A New Problem of Virtual Kaluza-Klein graviton exchanges

Tokyo Metropolitan U.
Haruka Namatame

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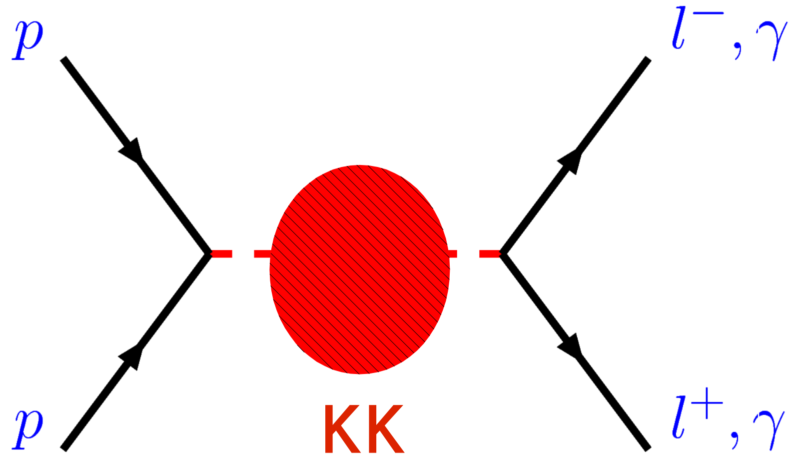
Motivation

"To probe ADD model at LHC"

Virtual Kaluza-Klein graviton exchange processes

↑ **massive particle**



- Analysises have performed **without the decay width !**

But: **Z boson** exchange process

$$\sigma \propto \left[\frac{1}{s - M_Z^2} \right]^2$$

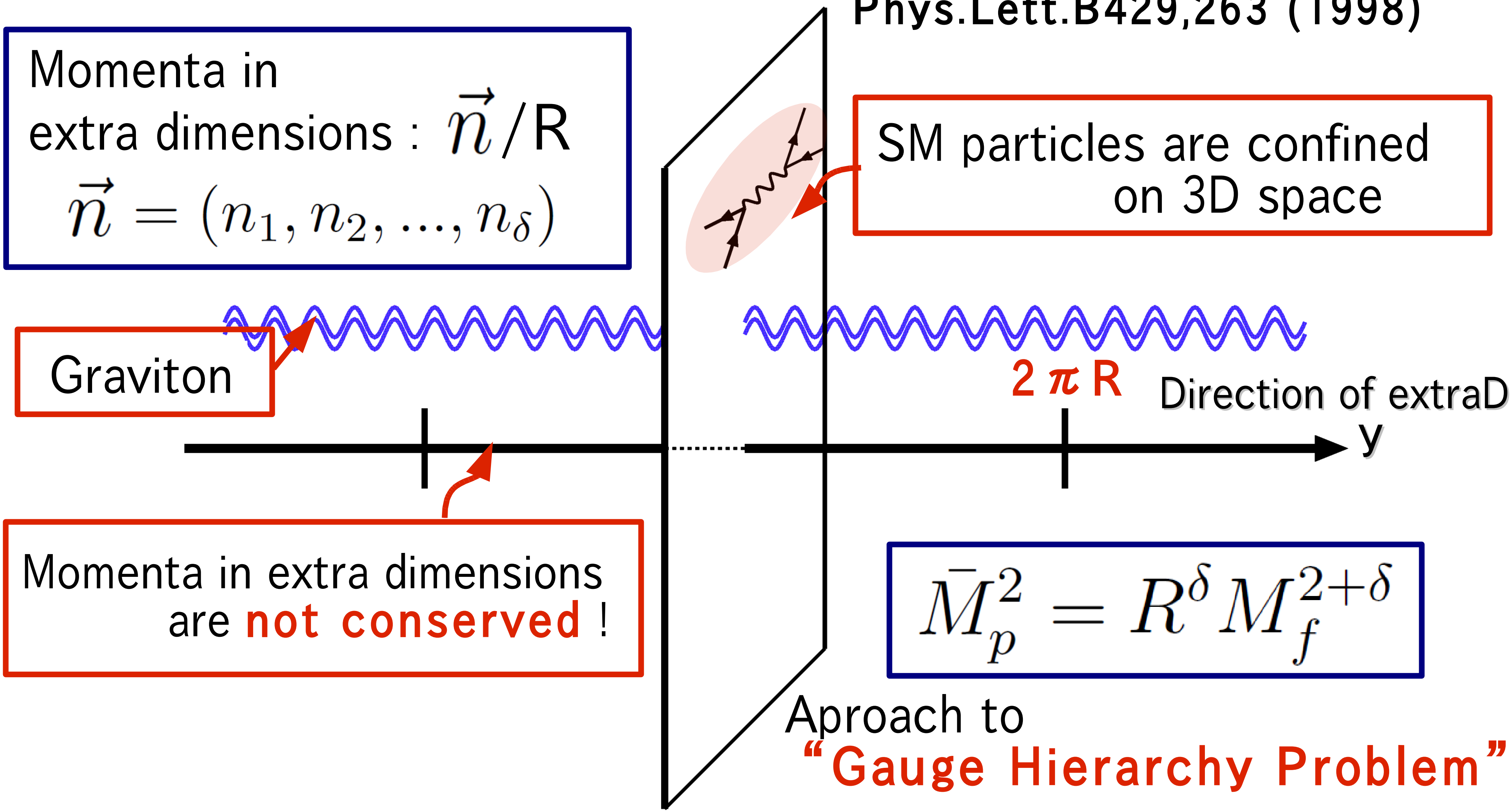
Diverge at $\sqrt{s} = M_Z$

⇒ Why can these cross section get a **finite value ?**

“Large Extra Dimension Scenario”

