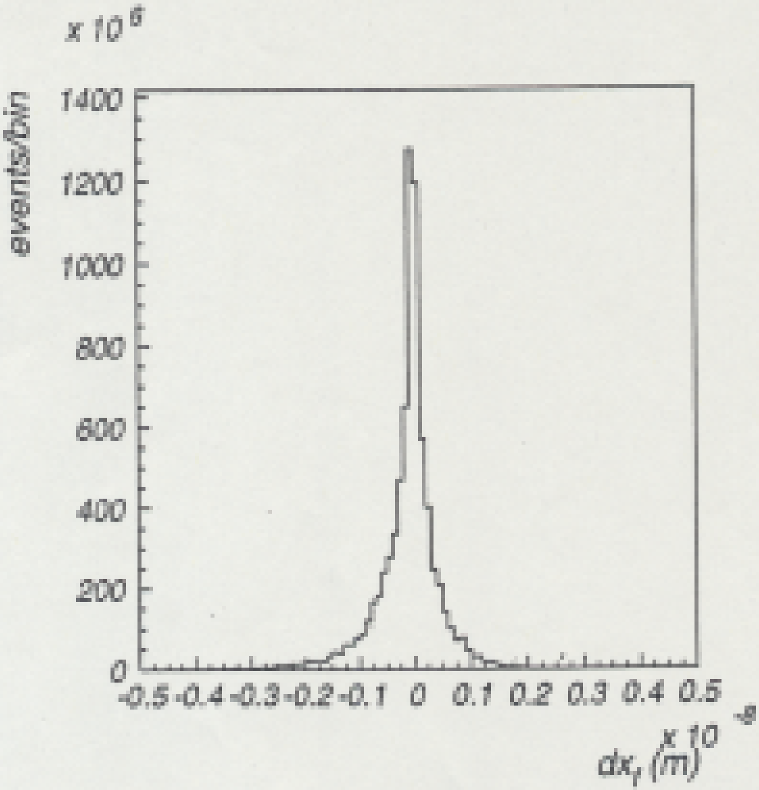


$|B_3/B_2|=10^{-3}$

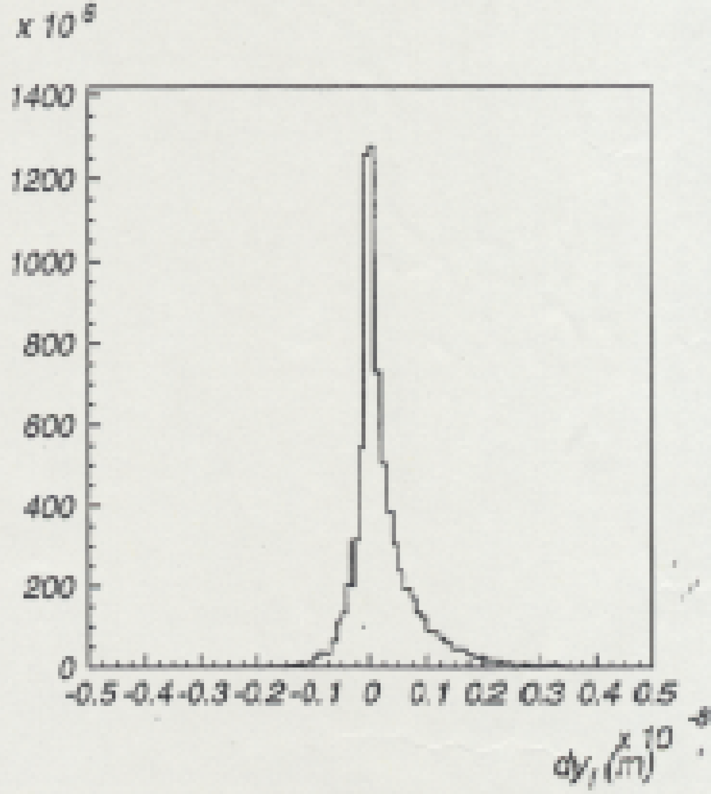
dx_f , dy_f distribution

dx_f , dy_f ; change of x_f , y_f due to the effect of the sextupole magnet

$$\begin{cases} dx_f = x_f(B_3 \neq 0) - x_f(B_3 = 0) \\ dy_f = y_f(B_3 \neq 0) - y_f(B_3 = 0) \end{cases}$$



dx_f distribution



dy_f distribution

Distribution spread: ~ O(1) nm

➡ Influence on the beam core with its size of ~ μm is almost negligible.

