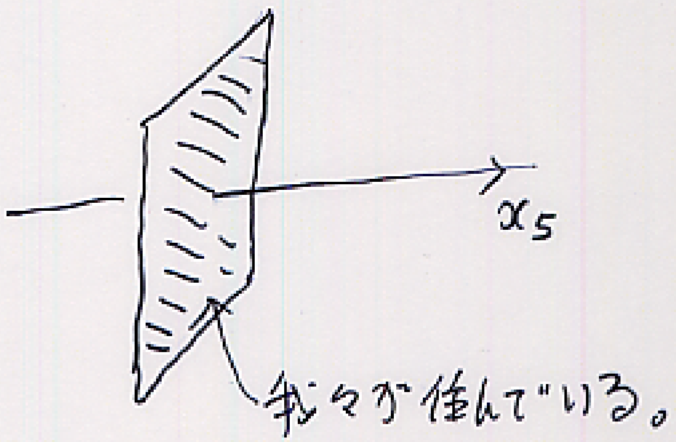
THE BRANE WORLD

1914~

By Nordström

Imamura, Watari, T.Y.

Watari, T.Y.



Neutrino Mass

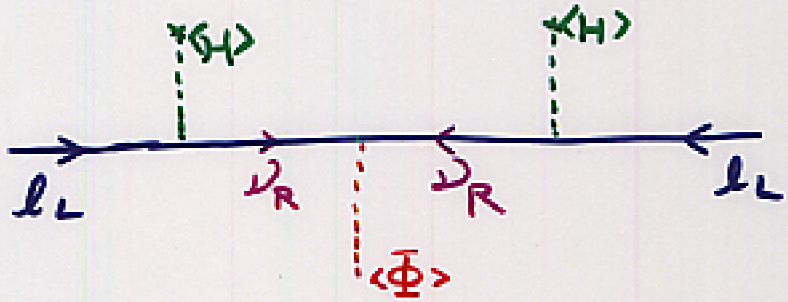
$$\mathcal{L} \sim \frac{1}{M_R} l_L l_L \langle H \rangle^2$$

$$l_L = \begin{pmatrix} e \\ \nu \end{pmatrix}_L$$

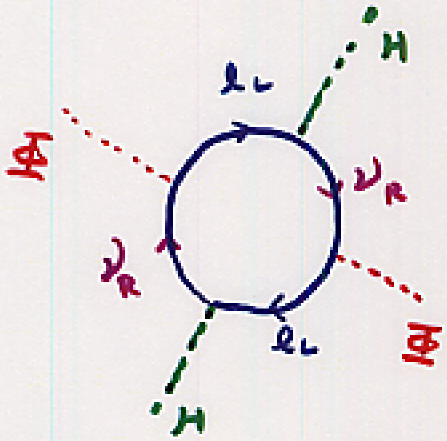
THE SEESAW MASS

$$m_\nu \sim \frac{1}{M_R} \langle H \rangle^2$$

$$m_\nu \simeq 0.1 \text{ eV} \rightarrow M_R \simeq 10^{15} \text{ GeV}$$



$$\langle \Phi \rangle \simeq M_R \simeq 10^{15} \text{ GeV}$$



$$\sim \frac{1}{16\pi^2} H^\dagger H \, \Phi^\dagger \Phi$$

$$\Downarrow$$

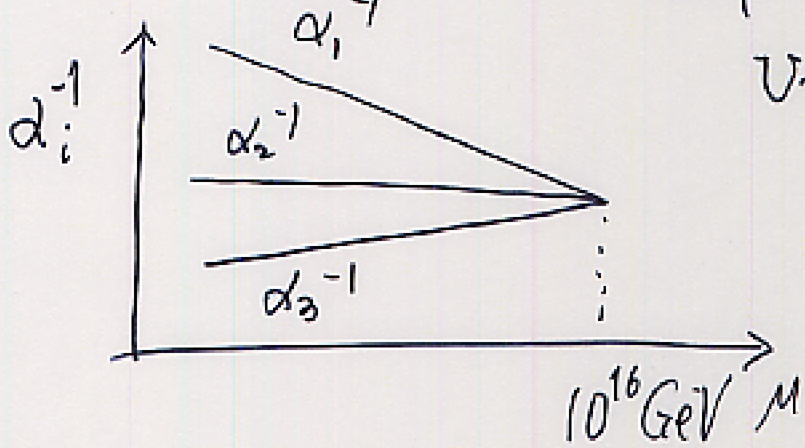
$$\frac{|\langle \Phi \rangle|^2}{16\pi^2} H^\dagger H$$