A model by Arkani-Hamed, Dimopoulos and Dvali

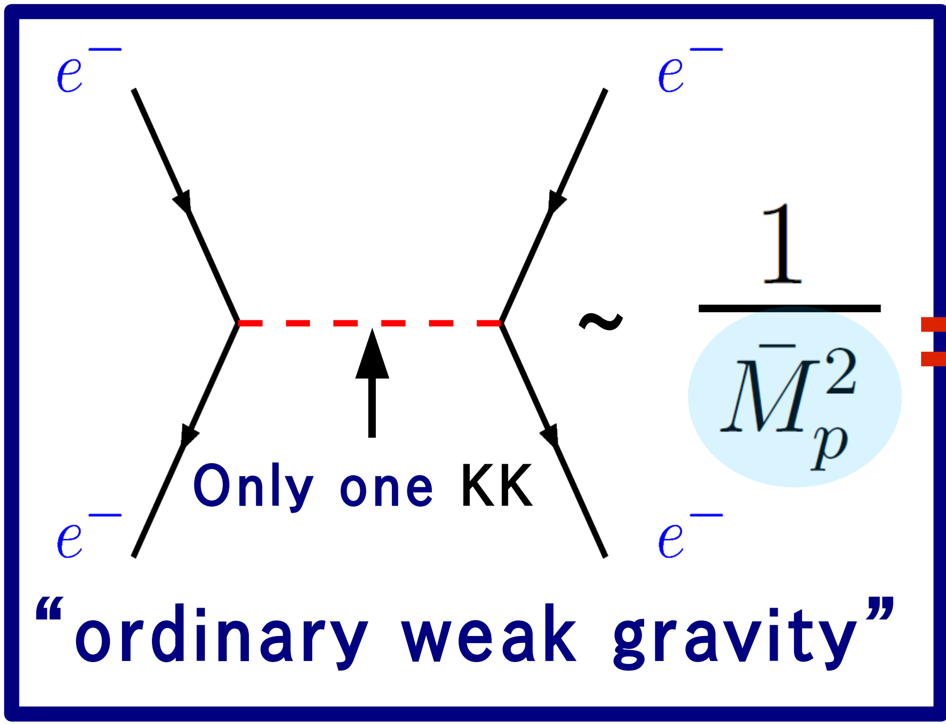
Phys.Lett.B429,263 (1998)

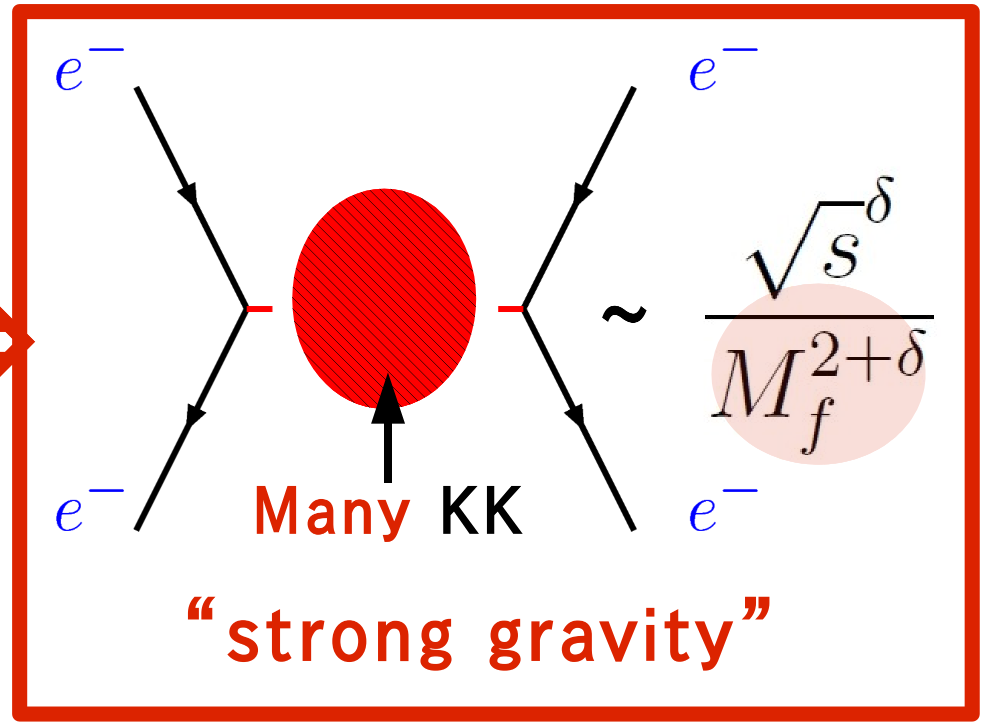


Kaluza-Klein gravitons

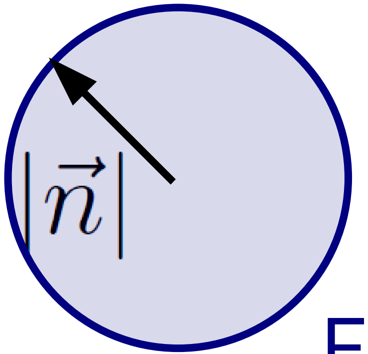
· Infinite number of massive gravitons exist in ADD model !

