# LHC実験のための
# シミュレーション・データの作り方

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ICEPP セミナー
2004年3月24日

# 最初に宣伝を少し

Mini-Workshop

Physics Simulation for LHC

5-6 April 2004 at KEK, Tsukuba, Japan

ここを見てね

http://www-conf.kek.jp/phsimLHC/index.html

# QCD 用語の基礎知識

- QCD Lagrangian
- QCD Feynman rules
- Renomarization scheme
- Renomarization energy scale
- Running Coupling
- $\Lambda_{QCD}$
- DGLAP equation
- Parton shower
- Sudakov form factor
- PDF
- Factrization energy scale

# QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavours}} \bar{q}_a (i\not{D} - m_q)_{ab} q_b + \mathcal{L}_{\text{gauge-fixing}}$$

$$F^A_{\alpha\beta} = \partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC} \mathcal{A}^B_\alpha \mathcal{A}^C_\beta$$

- QCD coupling strength is $\alpha_s \equiv g^2/4\pi$. Numbers $f^{ABC}$ $(A, B, C = 1, \ldots, 8)$ are structure constants of the SU(3) colour group. Quark fields $q_a$ $(a = 1, 2, 3)$ are in triplet colour representation, while gluon fields $\mathcal{A}^A_\alpha$ are in adjoint representation.
- $D$ is covariant derivative:

$$\begin{aligned}(D_\alpha)_{ab} &= \partial_\alpha \delta_{ab} + ig \left(t^C \mathcal{A}^C_\alpha\right)_{ab} \\ (D_\alpha)_{AB} &= \partial_\alpha \delta_{AB} + ig (T^C \mathcal{A}^C_\alpha)_{AB}\end{aligned}$$

# Color factor

- $t$ and $T$ are matrices in the fundamental and adjoint representations of SU(3), respectively:

$$[t^A, t^B] = i f^{ABC} t^C, \quad [T^A, T^B] = i f^{ABC} T^C$$

where $(T^A)_{BC} = -i f^{ABC}$. We use the metric $g^{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1)$ and set $\hbar = c = 1$. $\not{D}$ is symbolic notation for $\gamma^\alpha D_\alpha$. Normalisation of the $t$ matrices is

$$\mathrm{Tr}\, t^A t^B = T_R\, \delta^{AB}, \quad T_R = \frac{1}{2}.$$

- SU(N) matrices obey the relations:

$$\sum_A t^A_{ab} t^A_{bc} = C_F\, \delta_{ac}, \quad C_F = \frac{N^2-1}{2N}$$

$$\mathrm{Tr}\, T^C T^D = \sum_{A,B} f^{ABC} f^{ABD} = C_A\, \delta^{CD}, \quad C_A = N$$

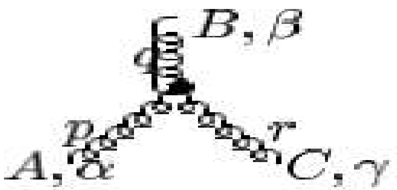
Thus $C_F = \frac{4}{3}$ and $C_A = 3$ for SU(3).

# Feynman rule


$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^2+i\varepsilon}\right]\frac{i}{p^2+i\varepsilon}$$

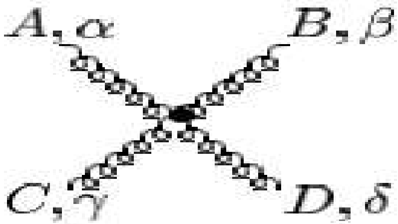

$$\delta^{AB}\frac{i}{p^2+i\varepsilon}$$


$$\delta^{ab}\frac{i}{(\not p-m+i\varepsilon)_{ji}}$$

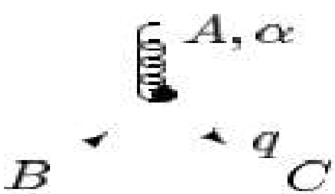



$$-gf^{ABC}\left[g^{\alpha\beta}(p-q)^{\gamma}+g^{\beta\gamma}(q-r)^{\alpha}+g^{\gamma\alpha}(r-p)^{\beta}\right]$$

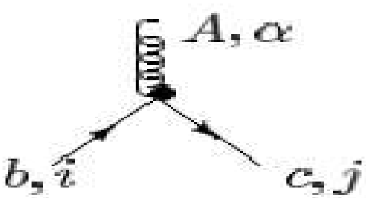
(all momenta incoming)




$$\begin{aligned}
&-ig^2 f^{XAC}f^{XBD}\left(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\delta}g_{\beta\gamma}\right)\\
&-ig^2 f^{XAD}f^{XBC}\left(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\gamma}g_{\beta\delta}\right)\\
&-ig^2 f^{XAB}f^{XCD}\left(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma}\right)
\end{aligned}$$




$$gf^{ABC}q^{\alpha}$$




$$-ig\left(t^{A}\right)_{cb}\left(\gamma^{\alpha}\right)_{ji}$$

$$\int \frac{\mathrm{d}^D k}{(k^2+m^2)^2}$$

$$\frac{\mathrm{d}^4 k}{(2\pi)^4} \longrightarrow (\mu)^{2\epsilon} \frac{\mathrm{d}^{4-2\epsilon} k}{(2\pi)^{4-2\epsilon}}$$

$$\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$$

replace bare coupling by renormalized coupling $\alpha_s(\mu)$

# Running coupling and $\Lambda_{QCD}$

$$\alpha_S(Q) = \frac{\alpha_S(\mu)}{1+\alpha_S(\mu)b\tau}, \quad \tau = \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$b = \frac{(11C_A - 2N_f)}{12\pi}$$

$$\alpha_S(Q) = \frac{1}{b\ln(Q^2/\Lambda^2)} \qquad \text{(LO)}$$

$\Lambda$ sets the scale at which $\alpha_S(Q)$ becomes large.

# DGLAP Equation

DGLAP Equation

Splitting function ← pQCD

$$\frac{dD(x,Q^2)}{d\ln Q^2} = \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y} P_+(x/y) D(y,Q^2)$$

Parton distribution

$$D(x,Q^2) = \Pi(Q^2,Q_s^2) D(x,Q_s^2) + \frac{\alpha}{2\pi}\int_{Q_s^2}^{Q^2} \frac{dK^2}{K^2} \Pi(Q^2,K^2) \int_x^{1-\epsilon} \frac{dy}{y} P(y) D(x/y,K^2)$$

$$\Pi(Q^2,Q'^2) = \exp\left(-\frac{\alpha}{2\pi}\int_{Q'^2}^{Q^2} \frac{dK^2}{K^2}\int_0^{1-\epsilon} dx P(x)\right)$$

Sudakov Factor

non-branch provability

# Introduction

LHC Experimental requirement

New Particle Search/Precision Measurement

LO-QCD Event generator+K-factor

↓

Obviously not enough!

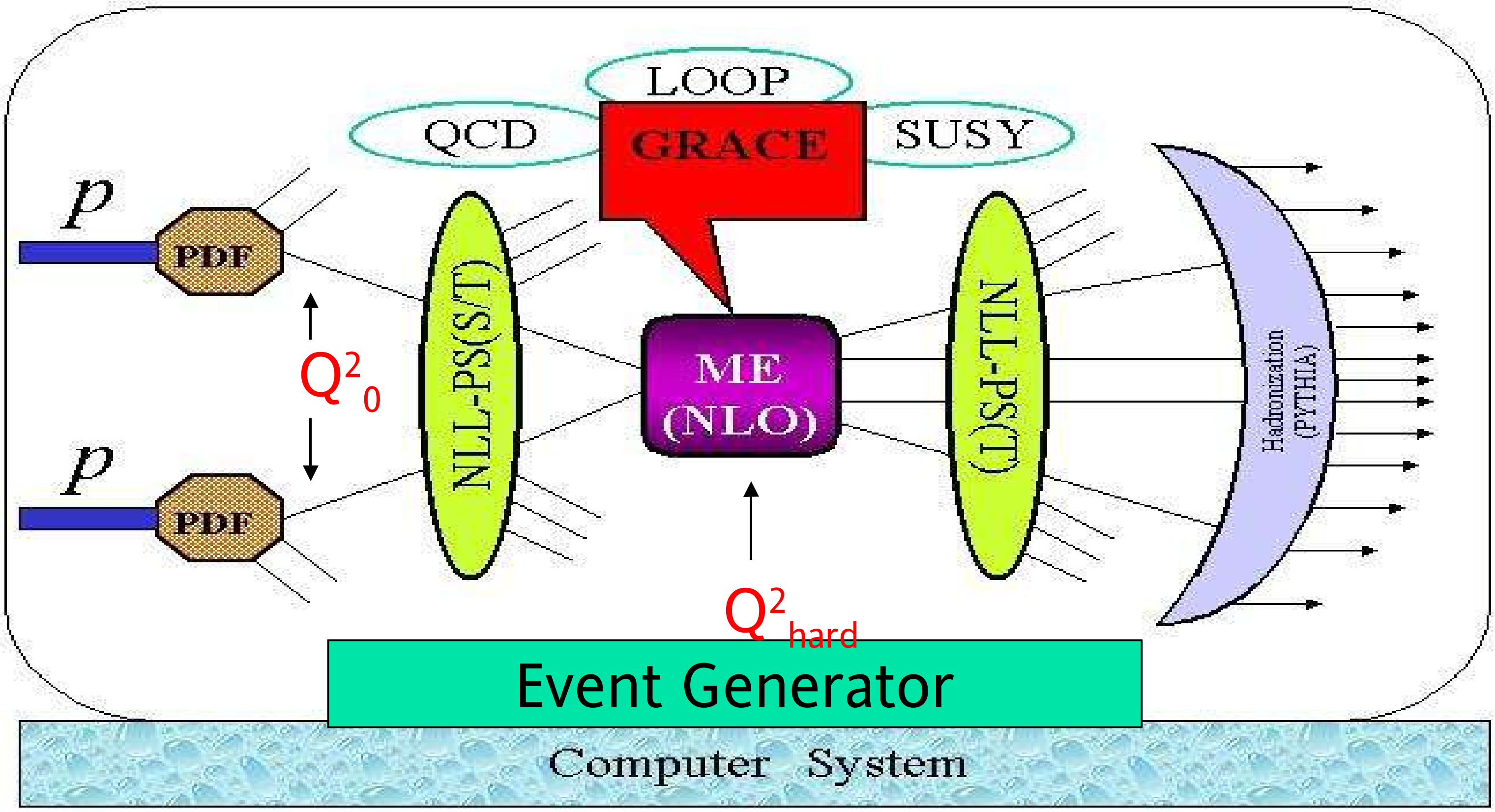
We need

NLO Event generator!