$$m_H \sim 10^{14}\,\mathrm{GeV} \gg 10^2\,\mathrm{GeV}$$

We need some cancellation mechanism !!

SUSY !!

Coupling Unification



$$\begin{cases} \mathcal{H}(x,\theta) = \mathcal{H}(x) + \psi_H(x)\,\theta \\ \text{chiral multiplet} \\ \bar{\mathcal{H}}(x,\theta) = \bar{\mathcal{H}}(x) + \psi_{\bar{H}}(x)\,\theta \end{cases}$$

Higgs mass term

$$W = m_H \cdot \bar{H} H \qquad \leftarrow \text{SUSY invariant}$$

WHY $m_H \ll M_{Pl}$?

SUSY is not sufficient.

R symmetry

$$\theta \to e^{i\alpha}\theta$$

$$H(x,\theta) = H(x) + \psi_H(x)\,\theta$$

$$\rightarrow H(x) + \psi_H(x)\, e^{-i\alpha} \cdot e^{i\alpha}\theta$$

$$R: \quad \begin{cases} H(x) \rightarrow H(x) \\ \psi_H(x) \rightarrow e^{-i\alpha}\,\psi_H(x) \end{cases}$$

$$\begin{cases} \bar{H}(x) \rightarrow \bar{H}(x) \\ \psi_{\bar{H}}(x) \rightarrow \psi_{\bar{H}}(x)\, e^{-i\alpha} \end{cases}$$

Mass term is forbidden :

$$\psi_H \psi_{\bar{H}} \rightarrow e^{-i2\alpha}\,\psi_H \psi_{\bar{H}} .$$

$\psi_H, \psi_{\bar{H}}$ are massless.

$\xrightarrow[\text{SUSY}]{}$ $H, \bar{H}$ are massless

$$W = m_H\, H(x,\theta)\, \bar{H}(x,\theta)$$

$$m_H = 0 .$$

$$V \simeq |F_{SUSY}|^2 - 3|\cancel{R}|^2$$

BUT.

$\Lambda_{cos} \simeq 0$:

$$\cancel{SUSY} \simeq \cancel{R}$$

$$m_H \simeq \text{SUSY-Breaking scale}$$

$$\simeq 100\,\text{GeV} \sim 1\,\text{TeV}$$

$\cancel{R}$ term

$$\underset{(2)}{W} = \cancel{m_H}\, \underset{(0)}{H}\, \underset{(0)}{\bar{H}}$$

$$\mathcal{L} = \int \underset{(-2)}{d^2\theta}\, \underset{(2)}{W}$$

R charge

Inconsistent with GUT !

$$\left\{ \begin{array}{ll} H(5) = \begin{pmatrix} H_c \\ H_f \end{pmatrix} & : \ Q_R = 0 \\[2ex] \bar{H}(5^*) = \begin{pmatrix} \bar{H}_c \\ \bar{H}_f \end{pmatrix} & : \ Q_R = 0 \end{array} \right.$$

$$\underset{(2)}{W} = m_c \underset{(0)}{H_c} \underset{(0)}{\bar{H}_c} + m_H \underset{(0)}{H_f} \underset{(0)}{\bar{H}_f}$$

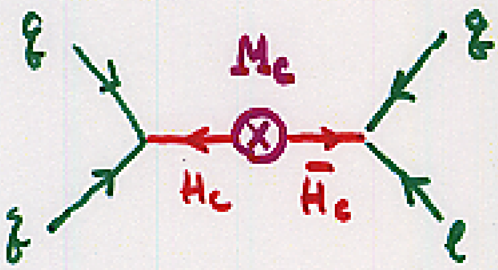
$$m_c \sim m_H \sim R\text{-breaking scale}$$
$$\sim \cancel{\text{SUSY}} \text{ scale}$$

But we need

$m_c \gtrsim 10^{16}$ GeV. $\leftarrow$ proton lifetime.