$$(p^0)^2 - \vec{p}\cdot\vec{p} = \left(\frac{|\vec{n}|}{R}\right)^2 = m^2$$





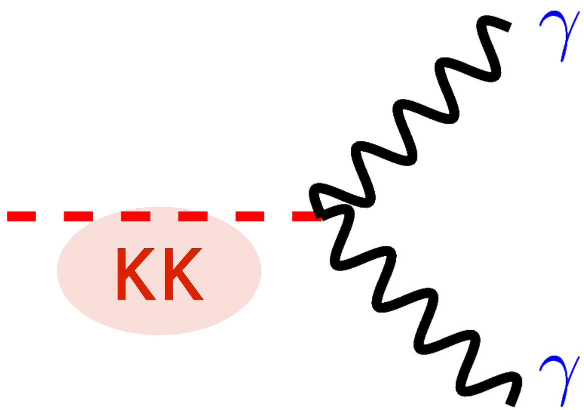
·**Degeneracy** of KK gravitons $\left(m = |\vec{n}|/R \right)$



$$\rho(|\vec{n}|) \propto |\vec{n}|^{\delta-1} = m^{\delta-1} R^{\delta-1}$$

Ex number of KK gravitons of which mass is 100 GeV $\sim 10^{14}$!

·**Decay width** of KK gravitons



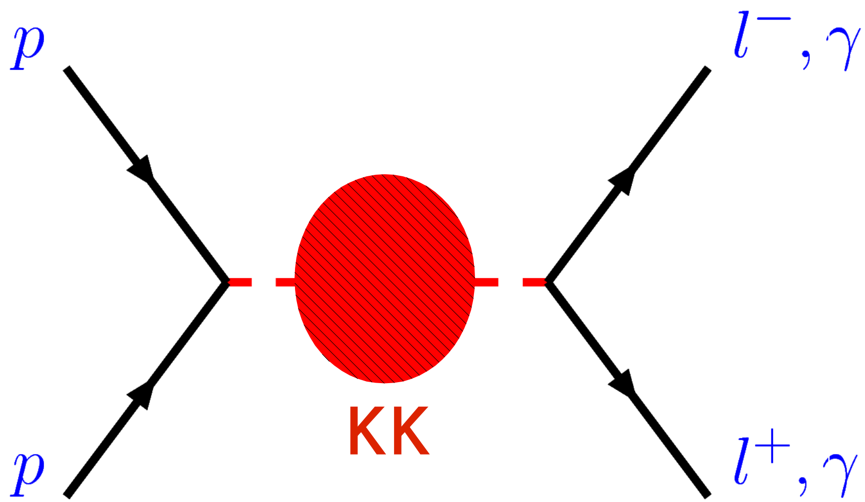
Main decay channels are

$$G \to \gamma\gamma,\, gg,\, f\bar{f}$$

$$\Gamma(m) = N(m)\frac{m^3}{\bar{M}_p^2}$$

Collider Signatures of the ADD model

Virtual Kaluza-Klein graviton exchange processes



$$pp \to l\bar{l}, \gamma\gamma$$

J.L.Hewett,
Phys. Rev. Lett. 82, 4765 (1999)

T. Han, J. D. Lykken and R. J. Zhang,
Phys. Rev. D 59, 105006 (1999)

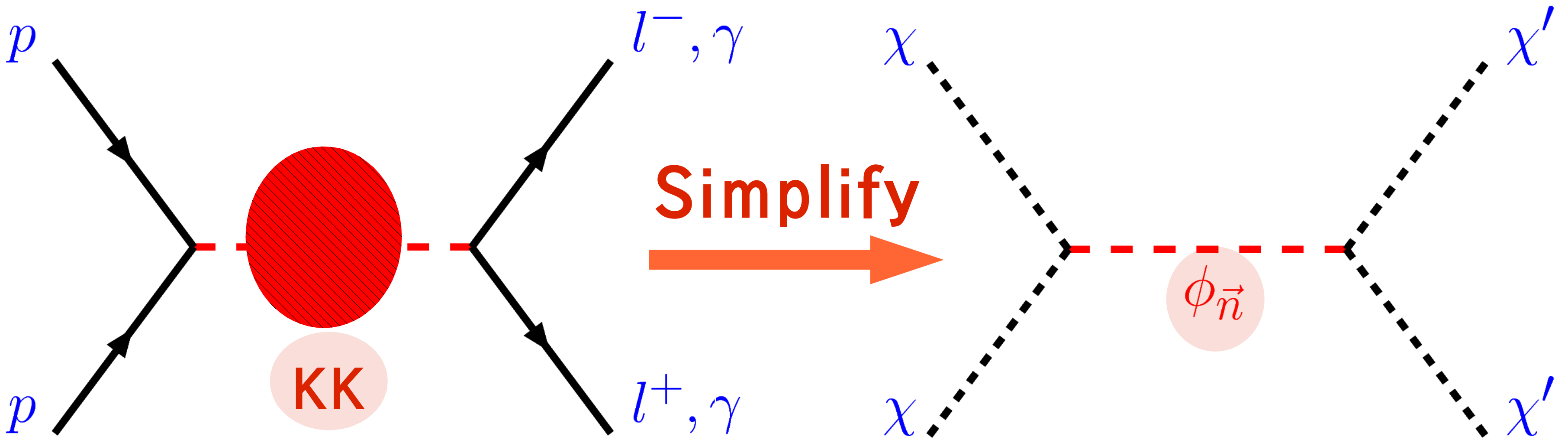
· On-shell KK gravitons must be in the intermediate state

⇒ Resonance effect is important

We should analyse these processes with the **decay width** of **KK gravitons** !