Emittance Blowup by the Sextupole Magnetic Field

The effect is negligible as long as $|B_3/B_2| < 10^{-1} - 1$.

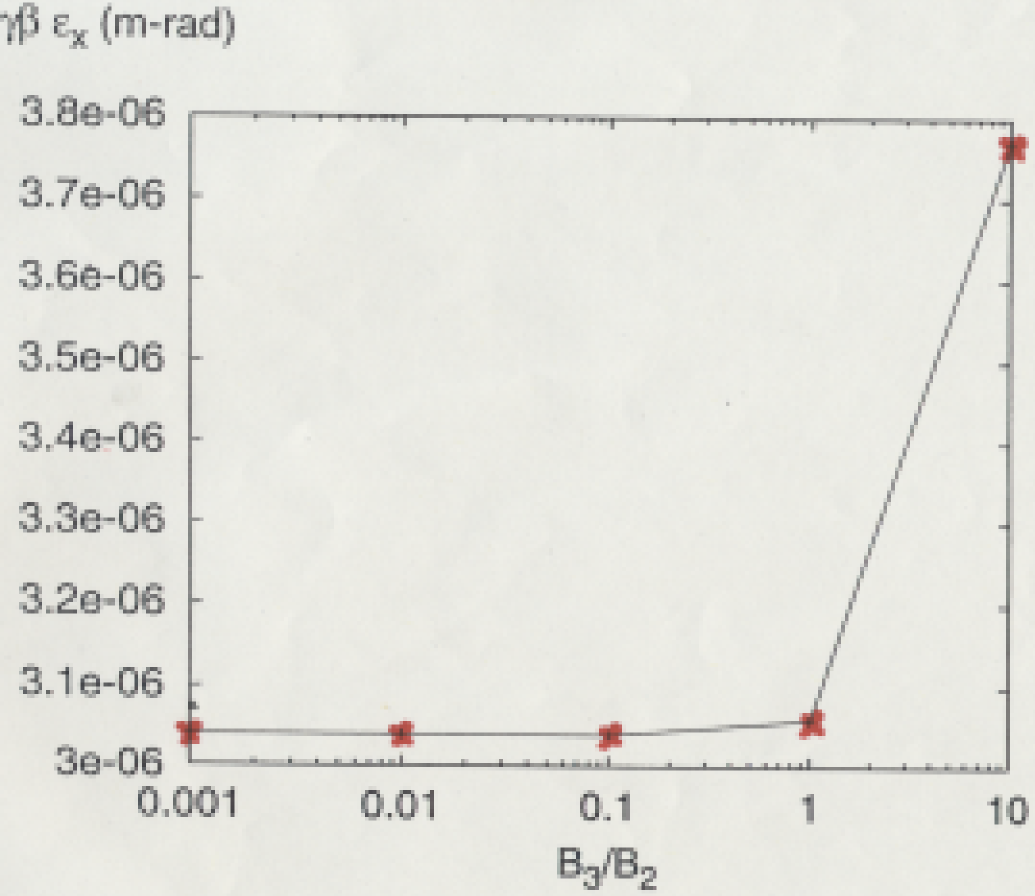


Fig : $\gamma \beta \varepsilon_x$ -blowup

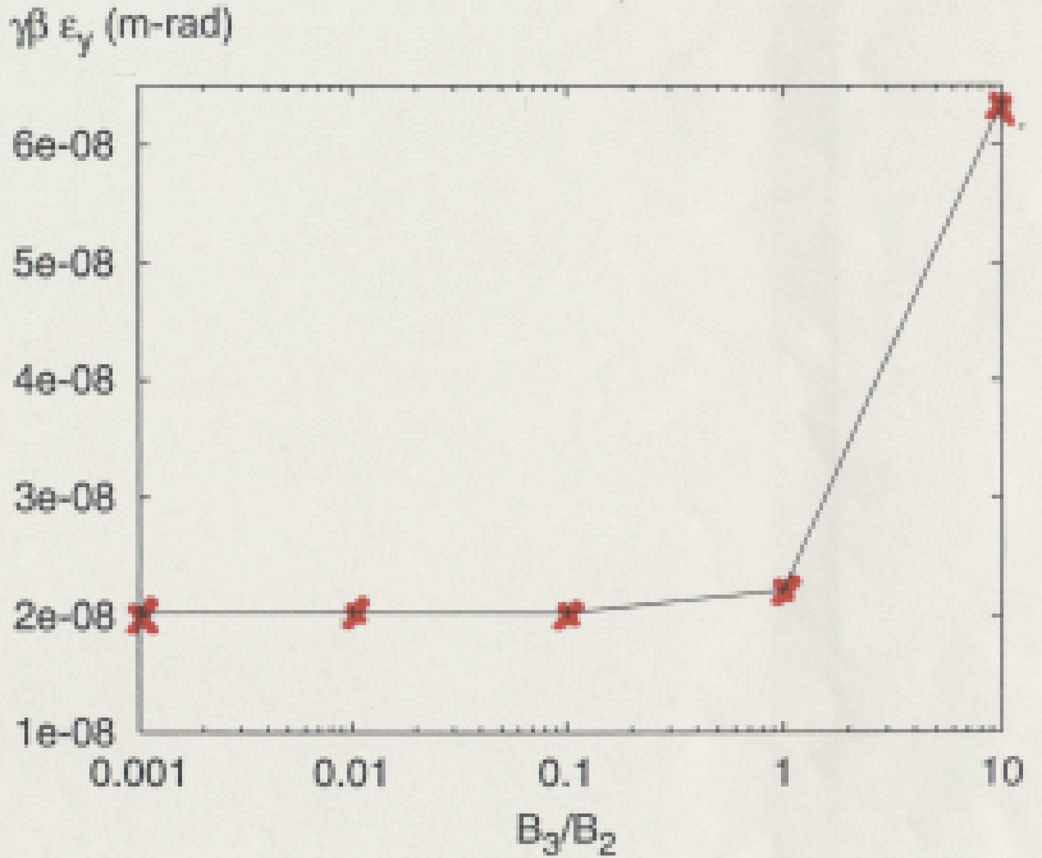


Fig : $\gamma \beta \varepsilon_y$ -blowup

Beam Scattering

◎Scattering process with large contribution in the linac

○ Interaction with residual gas

- elastic scattering
- bremsstrahlung

○ Compton scattering on thermal photon

◎ Assumptions

- Pressure ・・・ 10nTorr
- Temperature ・・・ 300K
- Gas component ・・・ N_2 only

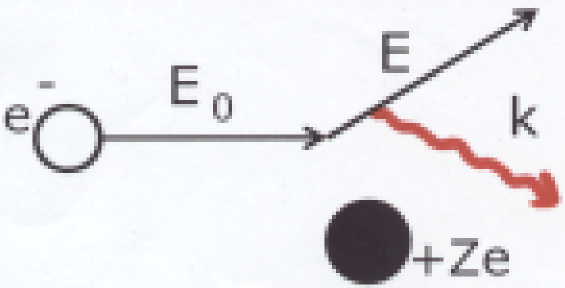
Number of Scattered Particles per Bunch in the Linac

◎Elastic scattering

$\sigma_{el} \sim O(10^5)$ barn

$N_{el} = 6.04 \times 10^5$

◎Bremsstrahlung



E_0 :energy before scat
E :energy after scat
k :energy of emitted γ

Infrared divergence ;