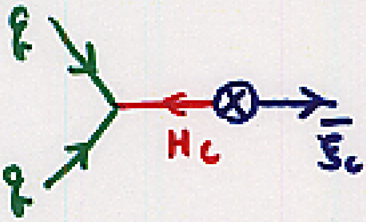
D=5 OPERATOR

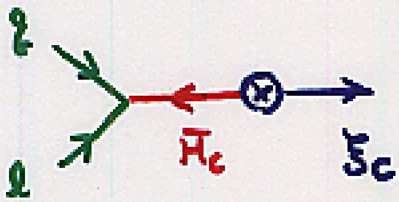
Sakai, T.Y.
Weinberg ('82)



$\tau(p \to K\nu) \lesssim 10^{31}$ years ×

Now Suppressed !!





THIS RESULT IS GENERIC.

" D=6 Proton Decay "

Fujii, Watari

SOLUTION :

T. Y. ('85)

$$\begin{pmatrix} H_c \\ H_f \end{pmatrix} \quad + \quad \bar{\xi}_c \leftarrow \text{color } 3^*$$

(0) (2)

$$\begin{pmatrix} \bar{H}_c \\ \bar{H}_f \end{pmatrix} \quad + \quad \xi_c \leftarrow \text{color } 3$$

(0) (2)

$$W = M_c \underset{(0)}{H_c} \underset{(2)}{\bar{\xi}_c} + \bar{M}_c \underset{(0)}{\bar{H}_c} \underset{(2)}{\xi_c}$$

H_f , $\bar{H}_f$: massless

How to introduce ξ_c and $\bar{\xi}_c$?

$SU(5)_{GUT} \times U(3)_H$

T. Y.

Hyperquarks $Q^i{}_\alpha$ $(5^*, 3)$ $\left(\begin{array}{l}\alpha = 1-3 \\ i = 1-5\end{array}\right.$

$\bar{Q}_i{}^\alpha$ $(5, 3^*)$

$$\langle Q \rangle = \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & 0 & 0 \end{pmatrix}$$

$$\langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & v \\ 0 & & \\ 0 & & \end{pmatrix}$$

$$SU(5)_{GUT} \times U(3)_H$$

$$\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

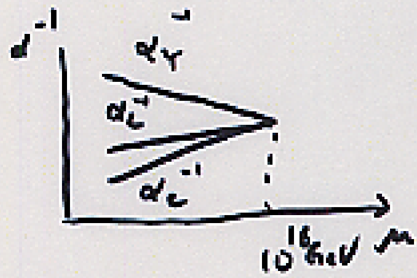
Gauge couplings

$$SU(3)_C \subset SU(3)_{GUT} \times SU(3)_H$$
$$U(1)_Y \subset U(1)_{GUT} \times U(1)_H$$
$$SU(2)_L \subset SU(5)_{GUT}$$

$$\alpha_C \simeq \frac{\alpha_{GUT}}{1 + \alpha_{GUT}/\alpha_{3H}}$$

$$\alpha_Y \simeq \frac{\alpha_{GUT}}{1 + \frac{1}{15}\alpha_{GUT}/\alpha_{1H}}$$

$$\alpha_2 = \alpha_{GUT}$$



" Unification "

$$\alpha_{1H} \sim \alpha_{3H} \gg \alpha_{GUT}$$

$$\alpha_c \simeq \alpha_L \simeq \alpha_Y$$

THE HYPERCOLOR $U(3)_H$ IS IN STRONG COUPLING REGION !