"Toy Model"

- **KK gravitons** → $\phi_{\vec{n}}$: real scalar, mass is $|\vec{n}|/R$
- **Light Fermion** → χ : real scalar, massless

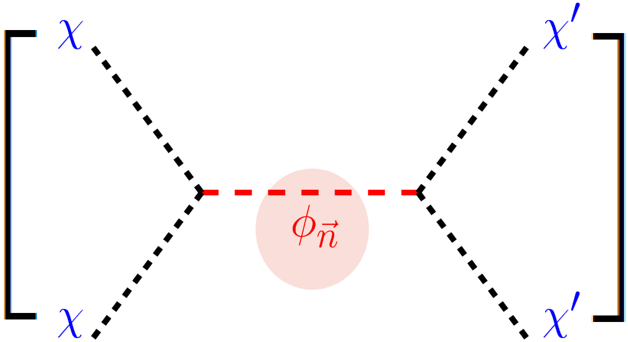


The process **in ADD model**

The process in **toy model**

2-body scattering via KK gravitons

$$\mathcal{M}_\Gamma = \sum_{\vec{n}}$$



$$\rho(m) \sim m^{\delta-1} R^\delta = m^{\delta-1} \frac{\bar{M}_p^2}{M_f^{2+\delta}}$$

$$\sim \left(\frac{g}{\bar{M}_p} s\right)^2 \int_0^{\alpha M_f} dm\, \rho(m) \frac{i}{s - m^2 + im\Gamma(m)}$$

$$\Gamma(m) = \frac{g^2}{128\pi} \frac{m^3}{\bar{M}_p^2}$$

$$\Rightarrow \sigma_\Gamma \propto \frac{s^3 g^4}{M_f^{2\delta+4}} \left| \int_0^{\alpha M_f} dm \frac{m^{\delta-1}}{s - m^2 + im\Gamma(m)} \right|^2$$

·In case $\delta > 2$

$$\sigma_\Gamma \sim \frac{s^3}{M_f^8} g^4 \alpha^{2(\delta-2)} \quad (M_f^2/\bar{M}_p^2, s/\bar{M}_f^2 \ll 1)$$

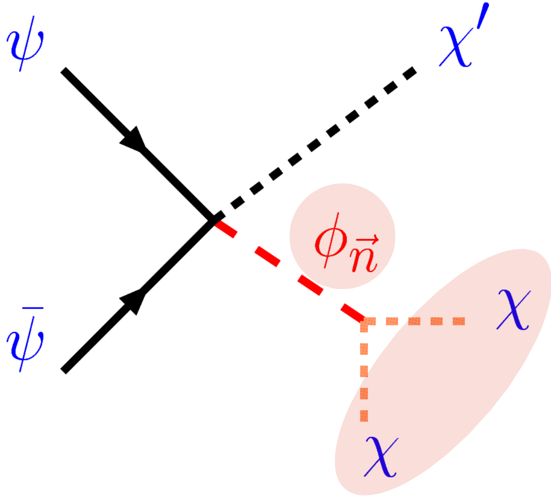
Strongly depend on the cutoff α

Not depend on δ (the number of extra dimensions)

$$\int_0^{\alpha M_f} dm \frac{m^{\delta-1}}{s - m^2 + im\Gamma(m)} \approx -\frac{(\alpha M_f)^{\delta-2}}{\delta - 2}$$

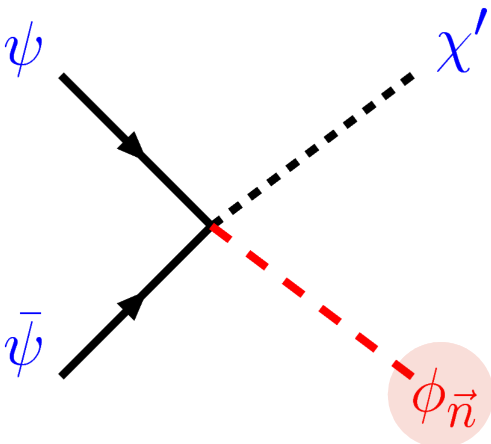
Leading term is **constant** ! (**not depend on** $\sqrt{s}$)

KK graviton productions



$$\sigma_\Gamma \sim \lambda^2 \frac{\sqrt{s}^{2\delta-1}}{M_f^{2(2+\delta)}} \frac{\bar{M}_p^2}{R}$$