$\sigma_{brems} \sim \ln(x_{min})$

$\left(x = \frac{k}{E_0} \right.$: energy loss rate $\left. \right)$

At $x_{min} = 10^{-2}$,

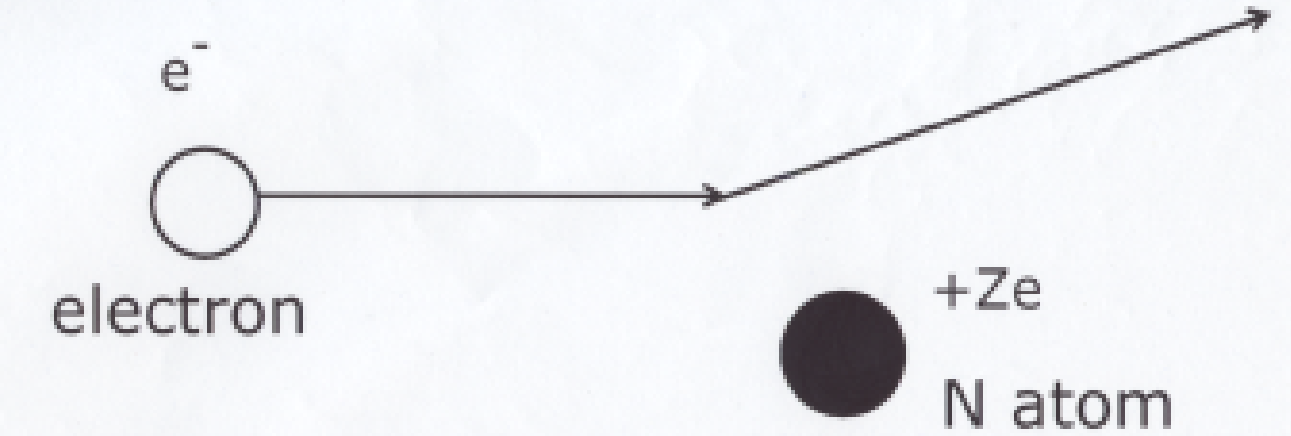
$\sigma_{brems} \sim O(1)$ barn

$N_{brems} \sim O(10)$

◎Compton Scattering

$N_{comp} = 3.15$

Simulation for Elastic Scattering



- Elastic scattering occurs uniformly in the linac.
- $\frac{d\sigma}{d\theta} \sim \frac{1}{\theta^3}$, $\theta_{min} = \frac{h}{pa}$ ⬅ due to screening effect

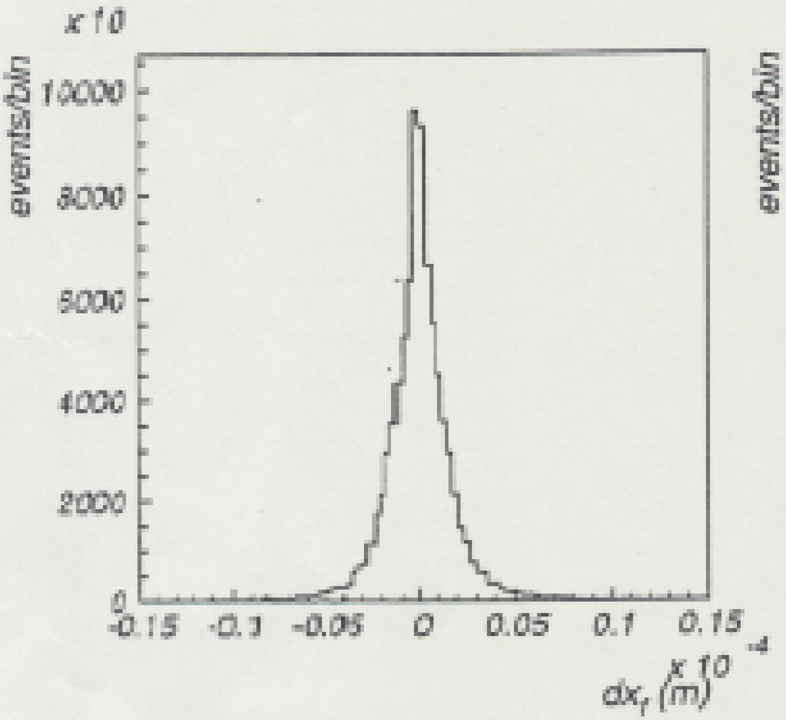
(⬌ $\theta \sim h/pb$
b: impact parameter)

(a: atomic radius of nitrogen
p: incident momentum of e^-)