# Difficulties → Solutions

- Large number of diagrams → GRACE
- Numerical instability due to a collinear singularity
- Double counting between ME and PDF/PS → LL-subtraction + Parton Shower
- Negative weight → Improved SPRING

# Grand Design



# Parton Shower

➢PYTHIA/HERWIG

Q2 evolution/x-distribution ➤ PDF(Parton distribution function)

Soft-Collinear jets ➤ (backward) Parton Shower

PS is used as a model to generate jets

➢ Our strategy

$Q^2$ evolution/x-distribution/Soft-Collinear jets

➤ LL/NLL Parton Shower (forward)

PS is legitimate child of pQCD
(no tunable parameters)

# Parton Shower

DGLAP Equation

Splitting function ← pQCD

$$\frac{dD(x,Q^2)}{d\ln Q^2}=\frac{\alpha}{2\pi}\int_x^1\frac{dy}{y}P_+(x/y)D(y,Q^2)$$

$$D(x,Q^2)=\Pi(Q^2,Q_s{}^2)D(x,Q_s{}^2)+\frac{\alpha}{2\pi}\int_{Q_s{}^2}^{Q^2}\frac{dK^2}{K^2}\Pi(Q^2,K^2)\int_x^{1-\epsilon}\frac{dy}{y}P(y)D(x/y,K^2)$$

$$\Pi(Q^2,Q'^2)=\exp\left(-\frac{\alpha}{2\pi}\int_{Q'^2}^{Q^2}\frac{dK^2}{K^2}\int_0^{1-\epsilon}dxP(x)\right)$$

Sudakov Factor

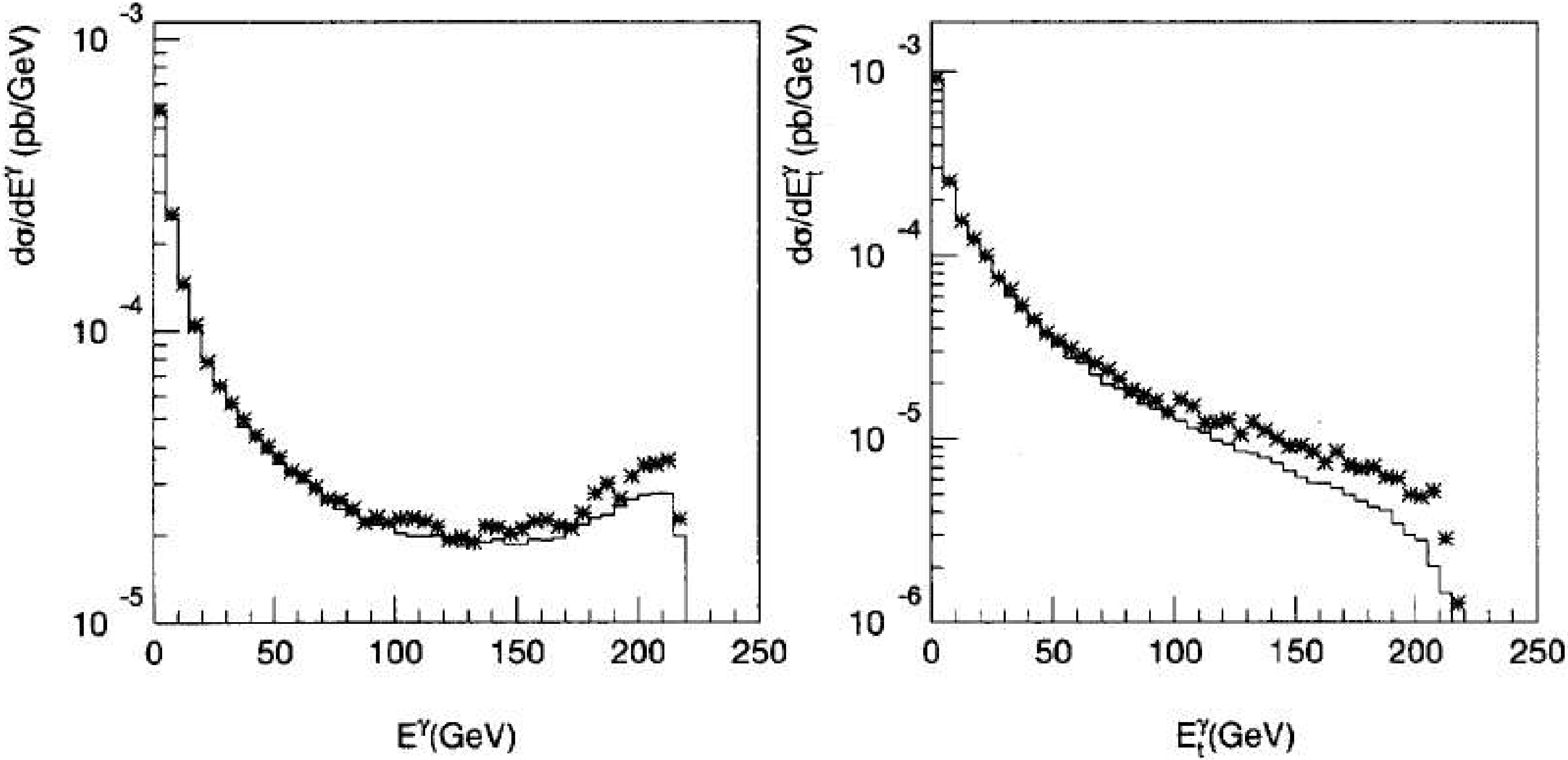
non-branch provability

# Example in QED (annihilation)

$$e^+e^- \longrightarrow ZH + \gamma$$

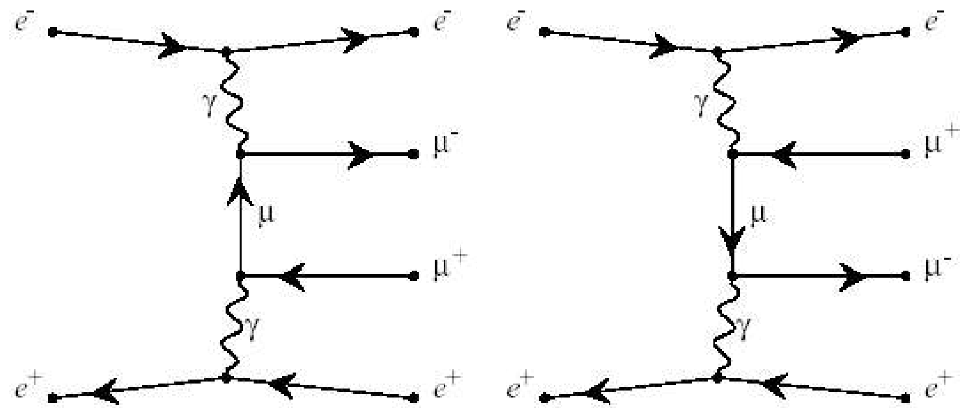
Zhγ ME

* ZH ME + γPS



# Example in QED (fusion)

$e^+e^- \to e^+e^-\mu^+\mu^-$




## Soft correction formula

$$\sigma_{\text{soft}} = \sigma_0(s)\left\{1 + \frac{\alpha}{\pi}\left[-2l(L-1) + \frac{3}{2}(L-1)\right]\right\}$$