$$\left(M_f^2/\bar{M}_p^2 , s/\bar{M}_f^2 \ll 1 \right)$$



$$\sigma \sim \lambda^2 \frac{\sqrt{s}^{\delta}}{M_f^{2+\delta}}$$

$$\left(\sigma = \int_0^{\sqrt{s}} dm\, \rho(m) \sigma(m) \right)$$

Hard to believe this as a **physical result !**

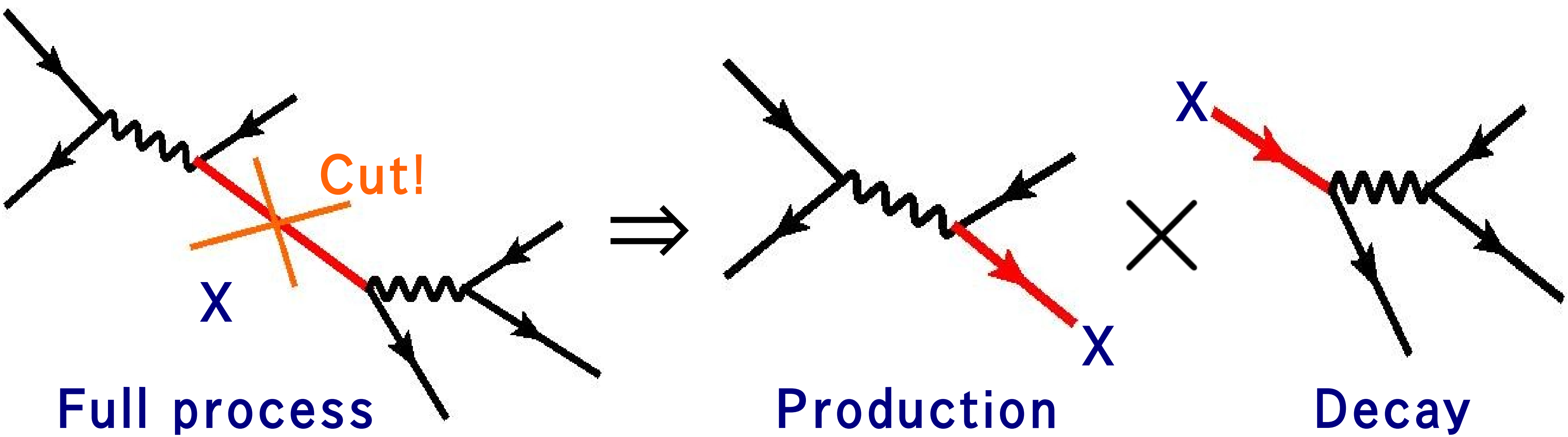
Hints

Narrow-width approximation

· In a process (a particle X is in intermediate state)

$\left\{ \begin{array}{l} \text{X can be on-shell in intermediate state} \\ \text{width of X is enough small } (\Gamma_X/m_X \ll 1) \end{array} \right.$

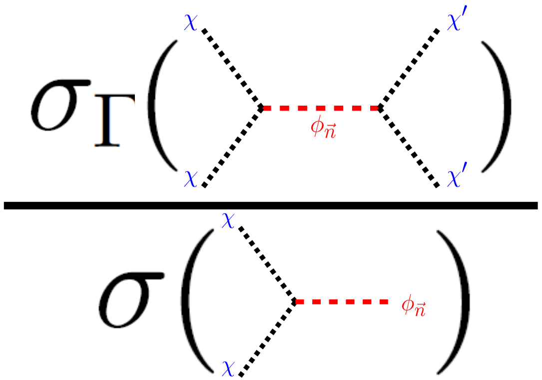
$$\Rightarrow \sigma_\Gamma\left(\begin{array}{l}\text{Full} \\ \text{process}\end{array}\right) \approx \sigma\left(\begin{array}{l}\text{On-shell X} \\ \text{production}\end{array}\right) \times Br\left(\begin{array}{r}\text{Decay channel} \\ \text{of X}\end{array}\right)$$



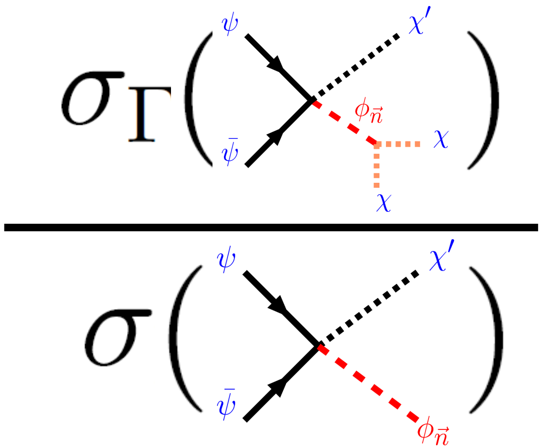
· In case of the **toy model** (or ADD model)

- $\phi_{\vec{n}}$ **can be on-shell** in intermediate state
- **width** of $\phi_{\vec{n}}$ is **enough small** $\left(\frac{\Gamma(m)}{m} \sim \frac{m^2}{\bar{M}_p^2} \ll 1 \right)$

But ! $\left(Br(\phi_n \to \chi\chi) = 1 \right)$



$$\sim g^2 \alpha^{2(\delta-2)} \left(\frac{\sqrt{s}}{M_f} \right)^{6-\delta} \ll 1$$



$$\sim \sqrt{s}^{\delta-1} R^{\delta-1} \gg 1$$

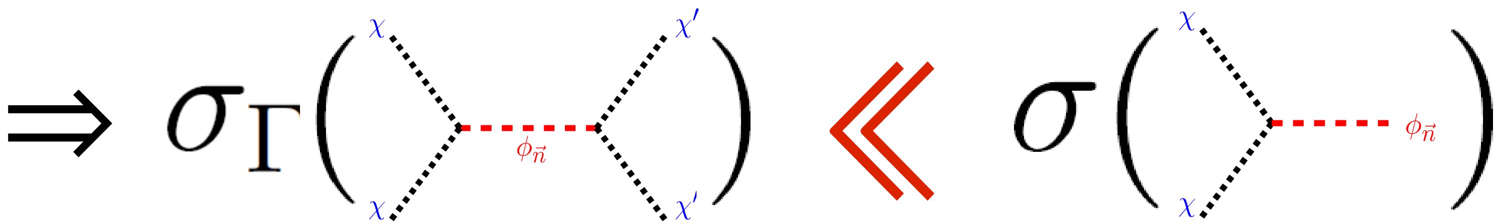
$$= \rho(|\vec{n}| = \sqrt{s} R)$$

Discussion

Why is the approximation not good ?