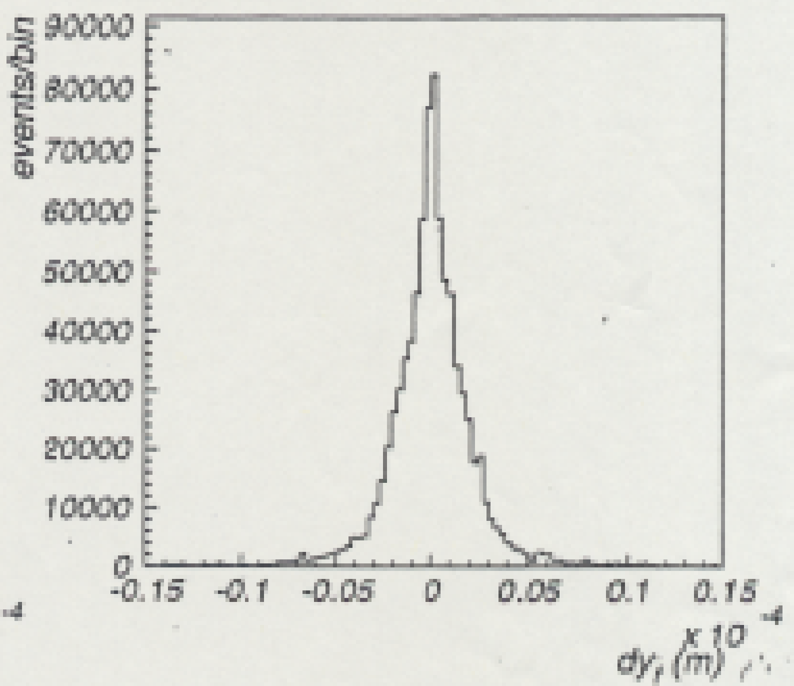
dx_f ,dy_f distribution of scattered electrons

dx_f ,dy_f : change of x_f ,y_f due to the scattering

$$\begin{cases} dx_f = x_{f,\text{scat-occurred}} - x_{f,\text{scat-not-occurred}} \\ dy_f = y_{f,\text{scat-occurred}} - y_{f,\text{scat-not-occurred}} \end{cases}$$



dx_f distribution



dy_f distribution

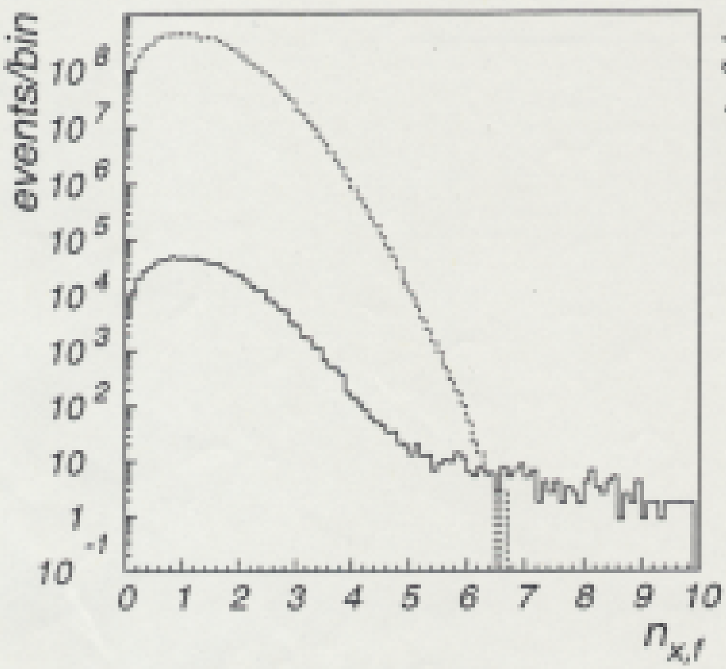
The distribution spread reaches as much as μm scale .