## O(α) correction

$$2\text{Re}F_1 + \delta_s \to \frac{\alpha}{\pi}\left(-2l(L_t-1) + \frac{3}{2}L_t - 2\right)$$

$$L_t = \ln(-t/m_e^2)$$

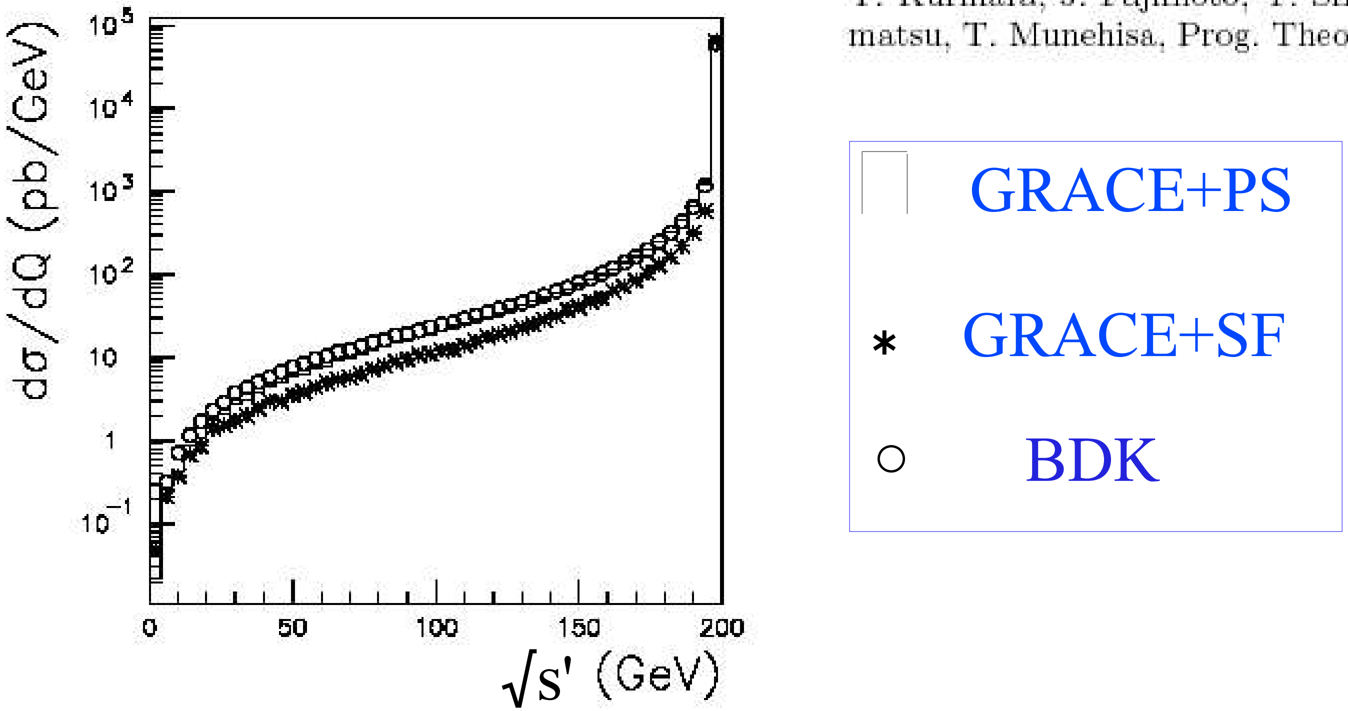
Factrization energy scale

F. A. Berends, P. H. Daverveldt and R. Kleiss, Nucl. Phys. **B253** (1985), 412.

# •$e^+e^- \rightarrow e^+e^-\mu^+\mu^-$ :Total cross sections

| $E_{CM}$ (GeV) | $\sigma_0$ (nb) | $\sigma_{\rm sf}$ (nb) | $\sigma_{\rm QEDPS}$ (nb) | $\sigma_{\rm BDK}$ (nb) |
|---|---|---|---|---|
| 20 | 97.0(1) | 96.3(1) | 96.0(2) | 96.0(1) |
| 40 | 137.5(3) | 136.3(1) | 135.9(3) | 135.9(1) |
| 100 | 202.8(4) | 201.0(2) | 200.5(4) | 200.5(2) |
| 200 | 262.0(6) | 259.8(3) | 259.1(6) | 258.8(2) |

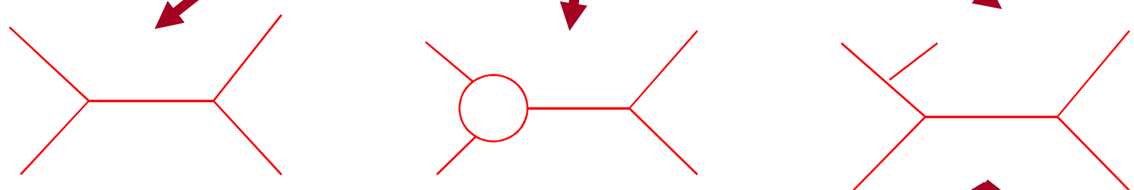
F. A. Berends, P. H. Daverveldt and R. Kleiss, Nucl. Phys. **B253** (1985), 412.




Y. Kurihara, J. Fujimoto, Y. Shimizu, K. Kato, K. Tobimatsu, T. Munehisa, Prog. Theor. Phys. **103**, 1199 (2000)

# NLO Cross sections

$$\sigma_{NLO}=\sigma_{tree}+\sigma_{loop}+\sigma_R$$



$$=\sigma_{tree}(1+\delta_V+\delta_{s/c})+\sigma_{vis}$$

$\delta_V$ : Virtual (loop) correction

$\delta_{s/c}$ : Soft/Collinear correction

$\sigma_{vis}$ : Visible jet cross section

# Matrix Elements: Tree

- Tree diagrams

  $M_0(1,2\to 1,2\ \cdots\ ,n)$ : Born

  $M_R(1,2\to 1,2\ \cdots\ ,n,n+1)$ : Real radiation

GRACE: Automatic generation up to n≅6

↑

Diagrams & FORTRAN source code

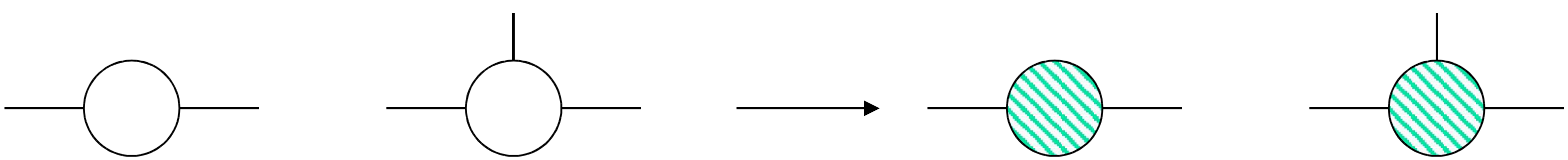
CHANEL: Numerical Helicity Amplitude

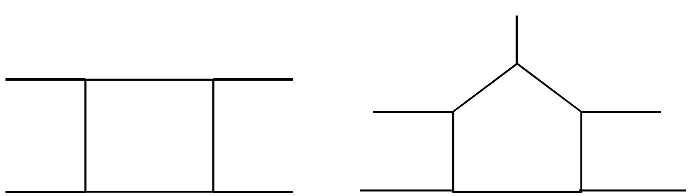
# Matrix Elements: Loop

- Loop diagrams

$M_V(1,2\to 1,2\cdots,n)$ : Effective vertices

(up to three point)

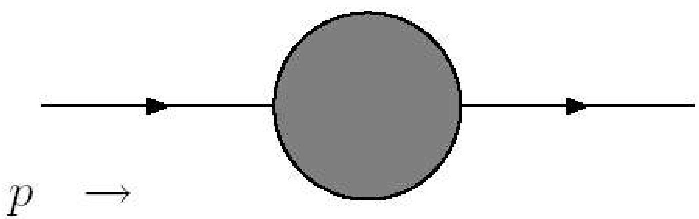




: Numerical calc. by CHANEL

# Matrix Elements: Effective coupling

- Quark self-energy

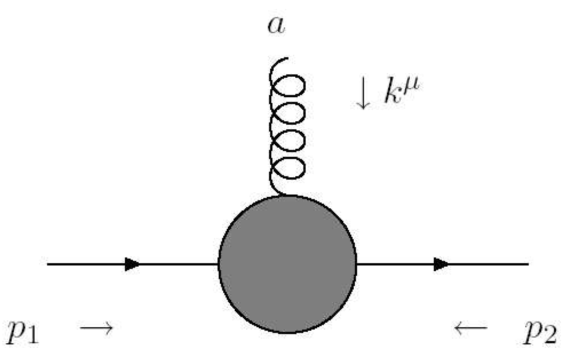



$$= \Sigma(p^2)$$

| on-/off-shell | self-energy |
|---|---|
| $p^2 \neq 0$ | $\Sigma(p^2) = C_F \frac{\alpha_s}{4\pi}(1 - \ln \frac{-p^2}{\mu^2})$ |
| $p^2 = 0$ | $\Sigma(0) = C_F \frac{\alpha_s}{4\pi} \frac{1}{\varepsilon_{IR}}$ |

# Matrix Elements: Effective coupling

- Quark-Quark-Gluon vertex




$$= \Lambda^a_\mu(p_1, p_2, k)$$

| | |
|---|---|
| $\mathcal{F}_1^I$ | $\frac{2}{\varepsilon_{IR}} + L - 4$ |
| $\mathcal{F}_2^I$ | $\frac{4}{\varepsilon_{IR}} + 4L - 10$ |
| $\mathcal{F}_1^{II}$ | $-\frac{2}{\varepsilon_{IR}^2} + \frac{3-2L}{\varepsilon_{IR}} + \frac{\pi^2}{6} - L$ |
| $\mathcal{F}_2^{II}$ | $-\frac{2}{\varepsilon_{IR}^2} - \frac{2L}{\varepsilon_{IR}} + \frac{12+\pi^2-6L^2}{6}$ |

$$\begin{aligned}\Lambda_\mu(p_1, p_2, k) &= g\mathbf{T}^a\left(C_F - \frac{1}{2}C_G\right)\frac{\alpha_s}{4\pi}\left(\mathcal{F}_1^I \gamma^\mu + \mathcal{F}_2^I \frac{p_j^\mu \not{k}}{-p_j^2}\right) \\ &+ g\mathbf{T}^a \frac{1}{2} C_G \frac{\alpha_s}{4\pi}\left(\mathcal{F}_1^I \gamma^\mu + \mathcal{F}_2^I \frac{p_j^\mu \not{k}}{-p_j^2}\right)\end{aligned}$$

$$k^2 = 0,\ p_i^2 = 0,\ p_j^2 = q^2 \neq 0, \text{ and } L = \ln\frac{-q^2}{\mu^2},$$