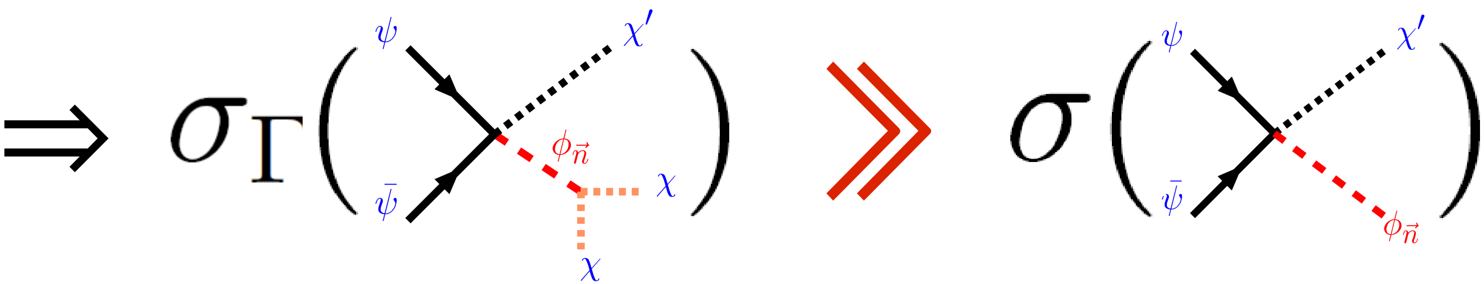
· **Cancellation** of **on-shell contributions**

(only for **2-body scattering**)



· **Interferences** between KK graviton

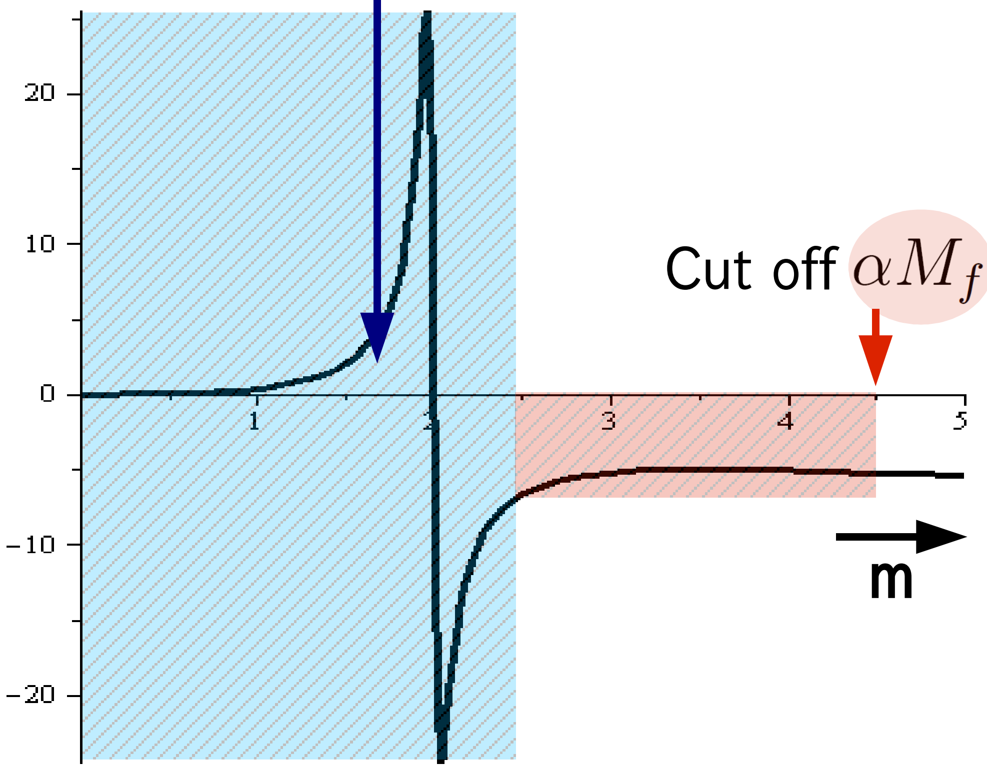
which have **same mass**



· Cancellation of on-shell contributions

$$\sigma_\Gamma \propto \left| \int_0^{\alpha M_f} dm \frac{m^{\delta-1}}{s - m^2 + im\Gamma(m)} \right|^2$$

Canceled out for any $\sqrt{s}$



$$\int_0^{\alpha M_f} dm \frac{m^{\delta-1}}{s - m^2 + im\Gamma(m)}$$

$$\simeq -\frac{(\alpha M_f)^{\delta-2}}{\delta - 2}$$

$$= \int_0^{\alpha M_f} dm \frac{m^{\delta-1}}{-m^2}$$

・**Interferences** between KK graviton which have **same mass**

The effect of **interferences** between $\phi_{\vec{n}}$ which have same mass is **unphysical !**