Introduce A pair of Q^6_α and $\bar{Q}^\alpha_6$.

$$
\begin{array}{cc}
 & Q^6 \\
\mathbf{5}^* \left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right. &
\begin{array}{c} Q^1 \\ Q^2 \\ Q^3 \\ Q^4 \\ Q^5 \end{array}
\end{array}
\qquad\qquad
\begin{array}{cc}
\bar{Q}_6 & \\
\begin{array}{c} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \\ \bar{Q}_4 \\ \bar{Q}_5 \end{array} &
\left. \begin{array}{c} \\ \\ \\ \\ \end{array} \right\} \mathbf{5}
\end{array}
$$

$$
\langle Q \rangle = \begin{pmatrix} v & & & \\ & v & & 0 \\ & & v & \end{pmatrix} \; ; \quad \langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & v \\ & 0 & \\ & & \end{pmatrix}
$$

Global Symmetry : $SU(6)_L \times SU(6)_R$

$$\Downarrow$$

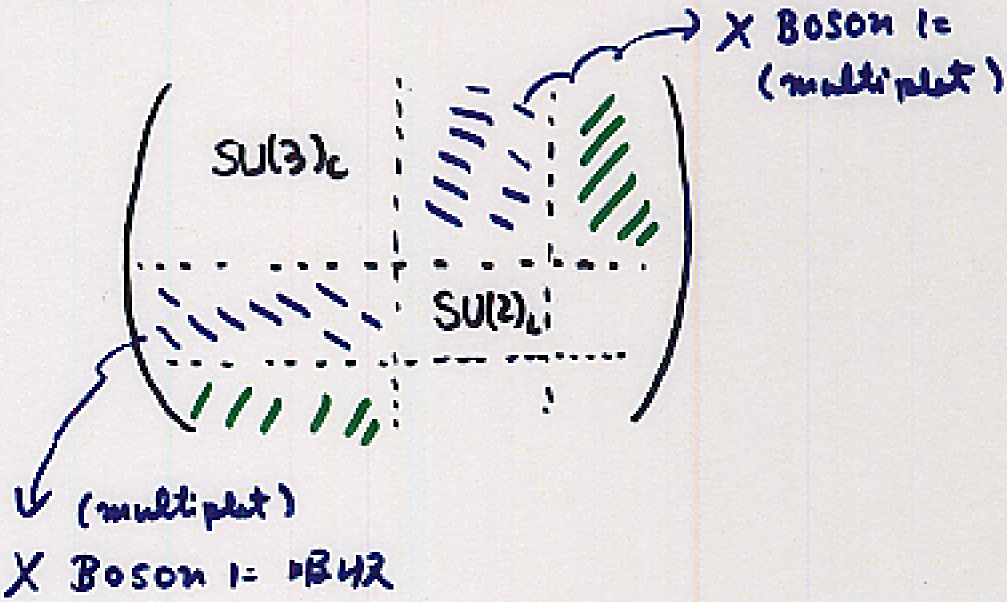
$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

Gauge Symmetry :

$$SU(5)_{GUT} \times U(3)_H$$

$$\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

Nambu - Goldstone multiplets



が massless として のこる.

color (3 + 3*)

ξ_c $\bar{\xi}_c$

H_c $\bar{H}_c$ と mass をくむ.

$H_f + \bar{H}_f$ が massless として のこる.

But more N-G multiplets appear,

since $G = SU(6)_L \times SU(6)_R$.

$\mathcal{N}=1$ $\underbrace{}^{\text{SUSY}}$ HYPER-COLOR SU(3)

$$Q^i_\alpha \qquad \bar{Q}^\alpha_i \qquad \alpha = 1,2,3 \quad i = 1 - N$$

$$\mathcal{L} = \int d^4\theta \, \mathcal{K}$$

$$\mathcal{K} = Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} Q$$

Global Symmetry : $U(N)_L \times U(N)_R$

$\mathcal{N}=2$ SUSY HYPER-COLOR SU(3)

$$Q^i_\alpha \,, \quad \bar{Q}^\alpha_i \qquad X^\beta_\alpha$$

$$\mathcal{L} = \int d^4\theta \, \mathcal{K} + \int d^2\theta \, W + h.c$$

$$W = g \, Q_\alpha X^\alpha_\beta \bar{Q}^\beta$$

Global Symmetry : $U(N)_{L+R}$!!