# IR-behavior

No IR-divergence

$$\sigma_{NLO} = [\sigma_{tree}(1 + \delta_V + \delta_{s/c}) + \sigma_{vis}] \otimes \boxed{\text{PDF/PS}}$$

$1/\varepsilon^2_{IR}, 1/\varepsilon_{IR}$ cancellation

PDF/Parton Shower

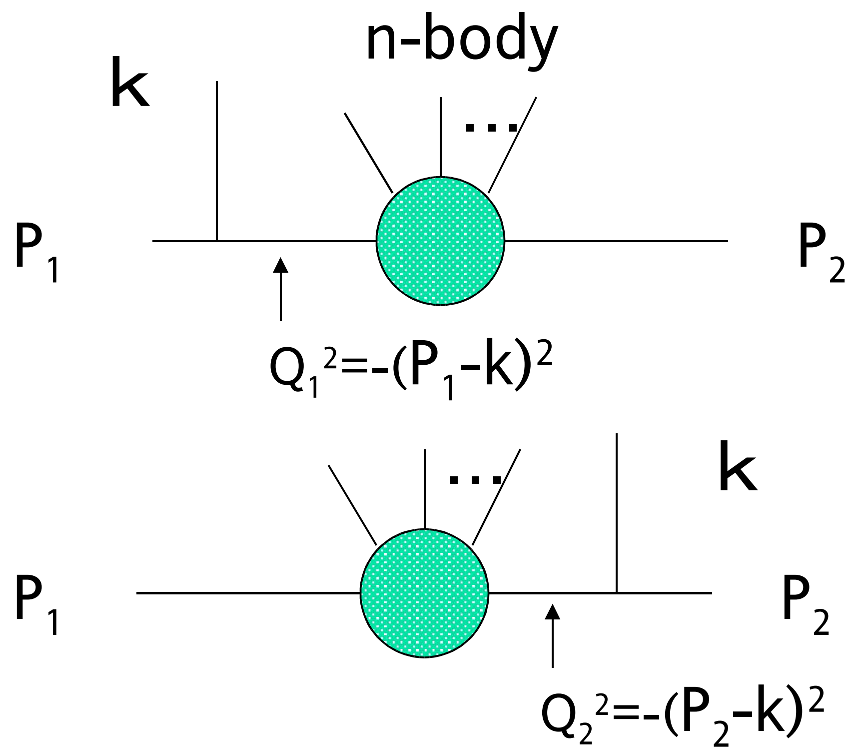
$$\boxed{\frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x)}$$

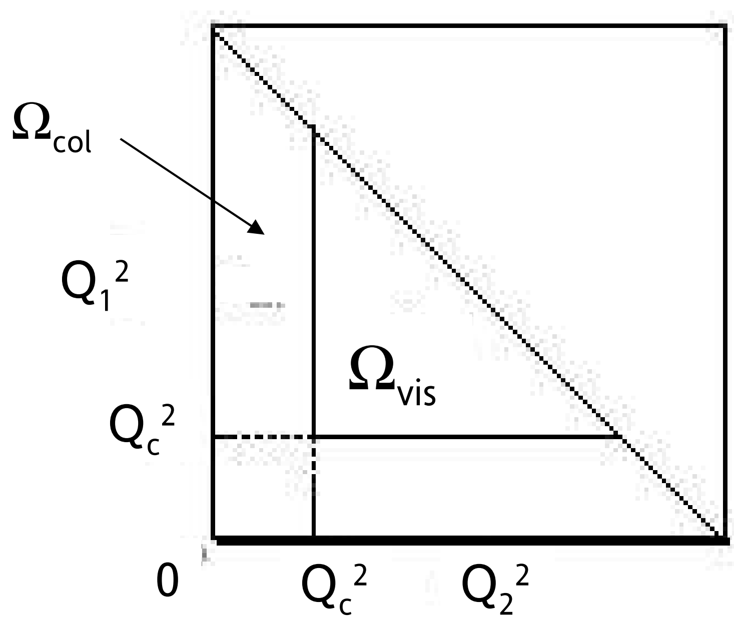
P(x) : Splitting function

Space/time dimension : $d=4+2\varepsilon_{IR}$

# Soft/Collinear Treatment

- Subtraction method — S.Catani, M.M. Seymour, hep-ph/9605323
- Phase-space slicing — W.B. Harris, J.F.Owens, Phys. Rev. D65 (2002) 094032



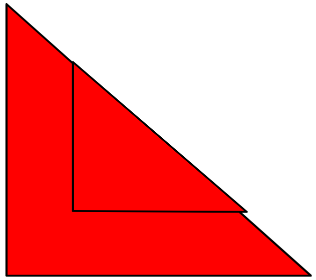



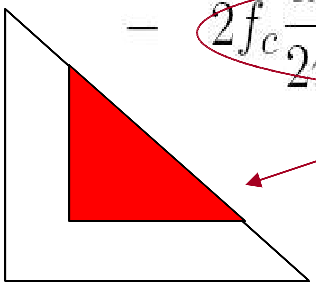
# Collinear Cross Section

Canceled with $\delta_V$

$$
\begin{aligned}
\sigma_{coll} &= \sigma_0(s)\frac{\alpha_s}{2\pi}f_c\left[\frac{2}{\varepsilon_{IR}^2}+\frac{2L-3}{\varepsilon_{IR}}-\frac{\pi^2}{2}+L^2\right] \\
&+ 2\int_0^1 dx\sigma_0(xs)\phi(x,\varepsilon_{IR}) \\
&+ 2f_c\frac{\alpha_s}{2\pi}\int_0^1 dx\sigma_0(xs)\left[L\frac{1+x^2}{(1-x)_+}+2\frac{(1+x^2)\ln(1-x)}{(1-x)_+}-\frac{1+x^2}{1-x}\ln x\right] \\
&- 2f_c\frac{\alpha_s}{2\pi}\int_0^1 dx\sigma_0(xs)\frac{1+x^2}{(1-x)_+}\ln\left(\frac{s}{Q_c^2}(1-x)-1\right)\Theta\left(1-\frac{2Q_c^2}{s}-x\right),
\end{aligned}
$$