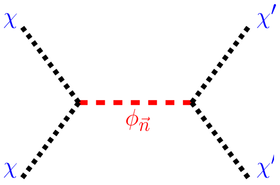
・in case of only **one** particle	$\Rightarrow \mathcal{M}_\Gamma$
	$\Rightarrow \sigma_\Gamma$

・in case of **N** particles	$\Rightarrow \mathbf{N} \times \mathcal{M}_\Gamma$
	$\Rightarrow \mathbf{N}^2 \times \sigma_\Gamma$

This is **not** the **quantum effect !**

- In case of **2-body scattering via KK gravitons**

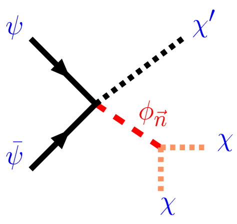
A contribution from **KK gravitons** of which mass is αM_f is **dominant**

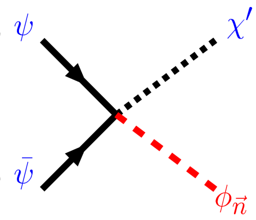
$\sigma_\Gamma\Big($ 

$\Big) \sim \left| \rho(|\vec{n}|=\alpha M_f R) \times \frac{1}{-(\alpha M_f)^2} \right|^2$

- In case of **KK graviton productions**

A contribution from **KK gravitons** of which mass is $\sqrt{s}$ is **dominant**

$\sigma_\Gamma\Big($ 

$\Big) \sim \rho(|\vec{n}|=\sqrt{s}R) \times \sigma\Big($ 

$\Big)$

Contribution from **on-shell KK gravitons**

Conclusion

The cross section of **virtual KK exchange process** can be **huge or small unphysically**

- This problem is a kind of **over counting**

$$\sigma_\Gamma \propto \int_0^{\alpha M_f} dm_1 \int_0^{\alpha M_f} dm_2 \; \rho(m_1)\rho(m_2) \left[\mathcal{M}_\Gamma(m_1)\mathcal{M}_\Gamma{}^\dagger(m_2)\right]$$

Double counting of phase spase in extra dimensions

- If there is **no double counting**, the problem **does not appear**

- An origin of this problem is the **explicit breaking** of **translational symmetry** in **extra dimensions**

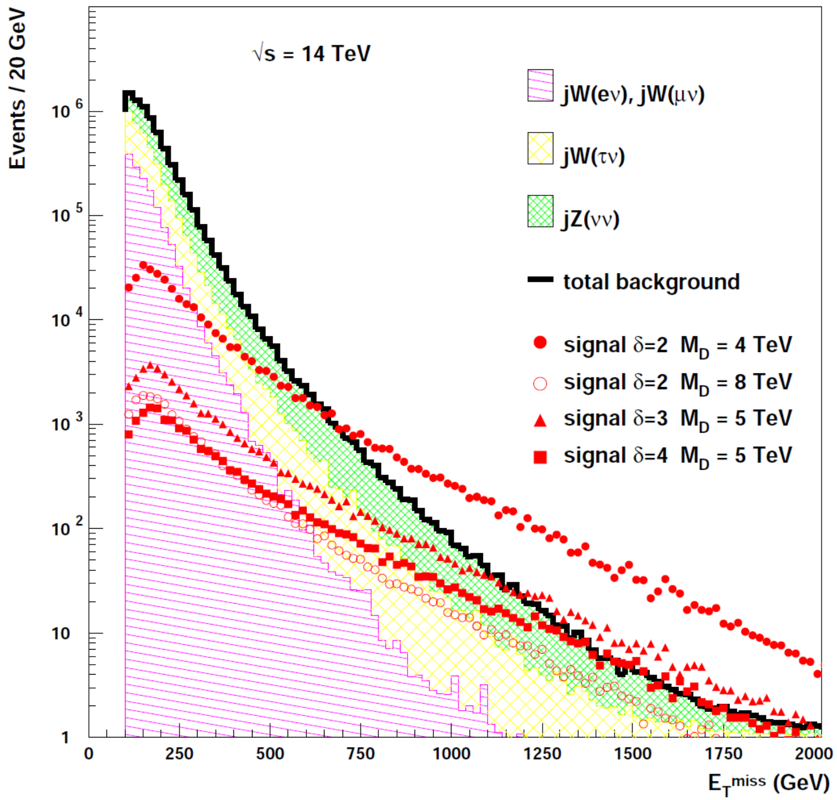
⇒ We need to consider some properties of **brane dynamics**

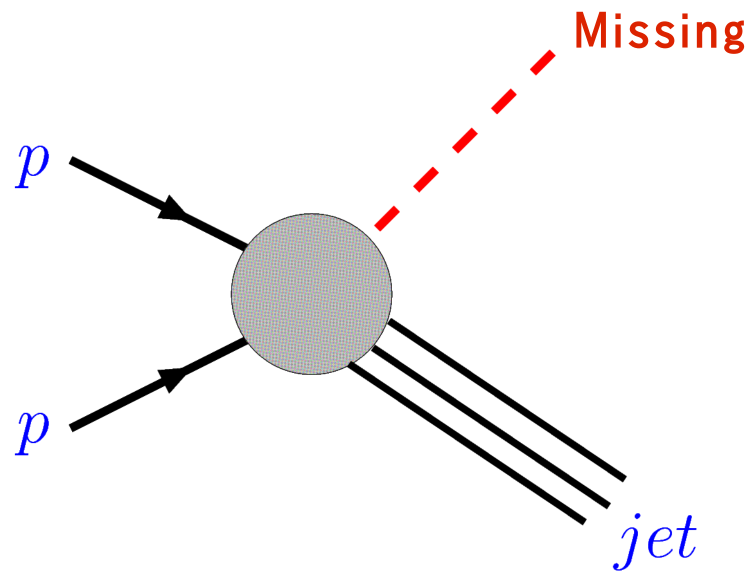
Thank you !