cf.) dx,dy ~ O(1)nm

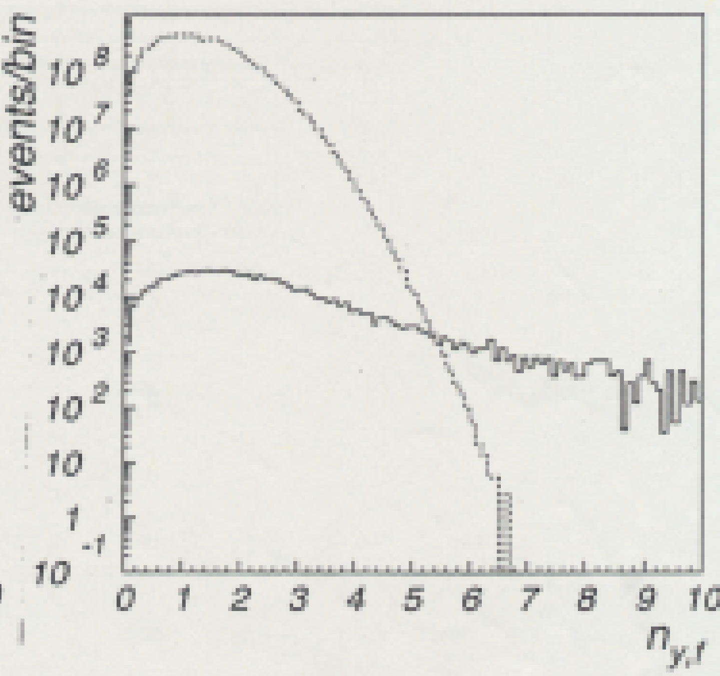
sextupole's effect

$n_{x,f}$, $n_{y,f}$ distributions

($n_{x,f}$, $n_{y,f}$: number of sigma at the exit)



$n_{x,f}$ distribution



$n_{y,f}$ distribution

(solid line: scattered electrons
dashed line: the others)

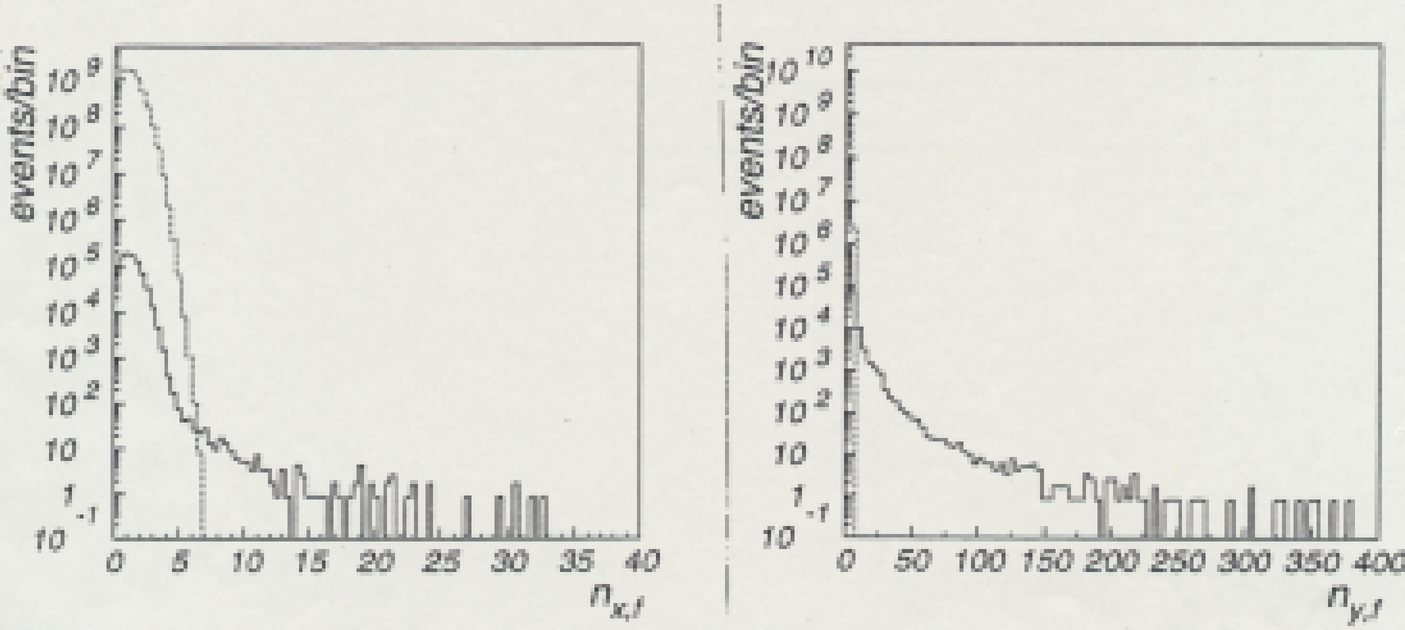
Long tails are produced in the distributions.