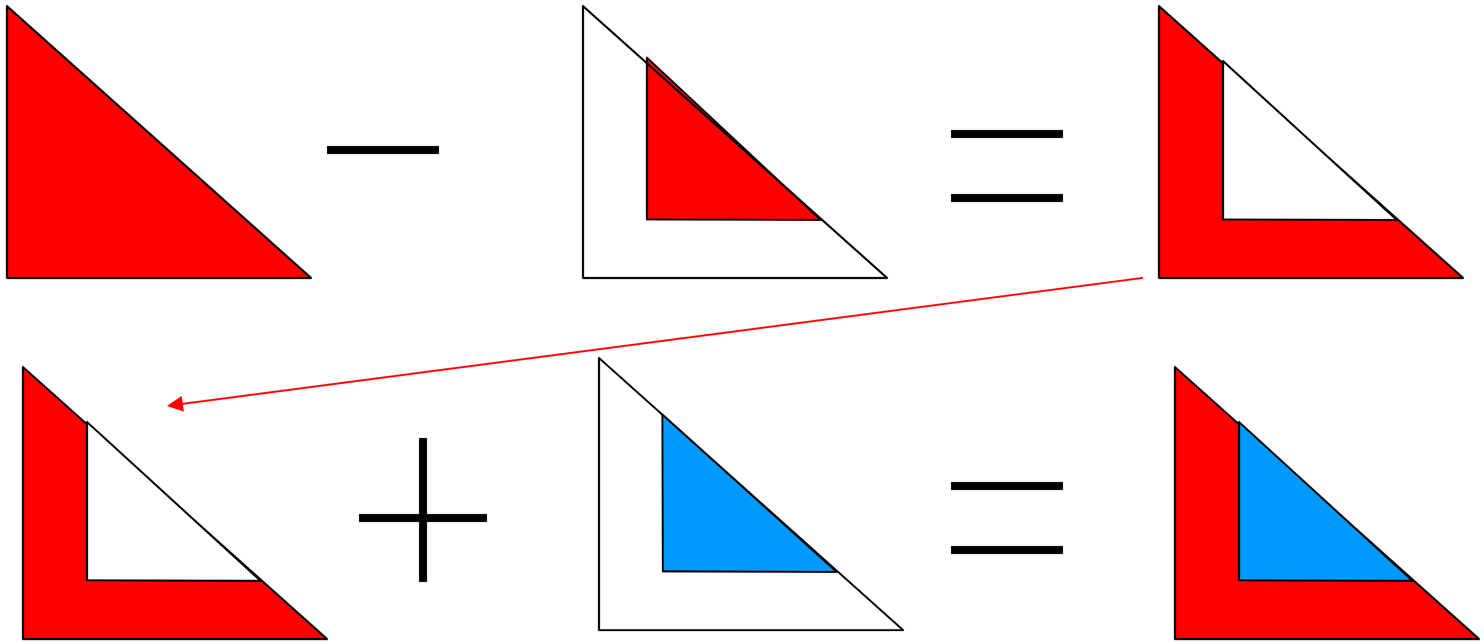
PDF/PS





$$L = \ln(s/\mu^2)$$

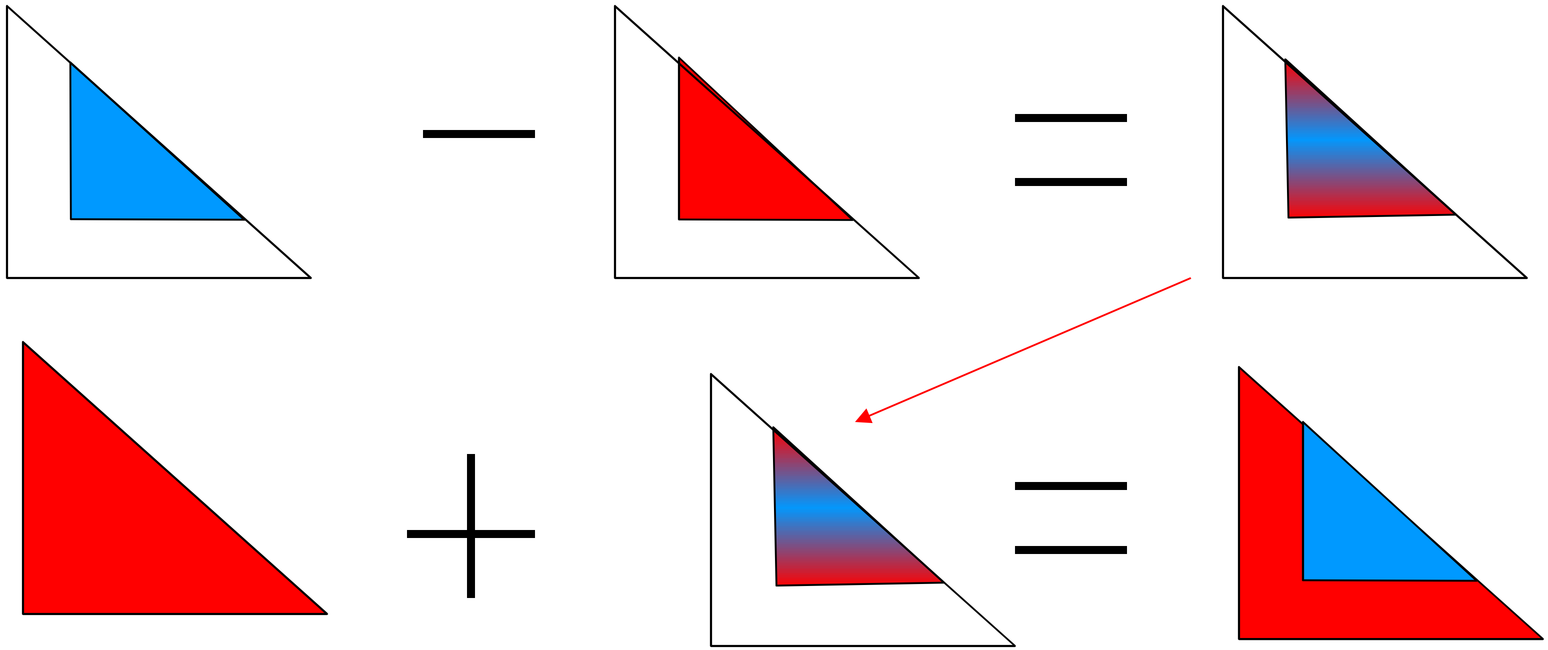
# Phase space slicing

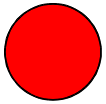


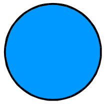
: Collinear (Leading Log) Approx.

: Exact Matrix Elements

# Leading Log Subtraction




: Collinear (Leading Log) Approx.


: Exact Matrix Elements

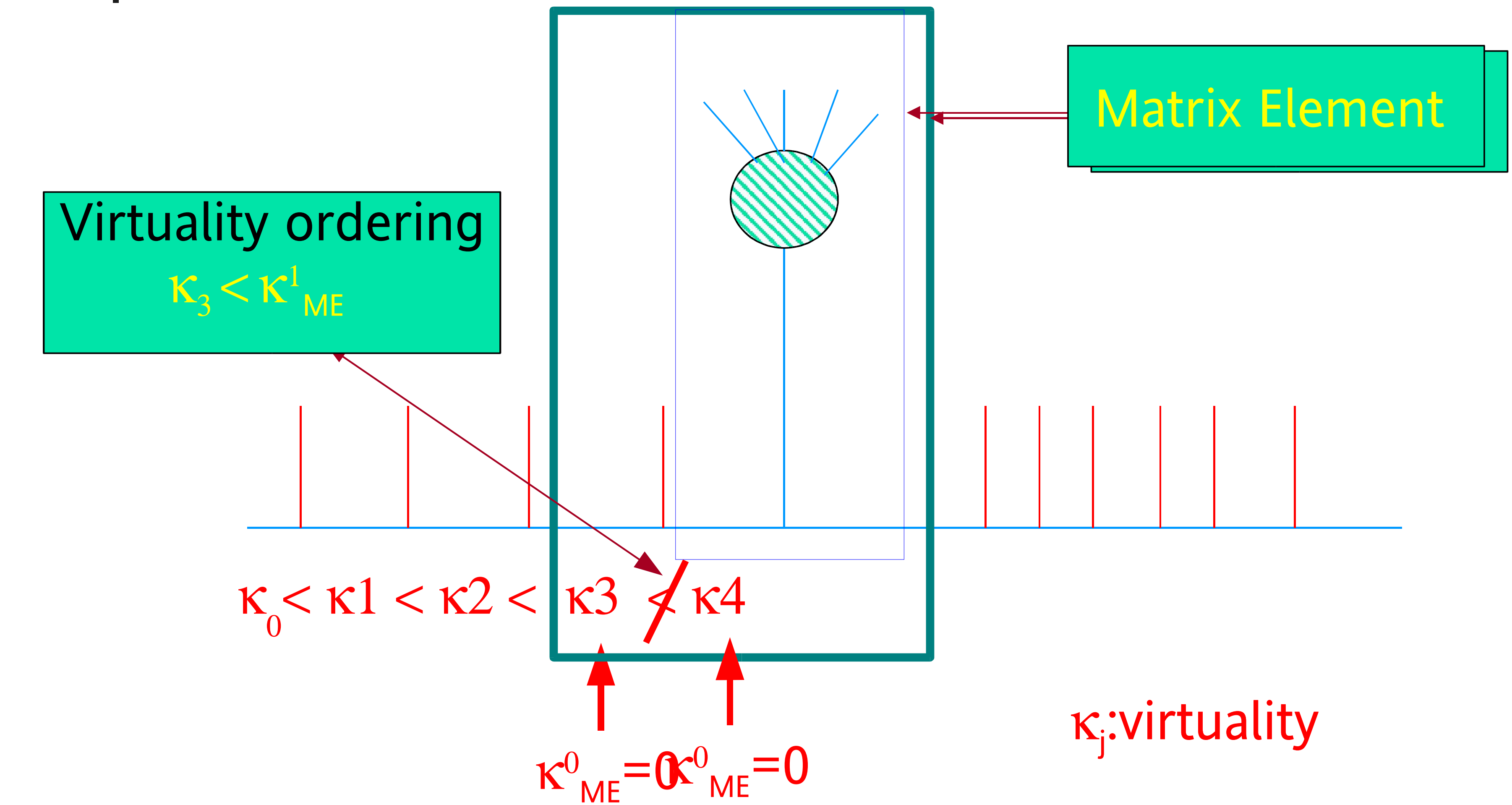
# Double Counting Problem

Double Counting

$$\sigma_{NLO} = [\sigma_{tree}(1 + \delta_V + \delta_{s/c}) + \sigma_{vis}] \otimes \boxed{\text{PDF/PS}}$$

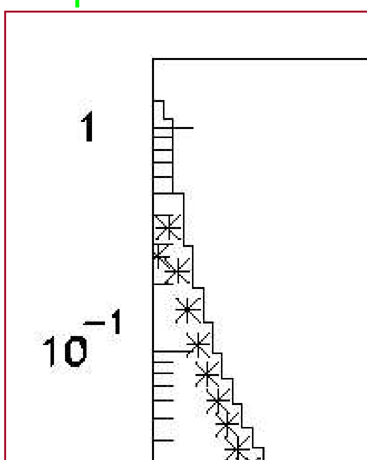
$$\boxed{\frac{1}{\varepsilon_{IR}} f_c \frac{\alpha_s}{2\pi} \mathrm{P}(x)}$$