$\mathcal{N}=2$ SUSY $U(3)_H$ THEORY

Izawa, T.Y.

$$W = Q^i_\alpha X^\alpha_\beta Q^\beta_i + Q^i_\alpha X_0 Q^\alpha_i - 3V^2 X_0$$

FI - F-term consistent with $N=2$ SUSY.

We have a unique vacuum

$$\langle Q \rangle = \begin{pmatrix} v & & \\ & v & \\ & & 0 \end{pmatrix} \; ; \quad \langle \bar{Q} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & 0 \end{pmatrix}$$

Global Symmetry: $U(6) \to SU(3) \times U(3)$

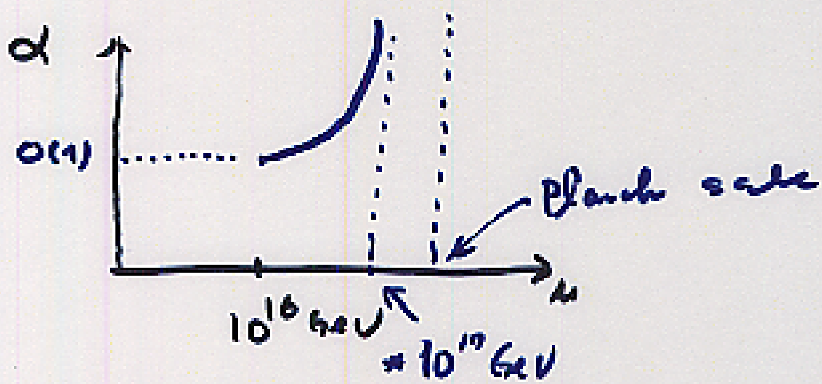
: Gauge symmetry:

$$SU(5)_{GUT} \times U(3)_H \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

We solved the problem, but new problems arise.

① $\alpha_H \gg \alpha_{GUT}$

α_H blows up below M_{pl}.



② Why $\mathcal{N}=2$ Theory for the hypercolor sector?

① → Cut-off Scale $M_* \sim 10^{17}$ GeV.

The Brane World.

solves the problems.

③ WHY $\underline{SU(5) \times U(3)}$?

④ Why $Q_\alpha^{\,i}$ $\bar{Q}_i^{\,\alpha}$ $\left(\begin{matrix} \alpha = 1-3 \\ i = 1-6 \end{matrix} \right)$?

⑤ Why $g_{iH}^2 \gg g_5^2$

Brane World Solves
All Problem.

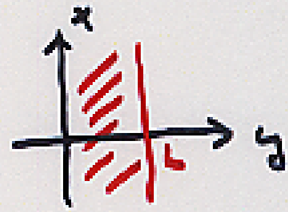
Imamura, Watari, T. Y.

$M_* \ll M_{pl}$ suggests
a higher dimensional theory.
Witten ('96)

$$\mathcal{L} = M_*^3 \int d^4x\, dy \sqrt{-g^{(5)}}\, R_{(5)}$$

$$g^{(5)} = \left(\begin{array}{c|c} g^{(4)} & \\ \hline & 1 \end{array} \right)$$

$$= \underset{\substack{\parallel \\ M_{pl}^2}}{M_*^3 L} \int d^4x \sqrt{-g}\, R$$

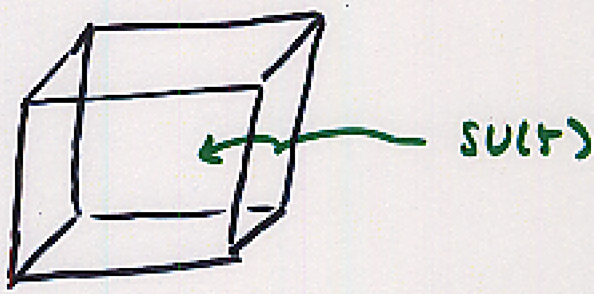


$$M_* L = \left(M_{pl} / M_* \right)^2$$

If $L \gg M_*^{-1} \rightarrow M_{pl} \gg M_*$

The Planck scale is only an effective scale.