$N(n_{x,f} > 7) =$ 160

$N(n_{y,f} > 7) =$ 24125

Tails

(solid line: scattered electrons
dashed line; the others)



$n_{x,f}$ distribution $n_{y,f}$ distribution

The tails extend to

~ { $30\text{-}40\sigma$ (for n_x)
   $300\text{-}400\sigma$ (for n_y)

Table; $N(n_{x,f} > n_{min})$ and $N(n_{y,f} > n_{min})$

n_{min}	10	50	100
$N(n_{x,f} > n_{min})$	75	7	2
$N(n_{y,f} > n_{min})$	24125	442	127

Emittance Blowup due to Elastic Scattering

Since the scattering probability is very small ($\sim 10^{-4}$), the effect on the whole beam does not become large.

	unscattered electrons	scattered electrons	all electrons
Number of electrons	7.50×10^{9}	6.02×10^{5}	7.50×10^{9}
$\gamma \beta \varepsilon_x$ (m-rad)	3.06×10^{-6}	3.14×10^{-6}	3.06×10^{-6}
$\gamma \beta \varepsilon_y$ (m-rad)	2.03×10^{-8}	6.00×10^{-8}	2.03×10^{-8}

Emittance-blowup vs. P_{linac} (P_{linac} : pressure in the linac)