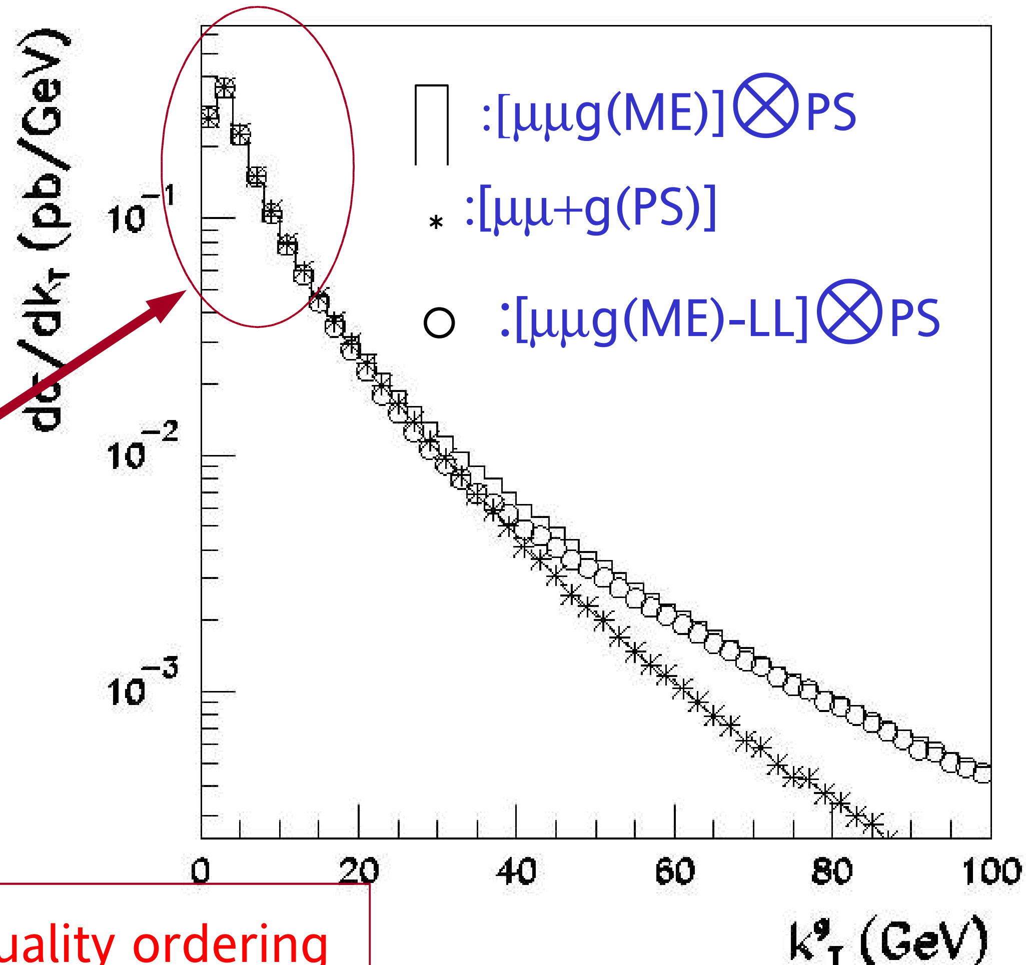
# Double Counting Rejection



# $k^g_T$ Test

- Process : $u\bar{u}\rightarrow\mu^+\mu^-$(+gluon) in $p\bar{p}$ collision
- Cuts: $\sqrt{s_{\mu\mu}}>40$GeV $k_T^g>1$ GeV



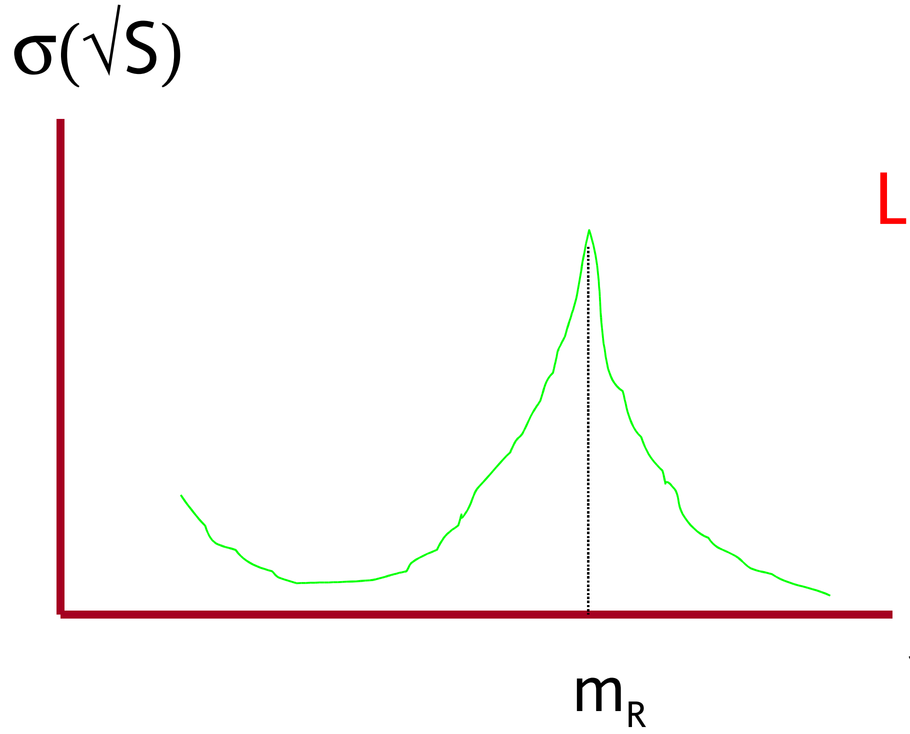




w/o virtuality ordering

# Parton Shower

forward evolution

Low efficiency?




$$\sqrt{S}=\sqrt{x_1 \cdot x_2 \cdot S_0}$$

# Parton Shower:
# x-deterministic Method

$Q^2$-evolution ← Sudakov form factor $\Pi(Q_2^2, Q_1^2)$

After the evolution (n-times)

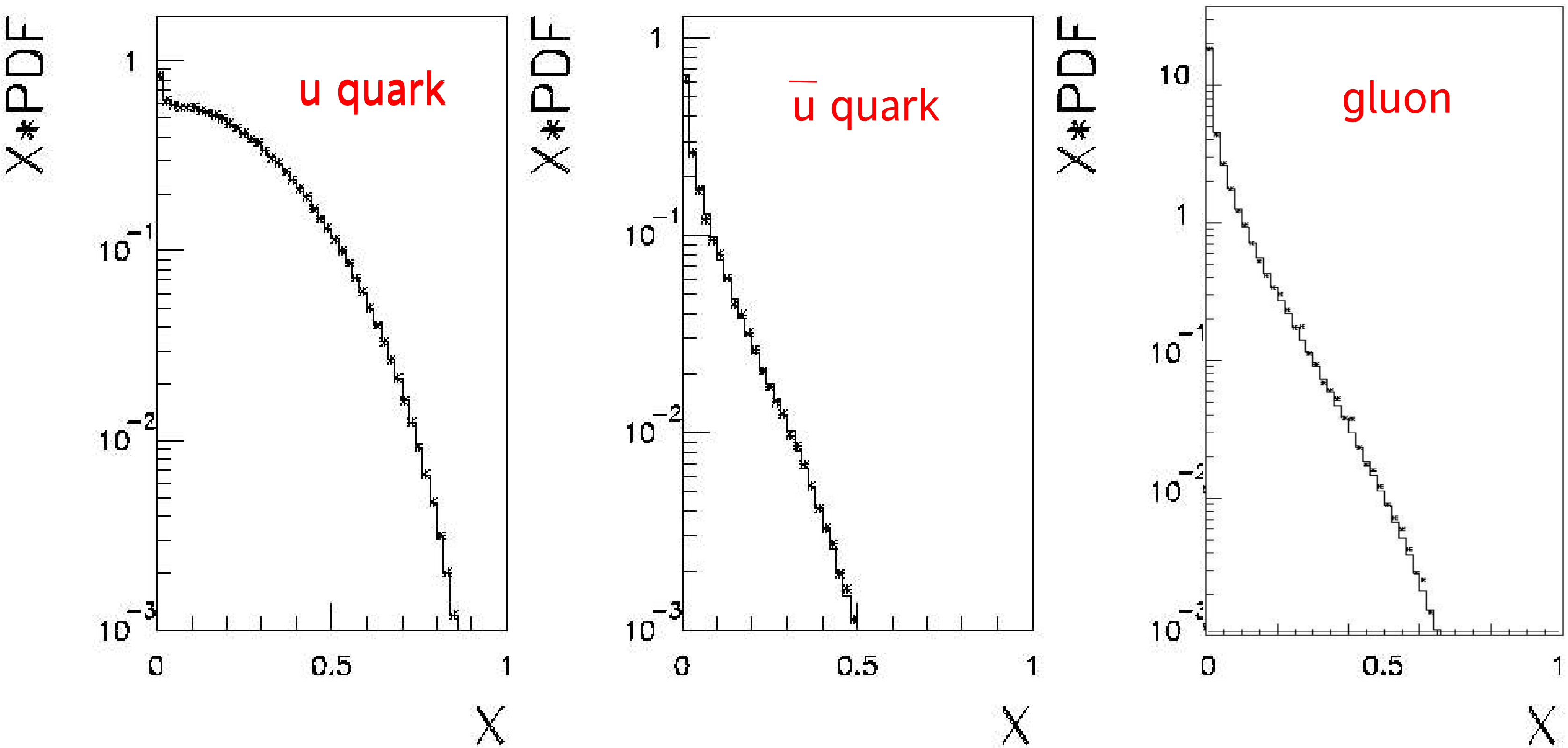
$$W = \frac{1}{W_0} \prod_{i=1}^{n} P(x_i) F_{PDF}(x_0) dx_i dx_0 \delta\left( x - x_0 \prod_{j=1}^{n} x_j \right) dx$$

$$= \frac{1}{W_0} \prod_{i=1}^{n} P(x_i) F_{PDF}(\tilde{x}_0) \frac{dx_i dx}{\prod_{j=1}^{n} x_j}$$

x-deterministic

$$W_0 = \prod_{j=1}^{n} \int_0^1 P(x_j) dx_j, \qquad \tilde{x}_0 = \frac{x}{\prod_{j=1}^{n} x_j}$$