Bulk SU(5) GUT.



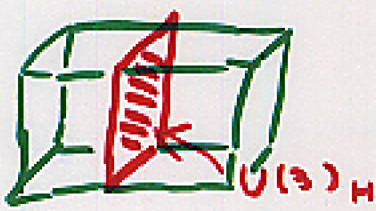
$$\mathcal{L} = \frac{M_*}{g_0^2} \int d^4x\, dy\; W_\alpha W^\alpha$$

SU(5) gauge multiplet

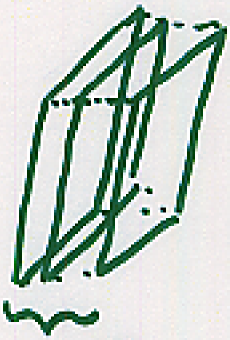
$$= \underbrace{\frac{M_* L}{g_0^2}}_{\frac{1}{g_{GUT}^2}} \int d^4x\; W_\alpha W^\alpha$$

$$\therefore \quad g_{GUT}^2 = \frac{g_0^2}{M_* L} \ll 1 \qquad \text{if } M_* L \sim \text{large}$$

$g_{GUT}^2 \ll g_H^2 \quad \rightarrow \quad$ HYPER-COLOR SECTOR on 3-brane



String Theory D-brane



N枚.

U(N) Gauge Theory

3枚の D-3 brane

Bulk SU(5)GUT と より高次元
時空の D-brane (5枚)

10次元 String Theory
(type II B)

D7-D3 brane

5枚の D7 ⊕ 3枚の D3 branes.

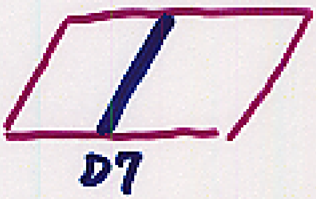
The type ⅡB String Theory
with D7-D3 Brane.

	0	1	2	3	4	5	6	7	8	9
D7		○	○	○	○	○	○	○		
D3		○	○	○						

SUSY Charges Q^i

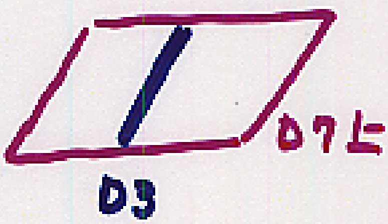
Q^i $(1-32)$ $\underline{32}$

D7 半分に落ちる $\underline{16}$



$\mathcal{N}=4$ SUSY

D3 さらに半分に落ちる $\underline{8}$



$\mathcal{N}=2$ SUSY

D3 Brane 上の $U(3)_H$ は $\mathcal{N}=2$ SUSY を持つ

D7 上の SU(5)GUT Gauge 理論

$\mathcal{N}=4$ SUSY

A_μ	χ_1	Σ
	χ_2	Σ'
	χ_3	Σ''
	χ_4	
	fermions	scalar bosons

$$SO(1,9) = SO(1,3) \times SO(6)$$

16 の super charge Q^i は

4つの Q_α^i に分解される

$$Q_\alpha^i = (\underline{2}, \underline{4}) \quad \text{complex}$$

SO(1,3) の spinor

SO(6) の $\underline{4}$

$$A_\mu \xrightarrow{Q^i} \begin{matrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{matrix}$$

残したいのは (A_μ, χ_1) の対

$\mathcal{N}=1$ SUSY

Orbifold をする。　$SO(6) \simeq SU(4)$ の subgroup

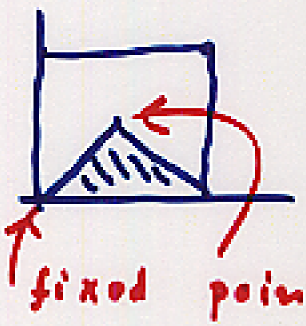
$$\begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix}$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} \to \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix}$$