ありがとうございました

Buck up

Signature of ADD at ATLAS

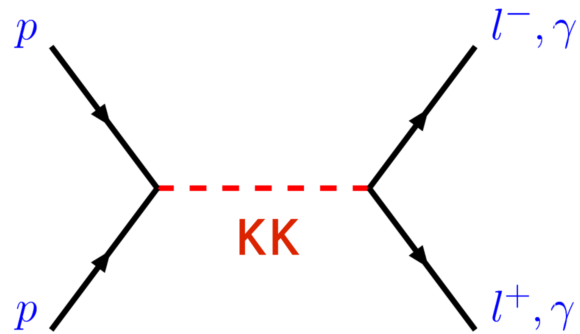




δ	$\sqrt{s} = 14$ TeV 100 fb^{-1}	$\sqrt{s} = 14$ TeV 1000 fb^{-1}	$\sqrt{s} = 28$ TeV 100 fb^{-1}	$\sqrt{s} = 28$ TeV 1000 fb^{-1}
2	9	12	15	19
3	7	8	12	14
4	6	7	10	12

Signature of ADD at ATLAS

channel	n		2	3	4	5
$\gamma\gamma$	luminosiy					
	10 fb^{-1}	M_S^{max} (TeV)	6.3	5.6	5.1	4.9
		S/B	36/18	36/18	39/25	34/13
	100 fb^{-1}	M_S^{max} (TeV)	7.9	7.3	6.7	6.3
		S/B	50/53	62/96	55/72	51/53
l^+l^-	10 fb^{-1}	M_S^{max} (TeV)	6.6	5.9	5.4	5.1
		S/B	33/11	31/8	30/6	30/6
	100 fb^{-1}	M_S^{max} (TeV)	7.9	7.5	7.0	6.6
		S/B	49/48	38/21	36/16	29/6
$\gamma\gamma + l^+l^-$	10 fb^{-1}	M_S^{max} (TeV)	7.0	6.3	5.7	5.4
	100 fb^{-1}	M_S^{max} (TeV)	8.1	7.9	7.4	7.0



$$\begin{bmatrix} |\mathcal{M}_1(m)|^2 & \mathcal{M}_1(m)\mathcal{M}_2{}^\dagger(m) & \mathcal{M}_1(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_1(m)\mathcal{M}_4{}^\dagger(m') \\ \mathcal{M}_2(m)\mathcal{M}_1{}^\dagger(m) & |\mathcal{M}_2(m)|^2 & \mathcal{M}_2(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_2(m)\mathcal{M}_4{}^\dagger(m') \\ & & |\mathcal{M}_3(m')|^2 & \mathcal{M}_3(m')\mathcal{M}_4{}^\dagger(m') \\ & & \mathcal{M}_4(m')\mathcal{M}_3{}^\dagger(m') & |\mathcal{M}_4(m')|^2 \end{bmatrix}$$

Dominant terms

All of each 4 terms are **same**

Ordinary quantum corrections

$$\begin{bmatrix} |\mathcal{M}_1(m)|^2 & \mathcal{M}_1(m)\mathcal{M}_2{}^\dagger(m) & \mathcal{M}_1(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_1(m)\mathcal{M}_4{}^\dagger(m') \\ \mathcal{M}_2(m)\mathcal{M}_1{}^\dagger(m) & |\mathcal{M}_2(m)|^2 & \mathcal{M}_2(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_2(m)\mathcal{M}_4{}^\dagger(m') \\ & & |\mathcal{M}_3(m')|^2 & \mathcal{M}_3(m')\mathcal{M}_4{}^\dagger(m') \\ & & \mathcal{M}_4(m')\mathcal{M}_3{}^\dagger(m') & |\mathcal{M}_4(m')|^2 \end{bmatrix}$$

Dominant terms
All of each 4 terms
are **same**

Ordinary quantum corrections

$$
\begin{bmatrix}
|\mathcal{M}_1(m)|^2 & \mathcal{M}_1(m)\mathcal{M}_2{}^\dagger(m) & \mathcal{M}_1(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_1(m)\mathcal{M}_4{}^\dagger(m') \\
\mathcal{M}_2(m)\mathcal{M}_1{}^\dagger(m) & |\mathcal{M}_2(m)|^2 & \mathcal{M}_2(m)\mathcal{M}_3{}^\dagger(m') & \mathcal{M}_2(m)\mathcal{M}_4{}^\dagger(m') \\
 & & |\mathcal{M}_3(m')|^2 & \mathcal{M}_3(m')\mathcal{M}_4{}^\dagger(m') \\
 & & \mathcal{M}_4(m')\mathcal{M}_3{}^\dagger(m') & |\mathcal{M}_4(m')|^2
\end{bmatrix}
$$

Dominant terms

All of each 4 terms are **same**