# Parton Shower:Test




□ : Cteq5L　　* : PS　　Q2: 5GeV→100GeV

# NLL Parton Shower

## ✓Splitting function

$$P_{ij}(\alpha_s, z) = \frac{\alpha_s}{2\pi} P_{ij}^{(0)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(1)}(z)$$

i,j : quark or gluon

LL order

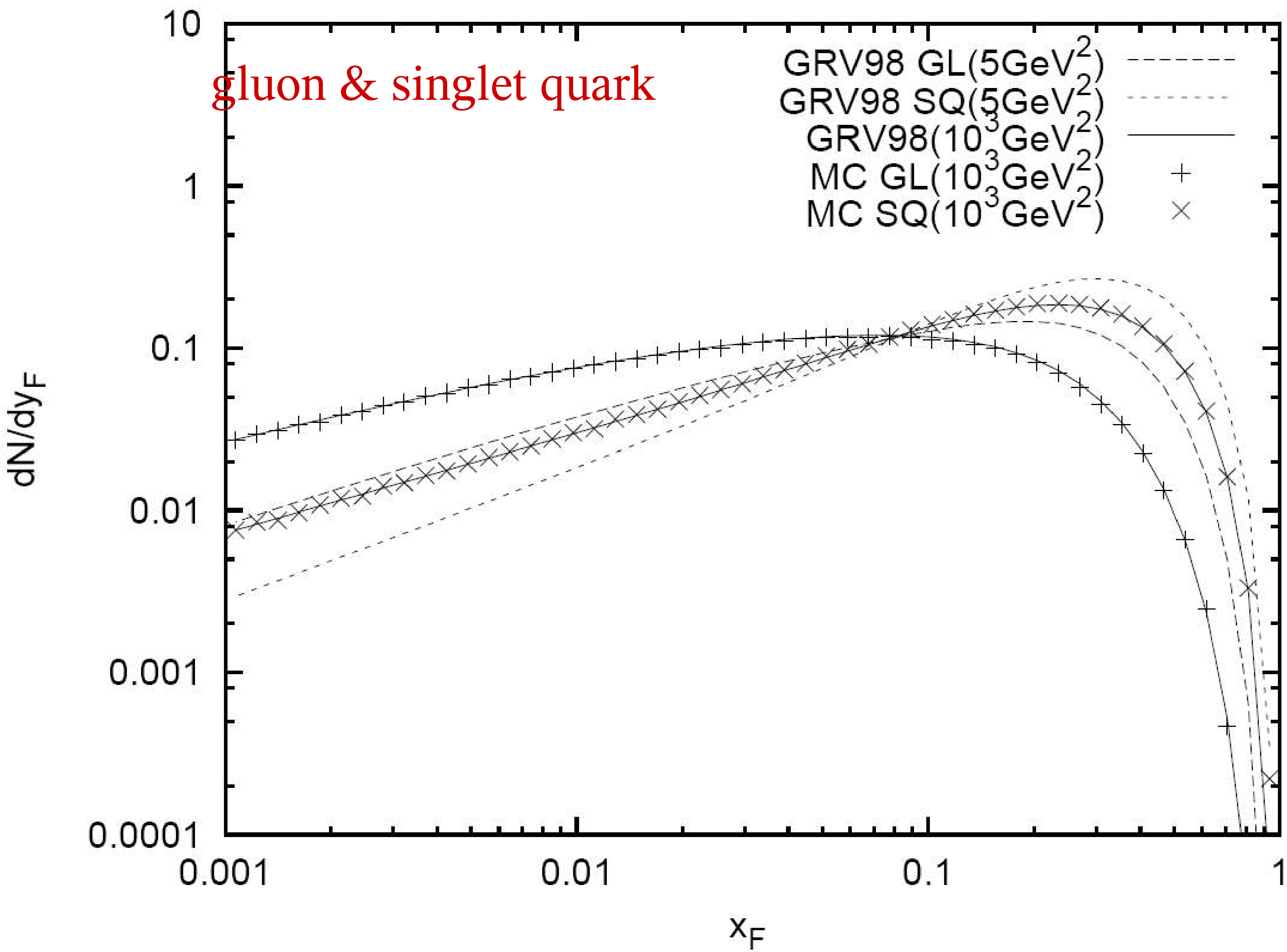
NLL order

$$P_q(\alpha_s, z) = P_{qq}(\alpha_s, z) + P_{gq}(\alpha_s, z),$$

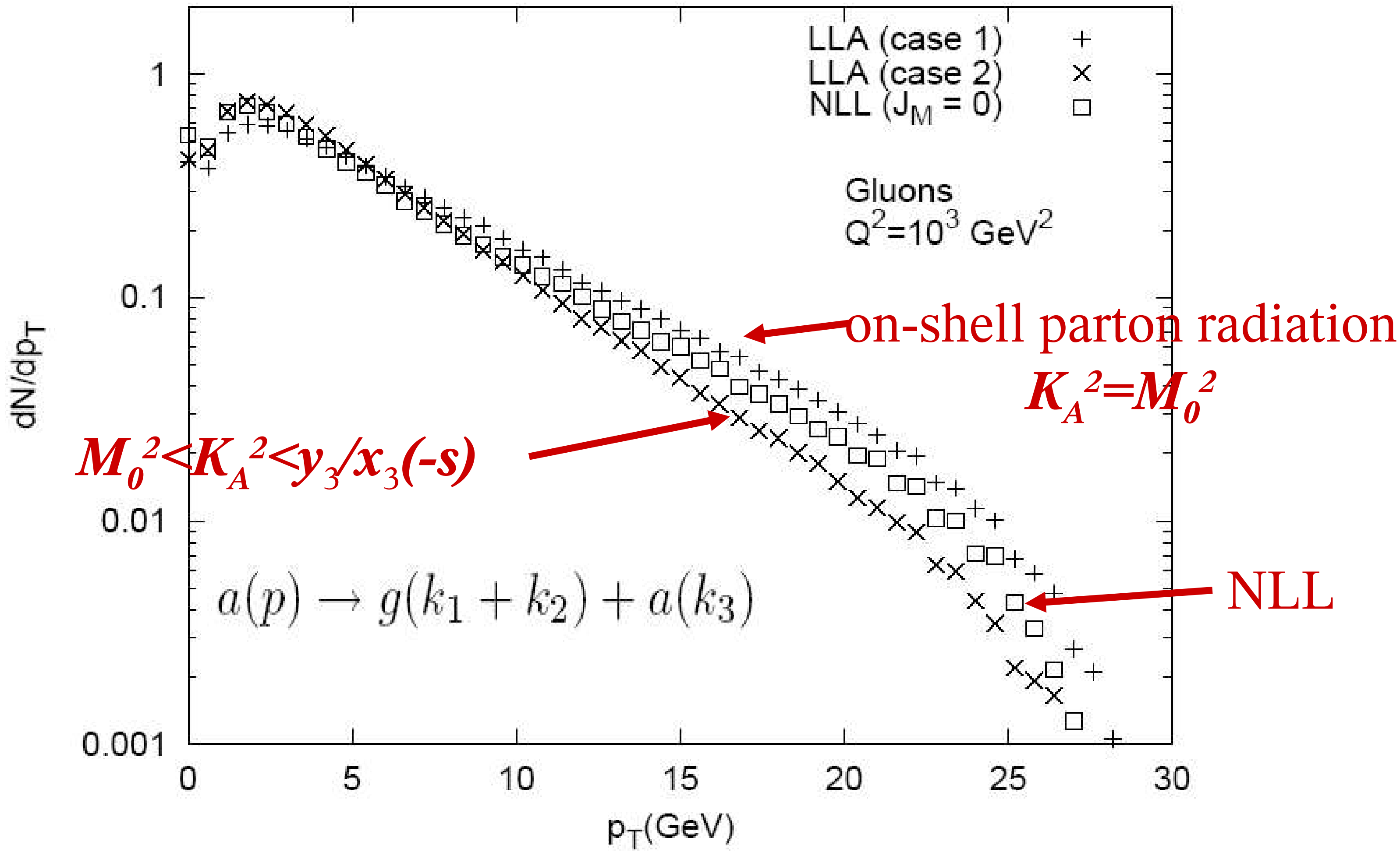
$$P_g(\alpha_s, z) = 2N_f P_{qg}(\alpha_s, z) + P_{gg}(\alpha_s, z)$$