The effect is negligible for $P_{linac} \lesssim 10^{-6} - 10^{-5}$ Torr

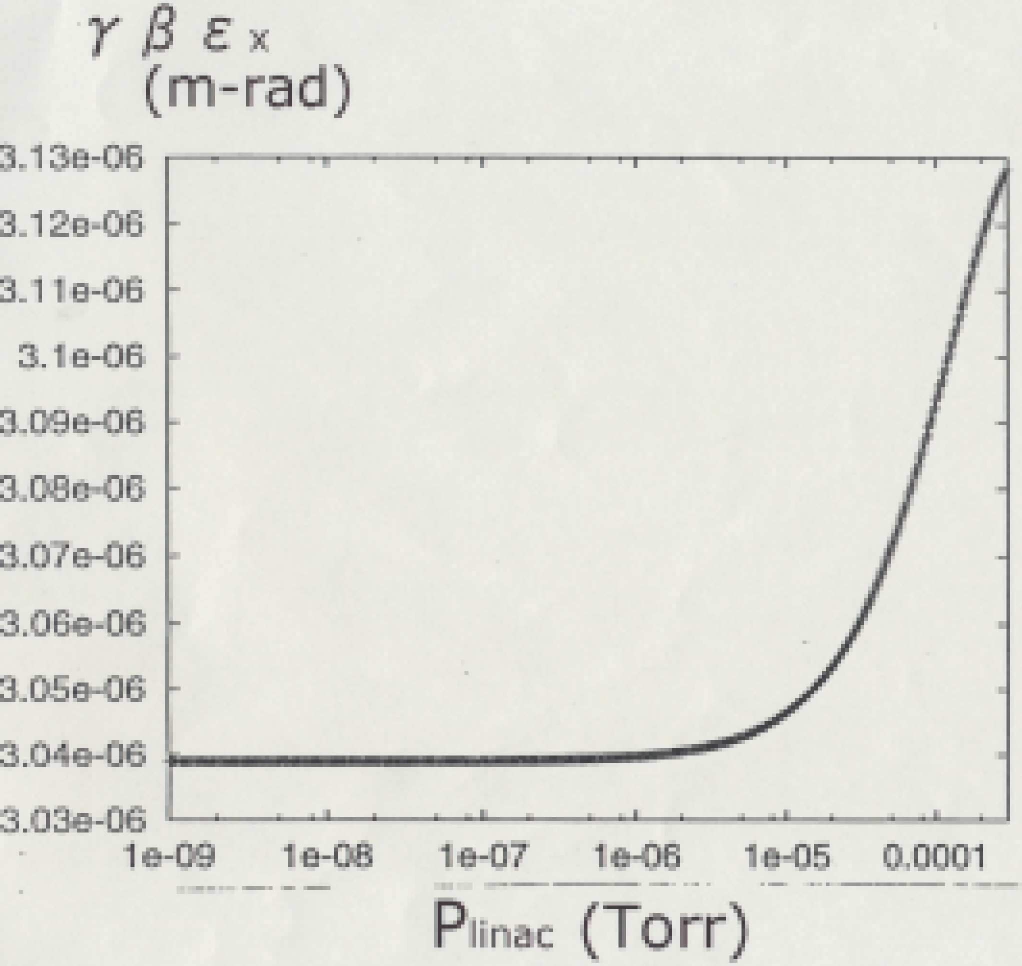


Fig: $\gamma\beta\varepsilon_x$ vs P_{linac}

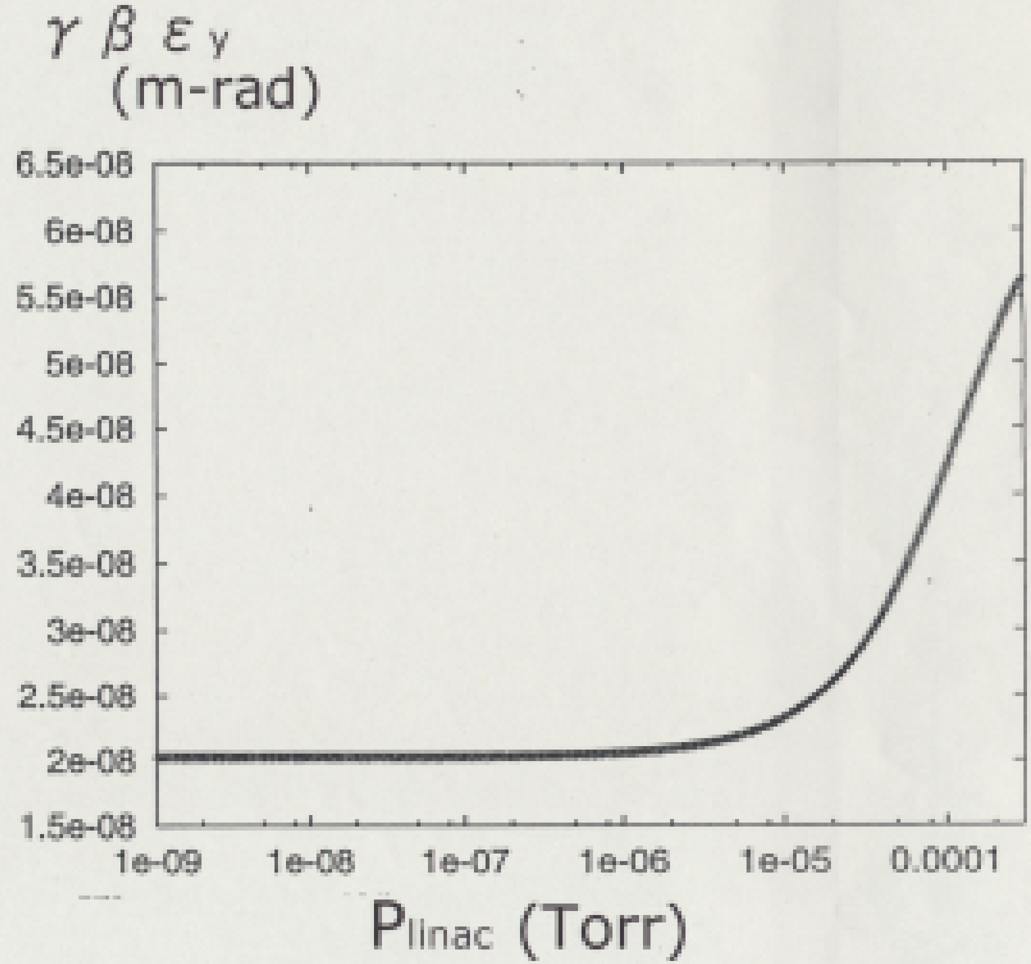


Fig: $\gamma\beta\varepsilon_y$ vs P_{linac}

Summary

Simulation for the Electron Beam in the Main Linac

- **Effect of sextupole magnet**

  Scale of the orbit spread : ~ O(1) nm

- **Effect of elastic scattering**

  Although the scale of the orbit spread reaches μm-order, the scattering probability is very small (~ 10^{-4}).

→ Neither of them cannot have a large contribution to emittance-blowup.