θ_1 のみが残る。

T^6/Z_4 orbifold を構成する。

T^2



Two fixed point

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \to \begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$x_2, x_3\ x_4$ は fixed point で 0 になる。

zero mode をもつのは x_1 のみ。　x_1 のみ生きのこる。

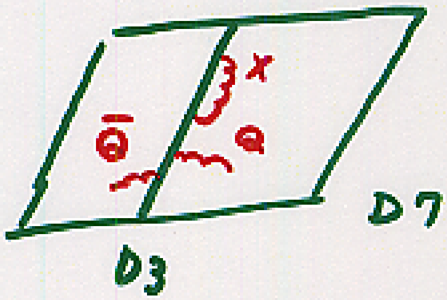
→ $(A_\mu\ x_1)$ $\mathcal{N}=1$ SUSY.

Fixed Point 上に quark, lepton ($5^* + 10$)

Higgs chiral multiplet をおく。

- R-symmetry は 8-9 plane の回転.
- FI term の origin は B-field.
- Q^α_i, $\bar{Q}^i_\alpha$ は D3 − D7 string

  X^α_β, X_0 は D3 − D3 string



T^6/Z_{12} Orientifold

Watari, T.Y.

But Fixed Point 上の $5^* + 10$ は、今の所

理解できていない。 fixed point の特殊性か?

5枚の D7-Branes.

$$U(5) = SU(5)_{GUT} \times U(1)$$

B-L と考える.

Anomaly Cancellation

↳ ν_R is required !!!

Neutrinos are massive.

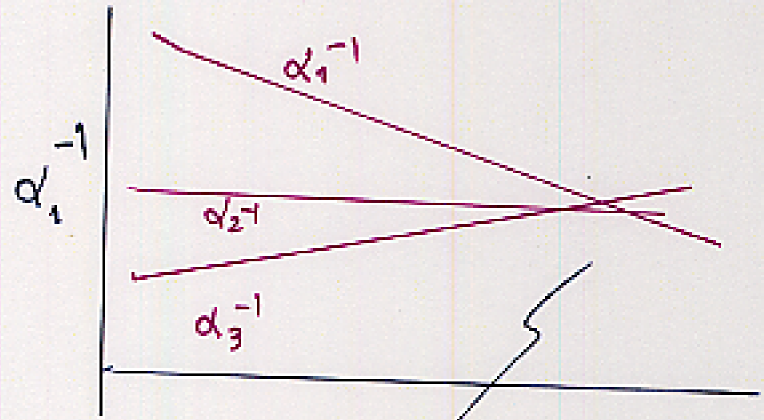
Seesaw mechanism

T.Y. (79)

G.M.R.S.

TEST OF THE THEORY.

1. $\alpha_c \; , \; \alpha_1 \lesssim \alpha_2$ at $\mu \simeq M_{GUT}$



$\hookrightarrow \; \alpha_3^{-1}, \alpha_1^{-1} > \alpha_2^{-1}$

GOOD!!

2. R-symmetry ← Anomaly Free

$\mathbb{Z}_{4R}$

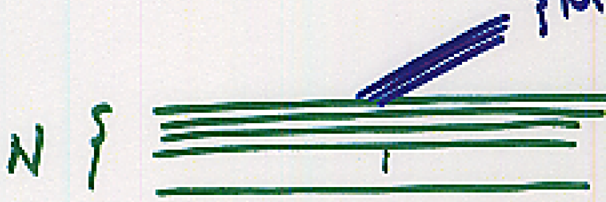
$\oplus$

$(5 + 5^*)_{\text{Matter}}$ at $E \sim 1\,\text{TeV}$

A New Pair of $(5+5^*)$ Family

Brane World

$U(N)$



$U(N) \times U(M)$
Gauge Theory

$$\underbrace{U(5)_{GUT} \times U(3)_H}_{SU(5)_{GUT} \times U(1)_{B-L}}$$

$\Downarrow$

Massive ν

Experimentaly supported.