# ✓Numerical results

H. Tanaka, PTP **110** (2003) 963.



# $p_T$ distribution Gluon



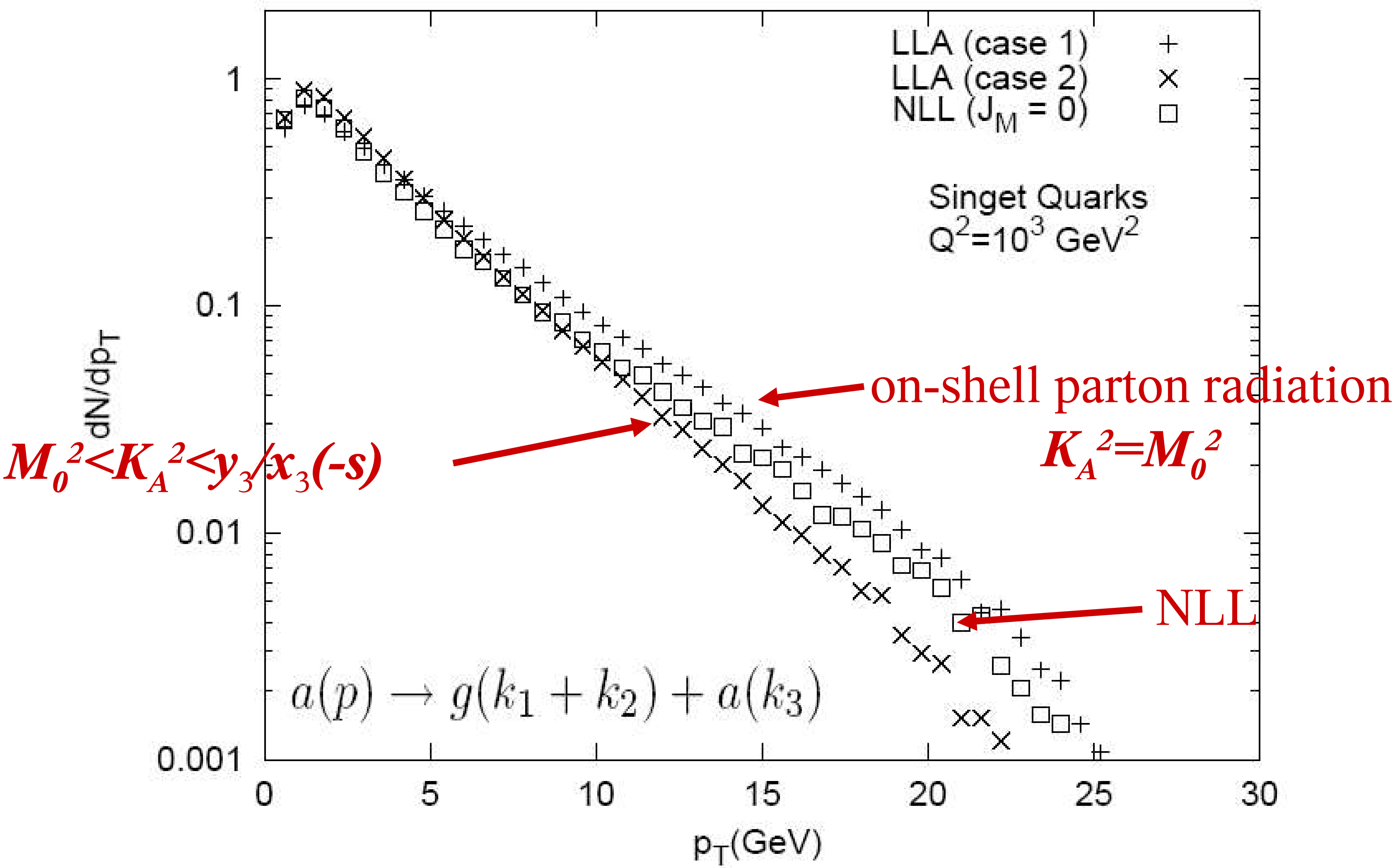

$$p_T^2 = x_3 y_3 \left[ p^2 + \frac{-s}{x_3} - \frac{K_A^2}{y_3} \right]$$

$K_A^2 = (k_1+k_2)^2$, $s = k_3^2$ and $p_T^2 = \vec{k}_{3T}^2$

$y_3 = 1 - x_3$

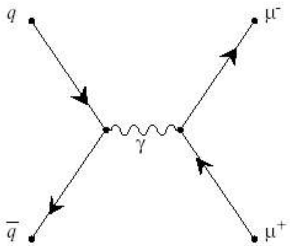
H. Tanaka, PTP **110** (2003) 963.

# $p_T$ distribution : singlet quarks

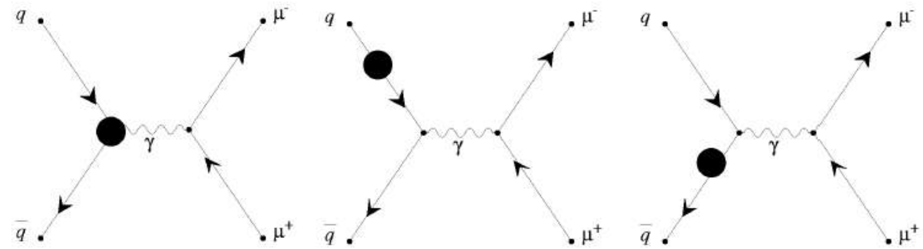



H. Tanaka, PTP **110** (2003) 963.

# Drell-Yan Process

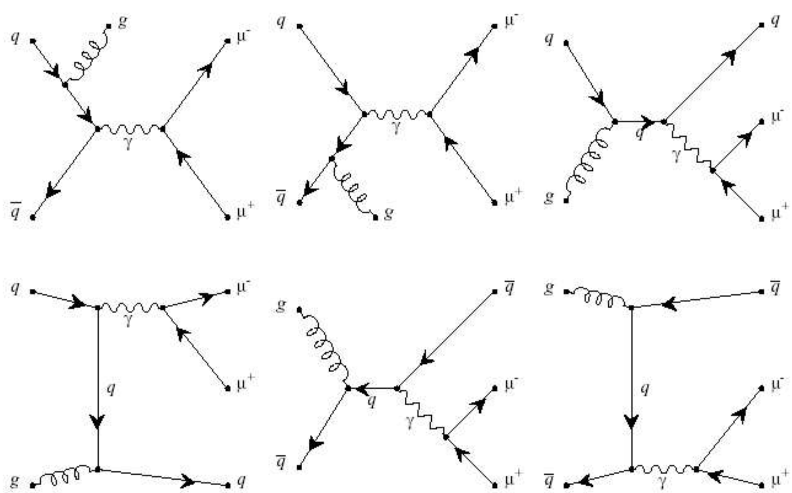



Born diagram




Loop diagrams

Real radiation diagrams



# Drell-Yan Process

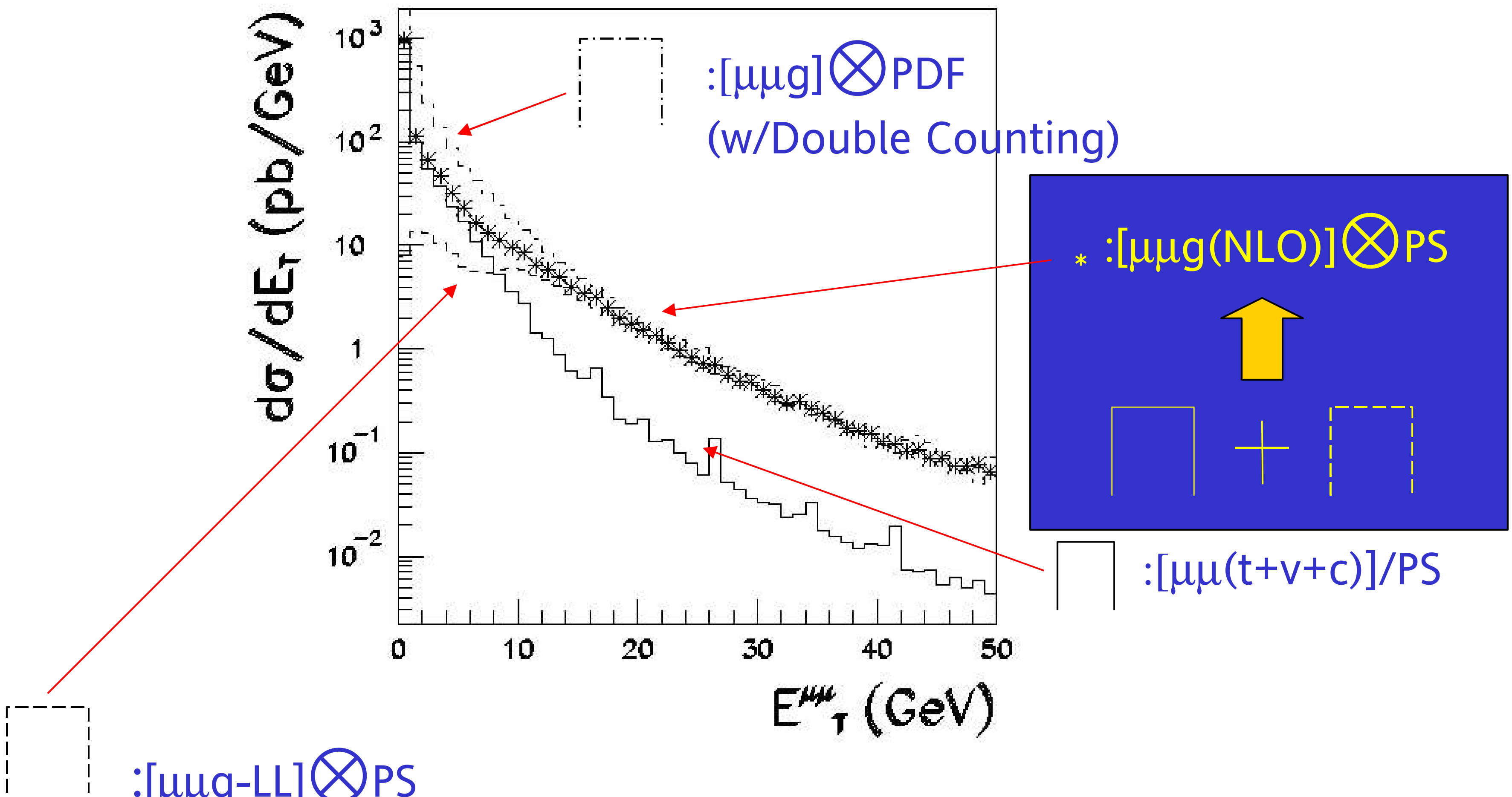
## Parameters

- Process: $p\bar{p} \rightarrow \mu^+\mu^-$
- $\sqrt{s}_{pp}$=2 TeV
- PDF : Cteq5L ($E_{scale}=\sqrt{s}_{\mu\mu}$)
- Cuts: $\sqrt{s}_{\mu\mu}$>10GeV

## Results

- $\sigma_{tree}$=1.026 nb
- $\sigma_{NLO}$=1.288 nb (K-factor=1.25)
- $\sigma_{P/S}= \sigma_{LL\text{-}Sub}$
- $Q^2_c$ independent

# $E_T$ of μ-pair system



# Summary

(1) Matrix Elements

◦Automatic generation by GRACE

(2) Parton Shower

◦x-deterministic PS at LL

◦NLL-PS is under development

(3) Soft/Collinear treatment

◦LL-subtraction method

(4) Application

◦Drell-Yan process

◦W,Z+jets processes in future